Mr. *Wingate's Arithmetick*,

Containing

A PLAIN AND FAMILIAR METHOD,

For attaining the

KNOWLEDGE and PRACTICE

Of

COMMON ARITHMETICK.

The fifth Edition.

First composed by *Edmund Wingate* late of *Grayes-Inne* Esquire.

Afterwards upon Mr. *Wingate's* request, enlarged in his life time: Also since his decease carefully revised, and much improved, as will appear by the Preface and Table of Contents.

By *JOHN KERSEY*, Teacher of the Mathematicks, at the sign of the *Globe* in *Shandois street* in *Covent-Garden*.

Boetius Arith. lib. 1. cap. 2.

Omnia quæcunque à primava rerum naturâ constructa sunt, Numerorum videntur ratione formata: Hoc enim fuit principale in animo Conditoris Exemplar.

LONDON.

Printed by *Thomas Roycroft*, for *Robert Stephens* at the Sign of the *Kings Arms* in *Chancery-Lane*, 1670.

TO THE

RIGHT HONOURABLE

THOMAS

Earl of *Arundel* and *Surrey*,

Earl Marſhal of

ENGLAND, &c.

Right Honourable,

THe good affection you bear to all kind of learning, *and in particular to the* Mathematicks, *makes me adventure to preſent your Lordſhip with this Tractate of* Arithmetick, *becauſe that Art, compared with*

other Mathematical Sciences, is as the Primum Mobile, *in reſpect of the other inferiour Orbs: For as the Poets uſed in times paſt to ſay of Venus,* Sine Cerere & Baccho friget Venus, *ſo may I alſo confidently aver of them, without* Arithmetick *they are poor, and without motion. Preſuming therefore that your Lordſhip, loving the Art, cannot diſaffect the Artiſt, nor his intention to do good in that kind, I am bold to ſhelter this Treatiſe under your Lordſhips protection, humbly intreating Your gracious acceptation, and earneſtly deſiring for ever to remain,*

Your Honours, in all
ſervice affectionately
devoted,

EDM. WINGATE.

THE PREFACE OF *JOHN KERSEY.*

ABout the year 1629 our learned Countreyman *Edmund Wingate* Esquire, publish'd a Treatise of *Arithmetick* divided into two Books, the one intituled *Natural Arithmetick*, the other *Artificial Arithmetick*; and in regard his principal design in that Treatise,

 was

was to remove the difficulties which ordinarily arise in the practice of *Common Arithmetick*, by the help of artificial, or borrowed numbers, called *Logarithmes*; (whose proper work is to perform *Multiplication* by *Addition*; *Division* by *Subtraction*, &c.) he did then in his said first Book omit divers pieces of *Common* or *Practical Arithmetick*, which for the perfect and universal understanding thereof, were necessary to have been inserted. But after the first impression of both those Books was spent our said Author being importuned to take care of the second Edition, he promised his assistance therein, yet his other necessary employments not permitting him to pursue his said purpose, he was pleased to impart his thoughts concerning the same unto me, together with his request, that I would peruse the said first Book, and supply it with such pieces of *Practical Arithmetick*, which for the reasons aforesaid were wanting in the first Edition.

In pursuance of which request, I have contributed my Talent towards perfecting this Tractate, upon our Authors foundation, partly in his life time to his good liking, and partly since his decease, in several Editions

com-

committed to my care to be prepared for the Press, wherein I have used my best endeavours, as well to preserve this Book as a Monument of our said Authors worth, as also to make it a compleat Store-house of *Common Arithmetick*; from whence the ingenious may be furnish'd with the excellencies of that Art, in reference both to common affairs, as also to the practical parts of the Mathematicks. And in order to those ends I have made these following alterations and Additions, namely

First, for the ease and benefit of such Learners, who desire only so much skill in Arithmetick, as is useful in Accompts, Trade, and such like ordinary employments; the Doctrine of whole Numbers, (which in the first Edition was intermingled with Definitions and Rules concerning broken Numbers, commonly called Fractions) is now entirely handled apart; and to the end the full knowledge of *Practical Arithmetick* in whole Numbers might more clearly appear, I have explained divers of the old rules in the first five Chapters, and framed anew, the Rules of *Division*, *Reduction*, and the *Golden Rule* in the sixth, seventh, eighth,

and ninth Chapters; so that now Arithmetick in whole Numbers is plainly and fully handled before any entrance be made into the craggy pathes of Fractions, at the sight whereof some Learners are so discouraged, that they make a stand, and cry out, *non plus ultra*, there's no progress further.

Secondly, to assist such young Students as desire to lay a good foundation for the attaining of a general knowledge in the Mathematicks, I have in a familiar method delivered the entire Doctrine of Fractions, both Vulgar and Decimal, which was omitted in the first Edition; and have also newly framed the Extraction of the Square and Cube roots, in a method which by experience is found to be much easier then that commonly used heretofore, and is exactly suitable to the Construction or Composition of Square and Cube numbers.

Lastly, I have added an *Appendix*, which is furnished with variety of choice and delightful knowledge in numbers, both Practical and Theoretical. In all which performances I have earnestly aimed at truth, perspicuity, and exact correction both of

the Text and Numbers; so that I hope this Book is now supplied with all things necessary to the full knowledge and practice of *Common Arithmetick*, the usefulness whereof is so generally known, that there will be no need of Arguments to excite any one that desires his own or the publick good, to be acquainted with so excellent an Art.

From my house at the sign of the Globe *in* Shandois-street *in* Covent-Garden, *the* 25th *day of* July, 1670.

JOHN KERSEY.

THE

The Table of Contents.

Where those Chapters of Mr. *Wingate's*, that have been altered and framed anew by *John Kersey*, are distinguished by this mark ☞, and those chapters that have been entirely composed by the said *J. K.* may be discovered by this Asterisk *

The Doctrine of whole numbers is contained in the first 15 chapters, the titles whereof are these following.

The

The Contents.

The Doctrine of Fractions both vulgar and decimal, is contained in the 16 chap. next following.

The

The Contents of the Appendix.

A

A TREATISE OF *Common Arithmetick.*

The First Book.

CHAP. I.

Concerning Notation of Numbers.

I. ARithmetick is the Art of accompting by Number. As magnitude or greatness is the subject of *Geometry*, so multitude or number is that of *Arithmetick.*

II. Number is that by which every thing is numbered; or that which an- swers

Number.

swers to the question, how many? (unless the question be answered by nothing:) So if it be demanded, how many dayes are in a week, the answer is seven, which is called Number.

The Characters by which number is expressed. *III.* The Notes or Characters, by which Number is ordinarily expressed, are these; 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 nothing.

IV. These Notes or Characters are either significant figures, or a Cypher.

V. The significant figures are the first nine; *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9. The first whereof is more particularly called an Unit, or Unity, and the rest are said to be composed of Unities, so 2 is composed of two unities, 3 of three Unities, &c.

VI. The Cypher is the last, which though of it self it signifies nothing, yet being annexed after any of the rest, it increaseth their value: As will appear in the following Rules.

VII. Arithmetick hath two parts, Notation and Numeration.

VIII. Notation teacheth how to express, read, or declare, the signification or value of any number written, and also to write down any number propounded, with proper Characters in their due places.

The places or degrees of any number. *IX.* A number is said to have so many places or degrees as there are Characters in the number; *viz.* when divers figures, whether they be intermixt with a Cypher or Cyphers or not, are placed together like letters in a word, without any point comma, line, or other note of distinction interposed,

posed, all those Caracters make but one number, which consists of so many places as there are Caracters so placed together: so this number 205 consists of 3 places, and this 30600 of five places, &c.

X. Notation consists in the knowledge of two things; *viz.* the order of places, and the value of every place in any number.

The order of places in any number.

XI. The order of the places is from the right hand towards the left: So in this number 465, the figure 5 standeth in the first place, 6 in the second, and 4 in the third, likewise in this number 7560, a Cypher stands in the first place, 6 in the second, 5 in the third, and 7 in the fourth.

The values of places in any number.

XII. The first place of a Number (which as before is the outermost towards the right hand) is called the place of Units or Unities; in which place any figure signifieth its own simple value; so in this number 465, the figure 5 standing in the first place signifieth five Unities, or five.

XIII. The second place of a number is called the place of Tens; in which place any figure signifieth so many Tens as the figure containeth unities: so in this number 465, the figure 5 in the first place signifieth simply five, but the figure 6 in the second place signifieth six tens, or sixty.

XIV. The third place of a number is called the place of Hundreds; in which place any figure signifieth so many hundreds as there are unities contain'd in the figure: So in this number 465, the figure 4 in the third place signifieth four hundreds; wherefore if it be required to read or pronounce this number 465, you are to begin on the left hand, and

and according to the aforesaid rules to pronounce it thus, four hundred sixty five; likewise this number 315 is to be pronounced thus, three hundred and fifteen: and this number 205, two hundred and five; also this number 500, five hundred. Whence it is manifest, that although a Cypher of it self signifies nothing, yet being placed on the right hand of a figure it increaseth the value thereof, by advancing such figure to a higher place then that wherein it would be seated, if the Cypher were absent.

The true reading or pronouncing the value of any number written, as also the writing down any number propounded, depends principally upon a right understanding of the three first places before mentioned, and therefore I shall advise the Learner to be well exercis'd therein, before he proceeds to the following Rules.

XV. The fourth place of a number is called the place of Thousands; (that is, any number of Thousands under ten thousand) the fifth place tens of thousands; the sixth place Hundreds of thousands; the seventh place Millions; (a Million being ten hundred thousand) the eighth place tens of Millions; the ninth place hundreds of Millions; the tenth place thousands of Millions; the eleventh place tens of thousands of Millions; the twelfth place hundreds of thousands of Millions: And in that order you may conceive places to be continued infinitely from the right hand towards the left, each following place being ten times the value of the next preceding place, but to give names to them would be both a troublesome and an unnecessary task.

XVI. *From the rules aforegoing, an easie way may be collected to read or express the value of a Number propounded, Viz.* Let it be required to read or pronounce this number 521426341, *First*, Distinguish by a Comma, or point, every three places, beginning at the right hand, and proceeding towards the left, so will the aforesaid number be distinguished into parts, which may be called *Periods*, and stand thus 521, 426, 341. where you may note the *first period* towards the right hand to consist of these figures 341, the *second* of these 426. and the *third* of these 521. *Secondly*, read or pronounce the figures in every *Period* as if they stood apart from the rest, so will the first *Period* be pronounced three hundred, forty one, the second four hundred twenty six: and the third five hundred twenty one. *Thirdly*, to every *Period* except the first towards the right hand, a peculiar *denomination* or *sirname*, is to be applyed, *Viz.* the *sirname* of the second *Period* is *Thousands*; of the third, *Millions*; of the fourth, *Thousands* of *Millions*, &c. Therefore beginning to pronounce at the highest *Period*, which in this Example is the third, and giving every *Period* its due *sirname*, the the said number will be pronounced thus, *Five hundred twenty one Millions, four hundred twenty six Thousands, three hundred forty one.*

A brief way of Notation.

A Period.

Note, When a number is distinguished into *Periods*, as before, the highest *Period* will not always compleatly consist of three places, but sometimes of one place, and sometimes of two, nevertheless after such *Period* is pronounced as if it stood apart, the due *sirname* is to be annexed; so this

number 3204689, after it is divided into *Periods*, will ſtand thus, 3, 204, 689, and to be pronounced thus, *Three Millions, two hundred and four thouſands, ſix hundred eighty nine.*

XVII. The aforeſaid Rules for the right pronouncing or reading of a Number which is written down, being well underſtood, will ſufficiently inform the Reader how to write down any number propounded to be written.

The Table of Notation.

	The order of Places.		The value of Places.
	&c.		&c.
Fourth Period	*Twelfth place*	3	*Hundreds of thouſand Millions.*
	Eleventh place	2	*Tens of Thouſand Millions.*
	Tenth place	1	*Thouſand Millions.*
Third Period,	*Ninth place*	9	*Hundreds of Millions.*
	Eighth place	8	*Tens of Millions.*
	Seventh place	7	*Millions.*
Second Period	*Sixth place*	6	*Hundreds of Thouſands.*
	Fifth place	5	*Tens of Thouſands.*
	Fourth place	4	*Thouſands.*
Firſt Period,	*Third place*	3	*Hundreds.*
	Second place	2	*Tens.*
	Firſt place	1	*Unites.*

Notation of Numbers by Latine Letters.

1	I.	21	XXI.
2	II.	30	XXX.
3	III.	40	XL.
4	IIII. or thus IV.	49	XLIX.
5	V.	50	L.
6	VI.	59	LVIIII. or thus LIX.
7	VII.	60	LX.
8	VIII or thus IIX.	89	LXXXIX.
9	VIIII. or thus IX.	100	C.
10	X.	200	CC.
11	XI.	300	CCC.
12	XII.	400	CCCC.
18	XVIII. or thus IIXX	500	D. or thus IↃ.
19	XVIIII. or thus XIX	600	DC. or thus IↃC.
20	XX.	700	DCC or thus IↃCC.

1000	CIↃ or thus M.
2000	CIↃ.CIↃ.
3000	CIↃ. CIↃ. CIↃ.
5000	IↃↃ.
10000	CCIↃↃ.

50000	IↃↃↃ.
100000	CCCIↃↃↃ. or thus CM.
500000	IↃↃↃↃ
1000000	CCCCIↃↃↃↃ.
1667	CIↃDCLXVII. or MDCLXVII

CHAP. II.

Concerning English Moneys, Weights, Measures, &c.

I. *THe things expressed by Numbers are principally, Money, Weight. Measure, Time, and things accompted by the dozen:* Of the three first of these, there are infinite kinds and varieties according to the diversity of the several *Common-wealths* in which they are used, all which here to produce were both endless and needless: wherefore we intend here to treat only of such *Moneys, Weights, Measures,* &c. as are used in this *Nation*, being indeed only necessary for our present purpose.

Of English Money.

II. *The least piece of money used in England is a Farthing, from whence this following Table is produced.*

1. *Farthing*	makes	1. *Farthing.*
4. *Farthings*	makes	1. *Penny.*
12. *Pence*	makes	1. *Shilling.*
20. *Shillings*	makes	1. *Pound.*

English (or sterling) Money is ordinarily written down with Figures after this manner,

l.	s.	d.	f.
34	13	05	2
09	05	10	1
06	00	06	3
00	12	11	0
00	00	07	2

The

The first Rank of the said Numbers signifies thirty four pounds, thirteen shillings, five pence, two farthings: the second rank expresseth nine pounds, five shillings, ten pence, one farthing: the third Rank, six pounds, no shillings, six pence, three farthings, *&c.*

III. The smallest Weight used in *England* is a grain, that is, the weight of a grain of Wheat well dryed and gathered out of the middle of the ear, whereof thirty two make another weight called a Penny-weight, and twenty penny weight make an Ounce Troy, Troy Troy Troy.

Vide Statute. de compositione ponderum, 51 Hen. 3.

Here observe, That by the *Statutes* quoted in the Margent, the weight of two and thirty grains of Wheat make a *penny weight*, which weight being once discovered by two and thirty such grains, the said penny weight (being the twentieth part of an ounce Troy) is usually subdivided into four and twenty parts onely, called also *Grains*, as appears by the ensuing Table;

31 Ed. 1. v. Rast. weights, 7 & 8. 12 Hen. 7. 5.

A Table of Troy Weights. *Troy Weight.*

32 *Grains of Wheat*	*make*	24 *Artificial Grains.*
24 *Grains*	*make*	1 *Penny Weight.*
20 *Penny weight*	*make*	1 *Ounce.*
12 *Ounces*	*make*	1 *Pound Troy.*

Troy Weight is ordinarily written down with Figures after this manner.

lb.	*oz.*	*p.w.*	*gr.*
17	05	13	23
00	11	07	06
00	00	05	20

The first rank of the said numbers expresseth seventeen pounds, five ounces, thirteen penny weight, thirteen grains, of *Troy weight*: the second rank, no pounds, eleven ounces, seven penny weight, six grains, and the third, no pounds, no ounces, five penny weight and twenty grains.

Malynes lex Mercat. pag. 49. *Malynes ib. pag.* 252.

Now this *Troy weight* serveth only to weigh Bread, Gold, Silver, and Electuaries. And here observe also by the way, that *Troy weight* regulateth and prescribeth a form how to keep the Money of *England* at a certain *Standard*. For about two hundred years before the Conquest, *Osbright* a Saxon, being then King of *England*, caused an ounce *Troy* of Silver to be divided into 20 pieces, at the same time called Pence, and so an Ounce of Silver at that time was worth no more than twenty pence, or one shilling eight pence, which continued at the same value until the time of *Henry* the sixth, who (in regard of the inhansing of Moneys in Forrain parts) valued the same at thirty pence, so that then there were accordingly thirty pieces made out of the Ounce, and the old pieces went then for three half pence, until the time of *Edward* the fourth, who valued the Ounce at forty pence, and then the old pieces went for two-pence a piece. After this, *Henry* the eight, valued the Ounce of sterling Silver at forty five pence, which value continued until Queen *Elizabeths* time, who valued the same old pence at three-pence the piece, so that all Three-pences coined by the same Queen, weighed but a penny weight, and every Six-pence two-penny weight; and so in like manner the Shilling and other pieces

accordingly; which made the Ounce *Troy* of Silver to be valued at sixty pence or five shillings, as it now remains at this day without alteration.

IV. The Weights used by Apothecaries are derived from a pound *Troy* which is subdivided as in the following Table. *Apothecaries Weights.*

A Table of Apothecaries Weights.

lb. *A pound Troy*	is equal unto	12	*Ounces.*
℥ *An Ounce*		8	*Drams.*
ʒ *A dram*		3	*Scruples.*
℈ *A Scruple*		20	*Grains.*

So that if you were to express in Figures 12 pounds 10 ounces, five drams, two scruples, and 16 grains: also three pounds, five ounces, seven drams, one scruple, and two grains, the ordinary way to write them down is briefly thus,

lb.	℥	ʒ	℈	*gr.*
12 —	10 —	5 —	2 —	16
03 —	05 —	7 —	1 —	02

V. Besides *Troy* weight before mentioned, there is another kind of weight used in *England*, called Averdupois weight, a pound whereof is equal unto 14. Ounces twelve penny weight *Troy*. This Averdupois weight serveth to weigh all kind of Grocery Ware, as also Butter, Cheese, Flesh, Tallow, Wax, and every other thing which beareth the name of *Garbel*, and whereof issueth a refuse or waste. *Malynes lib. p. 49.*

VI. Averdupois weight is either greater or less.

VII. The greater is, when one hundred and twelve pounds Averdupois are considered as one entire weight commonly *Averdupois greater weight.*

commonly called an hundred weight, and then such hundred weight is subdivided first into four quarters, and each quarter into eight and twenty pounds: again, each pound into four quarters, or (if you will be more exact) into 16 Ounces, and if you please each Ounce into four quarters. But ordinarily a pound is the least quantity that is taken notice of in Averdupois gross weights.

A Table of Averdupois greater Weight.

28 *pounds*	make	*a quarter of* 112 *lb.*
4 *quarters*	make	*an hundred weight, or* 112 *lb.*

So that if you were to express by Figures eight hundred, three quarters, and five pounds; likewise, seven hundred, one quarter, and seventeen pounds: the ordinary way to write them down is briefly thus,

C.	*q.*	*lb.*
8	3	5
7	1	17

Averdupois lesser weight. *VIII.* The lesser Averdupois weight is, when a pound is the highest name or Integer, each pound being subdivided into sixteen ounces, and each ounce again into 16 drams, and if you please, each dram into 4 quarters, as by the subsequent Table is manifest.

A Table of Averdupois lesser weight.

4 *Quarters of a Dram*	make	1 *Dram.*
16 *Drams*	make	1 *Ounce.*
16 *Ounces*	make	1 *Pound.*

So that if you were to express by Figures eighteen pounds, twelve ounces, five drams, and three quarters of a dram; likewise five pounds, no ounces twelve drams, and one quarter of a dram, the ordinary way to write them down is briefly thus.

lb.	*oz.*	*dr.*	*q.*
18	12	05	3
05	00	12	1

IX. The measures used in *England* are either of Capacity or Length.

X. The measures of Capacity are those which are produced from Weight, and they are either Liquid or Dry.

XI. The Liquid measures are those, in which all kind of Liquid substances are measured, and they are expressed in the Table following, *Liquid Measures.*

A Table of Liquid Measures.

	makes	
1 *Pound of Wheat Troy weight*		1 *Pint.*
2 *Pints*		1 *Quart.*
2 *Quarts*		1 *Pottle.*
2 *Pottles*		1 *Gallon.*
8 *Gallons*		1 *Firkin. of Ale, Sope, Herring.*
9 *Gallons*		1 *Firkin of Beer.*
10 *Gallons and an half*		1 *Firkin of Salmon or Eels.*
2 *Firkins*		1 *Kilderkin.*
2 *Kilderkins*		1 *Barrel.*
42 *Gallons*		1 *Tierce of Wine.*
63 *Gallons*		1 *Hogshead.*
2 *Hogsheads*		1 *Pipe or But.*
2 *Pipes or Buts*		1 *Tun of Wine.*

XII. Dry

Dry Measures. XII. Dry Measures are those, in which all kind of dry substances are meted, as Grain, Sea-coal, Salt, and the like; their Table is this that follows.

A Table of Dry Measures.

1 *Pinte*	make	1 *Pinte.*
2 *Pintes*		1 *Quart.*
2 *Quarts*		1 *Pottle.*
2 *Pottles*		1 *Gallon.*
2 *Gallons*		1 *Peck.*
4 *Pecks*		1 *Bushel land measure.*
5 *Pecks*		1 *Bushel water measure.*
8 *Bushels*		1 *Quarter.*
4 *Quarters*		1 *Chalder*
5 *Quarters*		1 *Wey.*

Long Measures. XIII. Long Measures are exprest in this Table following

3 *Barley Corns in length*	make	1 *Inch.*
12 *Inches*		1 *Foot.*
3 *Foot*		1 *Yard.*
3 *Foot nine Inches*		1 *Ell.*
6 *Foot*		1 *Fadome.*
5 *Yards and an half*		1 *Pole or Perch.*
40 *Poles*		1 *Furlong.*
8 *Furlongs*		1 *English Mile.*

Note, That a Yard, as also an Ell, is usually subdivided into four Quarters, and each Quarter into four Nails.

XIV. Super-

XIV. Superficial or square Measures of Land are such as are exprest in the Table following. *Land Measures.*

45 *Square Poles or Perches*	make	1 *Rood or quarter of an Acre.*
4 *Roods*	make	1 *Acre.*

So that if you would express by Figures these quantities of Land, *Viz.* Thirty six Acres, three Roods, twenty Perches : also seven Acres, no Roods, thirty two Perches , the ordinary way to write them down is thus.

A.	*R.*	*P.*
36 ——	3 ——	20
7 ——	0 ——	32

XV. *A Table of Time is this that follows.* *Time.*

1 *Minute*	make	1 *Minute.*
60 *Minutes*	make	1 *Hour.*
24 *Hours*	make	1 *Day natural*
7 *Dayes*	make	1 *Week.*
4 *Weeks*	make	1 *Moneth of twenty eight dayes.*
13 *Moneths* 1 *Day, &* 6 *Hours*	make	1 *Year very near.*

But in ordinary computations of time, the whole year consisting of three hundred sixty five dayes, is divided either into twelve equal parts or months, each moneth then containing thirty days and ten hours ; or else into twelve unequal *Kalender* moneths, according to the ancient Verse.

Thirty days hath September, April, June, and November :
February hath twenty eight alone, and each of the rest thirty one.

Note,

Note, That every *Leap-year* (which happeneth once in four years) containeth three hundred ſixty ſix dayes, and in ſuch year *February* containeth twenty nine dayes.

Of things accounted by the dozen.

XVI. Of things accounted by the dozen, a Groſs is the Integer conſiſting of twelve dozen, each dozen containing again twelve particulars: ſo that if you would expreſs in Figures, ſeven Groſs four Dozen, and five particulars; alſo four Dozen and eight particulars, they may be briefly written thus.

G.	*D.*	*P.*
7 ——	04 ——	05
0 ——	04 ——	08

CHAP. III.

Addition of whole Numbers.

I. Concerning notation of Numbers, and how thereby the quantities of things are uſually expreſt, a full Declaration hath been made in the preceding Chapters; Numeration enſueth, which comprehends all manner of operations by numbers.

II. In Numeration, the four primary or fundamental operations (commonly called Species) are theſe, Addition, Subtraction, Multiplication, and Diviſion.

III. Addition is that by which divers Numbers are added together, to the end that their ſum, aggregate, or total, may be diſcovered.

IV. In Addition, place the Numbers given, one

one above another in ſuch ſort, that like places or degrees in each number may ſtand in the ſame rank: that is Units above Units, Tens above Tens, Hundreds above Hundreds; *&c.* So theſe numbers 1213 and 462 being given to be added together, you are to order them as you ſee in the margent.

Addition of numbers of one denomination.

1213
462

V. Having thus placed the Numbers, and drawn a line under them, add them together, beginning with the Units firſt, and ſaying thus, 2 and 3 make 5, which write under the Rank of Units, then proceed to the ſecond Rank and ſay 6 and 1 make 7, which write under the ſecond Rank (being the place of tens) again 4 and 2 make 6, which write under the third Rank. Laſtly, write down 1 being all that ſtands in the fourth Rank, ſo the ſum of the ſaid given Numbers is found to be 1675, and the operation will ſtand as in the Margent.

1213
462
————
1675

In like manner the Numbers 2315, 7423, and 141, being given to be added together, their ſum will be found to be 9879; and the operation thereof will ſtand as you ſee in the Example.

2315
7423
141
————
9879

VI. When the ſum of the Figures of any of the Ranks amounts unto ten, or any number of tens without any exceſs, write down a Cypher under that Rank, but when the ſum of any Rank exceeds ten or any number of tens, write down the exceſs under ſuch Rank, and for every ten contained in the ſum of any Rank, reſerve an Unite or 1 in your mind, and add ſuch Unit or Units to the Figures

gures of the next Rank towards the left hand; so the Numbers 4937, 9878, and 394 being given to be added together, the operation will be thus, *viz.* beginning with the Rank of Units, I say 4, 8 and 7 make 19, wherefore I write down 9, the excess above 10, and carrying 1 in mind instead of the ten contained in the sayd 19. I say 1 and 9 (9 being the first figure of the second Rank) make 10, which added to 7 and 3, the other figures of the same Rank, the whole sum of them is 20, wherefore setting down a Cypher under the line in that Rank (because the excess above the two tens is nothing) I carry 2 to the third Rank, and say 2 and 3 (3 being the first figure of the third Rank) make 5, which being added to 8 and 9 (the other figures of the same Rank) the sum of them is 22, wherefore writing down 2 (being the excess above the two tens) under the line, in the third Rank, I carry 2 in mind (because there were two tens in 22) to the fourth Rank and say 2 and 9 make 11, which added to 4 makes 15, this 15 because it is the sum of the last Rank I write totally down under the line, on the left hand of the Figures before subscribed; so the sum of the three Numbers given is found to be 15209, as in the Example.

$$\begin{array}{r} 4937 \\ 9878 \\ 394 \\ \hline 15209 \end{array}$$

Addition of numbers of divers denominations.

VII. When the numbers propounded to be added, do express things of divers denominations, you must begin with the least denomination first, and when the sum of any of the denominations amounts unto an Integer or Integers of the next greater denomination, add such Integer or

or Integers to those of the next greater denomination, on the left hand; so these several sums 24 *l* — 13 *s* — 5 *d* — 3 *f*. Also 12 *l* — *os*. — 8*d*. and 5 *l* — 18 *s* — 2 *f*. being propounded to be added, their totall sum is 42 *l* — 12 *s* — 2*d* — 1 *f*. For having written them down orderly according to the 2 d. Rule of the 2 d. Chapter, and drawn a line underneath, I begin with the Farthings first, and say, two Farthings and three Farthings make five Farthings, that is, one Penny with a Farthing over and above; wherefore setting down 1 under the denomination of Farthings, I carry one Penny to the denomination of Pence, then I say 1, 8, and five Pence make 14 Pence, which contain one shilling and two Pence, wherefore writing two under the denomination of Pence, I likewise carry 1 shilling to the denomination of shillings: Then adding the said 1 shilling unto 18 shillings and 13 shillings, the sum will be found 1 pound and 12 shillings, wherefore setting down 12 under the denomination of shillings, I carry 1 pound in mind unto the denomination of pounds saying, 1 pound in mind, together with 5, 2, and 4 pounds which stand in the first Rank of pounds, make 12 pounds, wherefore (according to the sixth Rule of this Chapter) I write 2, the excess above 10, underneath the said first rank of pounds, and carry 1 in mind for the sayd 10 to the second Rank of pounds, then saying in like manner, 1 in mind, together with 1 and 2 which stand in the second Rank of pounds make 4, which I write underneath

l.	*s.*	*d.*	*f.*
24 —	13 —	05 —	3
12 —	00 —	08 —	0
05 —	18 —	00 —	2
42 —	12 —	02 —	1

derneath the line, that done, I find the total of the three sums propounded to be 42*l.*—12 *s.*—2*d.*—1*f.*

In like manner 3 lb.—05 *oz*—19 *p.w.* 15 *gr.* Also 2 *lb.*—0 *oz.*—3 *p.w.*—7 *gr.* Also 0 *lb.*—10 *oz.*—6 *p.w.* And 0 *lb.*—9 *oz.*—0 *p.w.*—17 *gr.* being given to be added together, their sum will be found to be 7 *lb.*—1 *oz.*—9 *p.w.*—15 *gr.* and the work will stand thus.

lb.	*oz.*	*p.w.*	*gr.*
03	05	19	15
02	00	03	07
00	10	06	00
00	09	00	17
07	01	09	15

Note, In adding together the Numbers in the last Example, it must be remembred that 24 grains make one Penny weight, also 20 Penny weight make one ounce, and 12 ounces make one pound *Troy*, as before declared in the second Rule of the second Chapter.

More Examples of this Rule are these following.

Addition of English Money.

lb.	*s.*	*d.*	*f.*	*l.*	*s.*	*d.*
230	17	10	1	0	13	05
175	12	11	3	0	17	08
052	05	06	0	0	00	10
009	00	08	1	0	10	03
506	13	00	2	0	15	06
974	10	00	3	2	17	08

Addition of Troy Weight.

lb.	oz	p.w.	gr.	oz.	p.w.	gr.
23	07	16	13	536	13	16
17	10	15	07	208	11	10
325	06	19	20	063	10	05
49	11	07	12	099	00	12
417	00	19	04	907	15	19

Addition of Averdupois Weight.

C.	q.	lb.	lb.	oz.	dr.
235	3	13	14	13	12
576	1	17	05	10	14
628	2	15	12	00	06
412	0	10	06	09	05
1852	3	27	39	02	05

Addition of Measures of Length.

yards	q.	nails	Ells	q.	n.
26	3	2	15	3	2
13	1	3	16	1	3
12	0	1	09	0	1
29	1	1	12	2	1
81	2	3	53	3	3

Addition of Superficial Measures of Land.

Acres	Roods	Per.	A.	R.	P.
136	3	13	240	2	17
513	1	26	500	3	13
212	2	10	249	1	36
517	0	00	006	0	40
1379	3	09	990	3	36

CHAP. IV.

Subtraction of whole numbers.

I. SUbtraction is that by which one number is taken out of another, to the end that the remainder, or difference, between the two numbers given may be known.

Subtraction of numbers of one denomination.

II. The number out of which the Subtraction is to be made, must be greater, or at least equal with the other. As you may Subtract, 4347 or 9478 out of 9478, so can you not subtract 9478 out of 4347.

III. In Subtraction rank the two given numbers one under the other as in Addition, with this caution, that the number placed uppermost may exceed or at least be equal unto the other. So if the number 4347 be given to be subtracted from 9478, I order them as in the Margent: then proceeding to the subtraction, I say, 7, taken out of 8, there remains one, which I place in the same rank under the line. In like manner 4 being taken out of 7, the remainder is 3, which likewise I set under the line in the next rank, again taking 3, from 4, the remainder is 1, which I likewise place under the third rank, lastly subtracting 4 from 9, there will remain 5 which I subscribe under the fourth rank, so the whole operation being finished I find, that if 4347 be taken out of 9478 the remainder is 5131, or (which is the same) the difference between the numbers

```
9478
4347
----
5131
```

numbers 9478 and 4347 is 5131 as in the example. In like manner if 106 be ſubtracted from 2856 the remainder will be found to be 2750; for after the numbers are orderly ranked I begin at the place of Units and ſay 6 from 6, there remains nothing, wherefore I ſubſcribe 0, then proceeding to the ſecond rank I ſay if 0 (or nothing) be taken from 5, there will remain 5 which I alſo ſubſcribe under the line, again 1 from 8, there remains 7; laſtly 0 from 2, there remains 2, See the work in the Margent.

2856
106
2750

IV. When any of the figures of the number given to be ſubtracted is greater than the upper figure out of which it is to be ſubtracted, you muſt borrow 10 of the next rank towards the left hand and add the ſaid 10 to the ſaid upper figure, then the figure of ſuch next rank which is to be ſubtracted muſt be eſteemed an unite greater than it is; wherefore in this caſe, keeping one in your mind add it to the next figure of the number given to be ſubtracted, and deducting all out of the figure above it, proceed in like ſort till you have finiſhed the whole operation. Example, let it be required to ſubtract 374 out of 8023. Having ranked them as before, I ſay four out of 3, that cannot be, wherefore borrowing ten of the next rank and adding the ſame to the ſaid 3, I ſay 4 out of 13, there remains 9, then writing 9 under the line, & carrying 1 in my mind, I ſay 1 and 7 make 8. 8 out of 2 that cannot be, but 8 out of 12 (12, becauſe 10 being borrowed, and added to 2, makes 12) there remains 4, which I ſubſcribe under the

8023
374
7649

line, again 1 in my mind being added to 3 makes 4, 4 out of nothing, that cannot be, but 4 out of 10 there remains 6 which I likewise subscribe under the line; lastly 1 in my mind being taken out of 8 there remains 7, thus you see that the remainder after 374 is subtracted from 8023 is 7649. Note diligently, that as often as 10 is borrowed, 1 must be kept in mind to be added to the figure standing in the next place of the lower number, and the sum of such Addition must be subtracted from the upper place; but if it happen that there is no figure in the next place of the lower number, then the 1 in mind must be subtracted from the upper place, (as in the last rank of the last Example) *Another Example.* Let it be required to subtract 92 from 62801. Having placed the greater number uppermost and the lesser orderly underneath, I begin at the place of units, and say, 2 from 1 I cannot take, but borrowing 10, and adding it to the sayd 1, I say 2 from 11, there remains 9, which I subscribe under the line, then I proceed and say, 1 in mind with 9 makes 10, 10 out of 0 I cannot take, but borrowing 10 I say 10 out of 10 and there remains 0, wherefore I subscribe 0 under the line; again, 1 in mind out of 8, there remains 7; then because there are no more Figures in the lower number I say 0 out of 2 there remains 2; lastly, 0 out of 6 there remains 6; therefore I conclude that 62801 exceeds 92 by 62709.

```
62801
   92
-----
62709
```

Subtraction of numbers of divers denominations.

V. If the numbers propounded have divers denominations, place them as before, and beginning with the

the least denomination first, subtract the lower number from the upper when it may be subtracted, and place the remainder underneath; but if it happen that the lower number cannot be taken out of the upper, you must borrow an integer of the next greater denomination on the left hand; which integer after it is converted into the same denomination with the said upper number must be added to it: then from the sum of such Addition you are to subtract the lower number and write down the remainder, keeping 1 (that is the integer borrowed) in your mind to be added to the next place of the number given to be subtracted, as before: so 90*l.*—14*s.*—10*d.*—3*f.* being subtracted from 124*l.*—11*s.*—7*d.*—1*f.* the remainder is 33*l.*—16*s.*—8*d.*—2*f.* For beginning with the farthings, I say, 3 farthings out of 1 farthing I cannot take where-fore borrowing 1 penny (that is an integer of the next greater denomination) and having converted this penny into four farthings, I adde them to the aforesaid 1 farthing, so the sum is five farthings, out of which subtracting 3 farthings there remains 2 farthings, which I place underneath the denomination of farthings, then I proceed to the next denomination, and say, 1 penny which I borrowed and 10*d.* make 11*d.* this 11*d.* out of 7*d* I cannot take, wherefore borrowing 1 shilling or 12*d.* and adding 12*d.* to the said 7*d.* the sum is 19 *d.* from which I subtract the said 11*d.* so there remains 8*d.* which I subscribe under the denomination of pence; again 1 shilling which I borrowed being added to 14*s.* makes

l.	s.	d.	f.
124	11	07	1
90	14	10	3
33	16	08	2

makes 15*s*. which I cannot ſubtract out of 11 *s*. and therefore I borrow 1 pound or 20*s*. which being added to the ſaid 11*s*. makes 31*s*. from which ſubtracting 15*s*. there remains 16*s*. which I ſubſcribe under the denomination of ſhillings, then carrying 1 pound which I borrowed to the lower place of pounds, I ſay 1 in mind with 0 make 1, which taken out of 4 there remains 3, again 9 out of 2, I cannot take, but 9 out of 12 (10 being borrowed and added to the ſaid 2 according to the fourth Rule of this Chapter) and there remains 3 laſtly 1 (for the 10 that was borrowed) being taken out of 1, there remains nothing, and ſo at laſt I find that if A being indebted to B in 124*l*. —— 11*s*. —— 7*d*. —— 1*f*. hath paid in part thereof 90*l*. —— 14*s*. —— 10*d*. —— 3*f* there remains yet undiſcharged 33*l*. —— 16*s*. —— 8*d*. —— 2*f*.

Subtraction of many numbers from one number.

IV. When many numbers are given to be ſubtracted from a number propounded, you muſt firſt add thoſe numbers together, according to the rules of the third Chapter, and then the ſum found is to be ſubtracted from the number firſt propounded. Example, A. being indebted to B. in 3240 *l*. paid thereof at one time 700 *l*. at a ſecond payment 1236 *l*. and at a third 305 *l*. the queſtion is how much of the debt remained undiſcharged? Firſt, I add together the ſeveral ſums paid, and find the total to be 2241 *l*. this I ſubtract from 3240 *l*. ſo there remains 999 *l*. undiſcharged as you ſee by the operation in the Margent.

l.	
3240	*The Debt.*
700	
1236	*Payments.*
305	
2241	*Total payd,*
999	*Reſt unpayd.*

Another

Another Example of like nature. A, being indebted to B an 500 *l.* paid in part thereof at one payment 340 *l.* — 12 *s.* — 06 *d.* at a second payment 13 *l.* — 18 *s.* — 3 *d.* at a third 17 *l.* — 16 *s.* — 10 *d.* the question is how much was in arrear? Here if the operation be prosecuted as before, it will appear that there was 127 *l.* — 12 *s.* — 05 *d.* unpaid, see the work in the Margent.

l.	*s.*	*d.*	
500	00	00	*The Debt.*
340	12	06	*Payments.*
13	18	03	
17	16	10	
372	07	07	*Paid in all.*
127	12	05	*Rest unp.*

VII. Addition is proved by subtraction and subtraction by Addition: For having added divers numbers together, if you subtract one of them out of the sum, the remainder must be equal to all the rest, as you may observe by the Example following, *viz.* supposing these 4 numbers are given to be added, *viz.* 236, 452, 29, 217 and that their sum is found to be 934 (by the Rules of the 3 d. Chap.) it is required to prove whether the said sum be true or not; to performe this I draw a line under the uppermost number 236, to seperate it from the rest, and seek the sum of all the numbers given, except that uppermost, which sum I find to be 698. Then I subtract the said uppermost number 236 from 934 (the total sum of all the numbers first found) and because the remainder 698 is the same

The proof of Addition and subtraction.

236	
452	934
29	236
217	698
934	
698	

with the sum of all the numbers excluding the uppermost, I conclude that the sum of all the numbers first found was truly computed.

In like manner is Subtraction proved by Addition, for if you add the remainder, and the number given to be subtracted together, the sum must must be equal to the number out of which the Subtraction is made: so if 4347 be subtracted from 9478 the remainder is 5131, for if 5131 be added to 4347, the sum is 9478, which is the same with the number out of which the Subtraction was made. Again, if a Servant receive 24 *l.* —— 13 *s.* —— 7 *d.* and lay out or disburse 19 *l.* —— 15 *s.* —— 08 *d.* there must remain in his hands 4 *l.* —— 17 *s.* —— 11 *d.* for this being added to 19 *l.* —— 15 *s.* —— 08 *d.* (which was the Money he expended, the sum will be equal to 24 *l.* —— 13 *s.* —— 07 *d.* (being the Money wherewith he was first charged.)

Example 1		Example 2 *l.*	*s.*	*d.*
out of	9478	24	13	07
subtr.	4347	19	15	08
Rest	5131	4	17	11
Proof	9478	24	13	07

More Examples of Subtraction are these that follow.

Subtraction of English Money.

	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*f.*
Rec.	3090	10	07	1	24	00	00	0
paid	0099	14	08	3	05	17	11	3
rest	2990	15	10	2	18	02	00	1
proof	3090	10	07	1	24	00	00	0

Sub-

Subtraction of Troy Weight.

	lb.	oz.	p.w.	gr.	oz.	p.w.	gr.
Bought	352	10	13	15	205	13	19
Sold	019	11	16	18	118	16	20
Rest	332	10	16	21	86	16	23
Proof	352	10	13	15	205	13	19

Subtraction of Averdupois Weight.

	C.	q.	lb.	lb.	oz.	dr.
Bought	256	2	23	25	13	12
Sold	079	3	26	00	14	23
Rest	176	2	25	24	14	15
Proof	256	2	23	25	13	12

Subtraction of Superficial Measures of Land.

	Acres	Roods	Perches	A.	R.	P.
Bought	780	2	35	2040	1	20
Sold	090	3	36	1919	3	30
Rest	689	2	39	120	1	30
Proof	780	2	35	2040	1	20

Questions to exercise Addition and Subtraction.

Quest. 1. Two persons, A. and B. owe several debts, the lesser debt being that of A. is 3045 *l.* the difference of their debts is 104 *l.* what is the debt of B? *Answer*, 3149 *l.*

Quest.

Queſt. 2. Two perſons A. and B. are of ſeveral ages, the age of the elder, being that of A. is 70, the differences of their ages is 19, what is the age of B? *Anſwer* 51.

Queſt. 3 What number is that which being added to 168 maketh the ſum to be 205? *Anſw.* 37.

Queſt. 4. The ſum of two numbers is 517, the leſſer is 40, what is the greater? *Anſw.* 477.

Queſt. 5. A certain perſon born in the year of our Lord 1616, deſires to know his age in the year 1667, what was his age? *Anſw.* 51.

Queſt. 6. The greater of two numbers is 130, their difference is 49, what is the leſſer number? *Anſw.* 81.

CHAP. V.

Multiplication of whole numbers.

I. Multiplication teacheth how by two numbers given to find a third, which ſhall contain either of the numbers given, ſo many times as the other contains 1 or unitie.

II. Of the two numbers given in Multiplication one (which you will) is called the Multiplicand, and the other the multiplicator (or both are called Factors.)

III. The number ſought, or ariſing by the multiplication of the two numbers given is called the product, the Fact, or the Rectangle: ſo if 5 be given

given to be multiplied by 3, or 3 by 5, the product is 15, that is 3 times 5, or 5 times 3 makes 15: and here 5 may be called the the Multiplicand, and 3 the Multiplicator, or 3 may be called the Multiplicand, and 5 the Multiplicator; and as 3, (one of the two numbers given) containeth 1 or unity thrice, so 15 the product containeth 5 (the other given number) thrice; likewise as 5 (one of the given numbers) contains unity 5 times, so 15 (the product) contains 3 (the other given number) five times.

IV. Multiplication is either single or compound.

Single multiplication. *V.* Single Multiplication is, when the Multiplicand and Multiplicator consist each of them of one only figure, as in the last Example; In like manner if you multiply 9 by 5, the product is 45, this is likewise single multiplication: now the several varieties of single multiplication are well exprest in the Table following, usually called *Pythagoras his Table.*

The Table of Multiplication.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The use of the *Table* is this, having one figure given

given to be multiplied by another to know the product of them. find the multiplicand in the top of the Table, and the multiplicator in the first Column thereof towards the left hand; this done, in the angle of position just against those two figures you shall find the product. So 9 being given to be multiplied by 5, I find 9 in the top of the table, and 5 in the first column towards the left hand then carrying my eye from 5 in a right line equidistant to the upper side or top line of the Table, until I come to that square which is directly under 9, I find 45, which is the Product required. The particular varieties of this *Table* ought to be learned by heart, (that is a man must be able to give the Product of any single multiplication without the least pause or stay) before he can readily work compound multiplication, as will farther appear hereafter.

Compound Multiplication. VI. Compound multiplication is, when the multiplicator and multiplicand, either one or both consist of more figures than one.

VII. In compound Multiplication, when the numbers given do end with significant figures, place them as in Addition and Subtraction. So 134 being given to be multiplyed by 2, place them thus; then proceeding to the multiplication say thus, two times 4 is 8, which write under the line in the rank of your multiplying figure; again, say two times 3 is 6, which likewise write under the line in the next rank; Lastly, two times 1 is 2, which being likewise written down under the line in the next rank, the Product is discovered to be 268, and the work will stand as in the Margent.

134
2
268

When

VIII. When the multiplicator consists of more figures than one, as many figures as it hath, so many several products must be subscribed under the line, which at last being added into one sum, gives you the total product of all. So 1232 being given to be multiplyed by 23, the operation thereof wil stand thus, for 1232 being multiplyed by 3 (according to the last rule) the product is 3696. Again, 1232 being multiplyed by 2, the product is 2464, which several products, after they are placed in their due order (that is, the first figure arising in each product under his respective multiplying figure) and added together produce 28336, the product required: In like manner 1321 being given to be multiplyed by 123, the product is 162483, and the operation will stand as you see in the Margent.

```
  1232
    23
  ----
  3696
 2464
 -----
 28336

   1321
    123
   ----
   3963
  2642
 1321
 ------
 162483
```

IX. When the product of any of the particular figures exceeds ten, place the excess under the line as before, and for every ten that it so exceeds, keep one in mind to be added to the next Rank.

Example, 3084 being given to be multiplyed by 36, the work will stand thus; for 6 times 4 being 24, I write 4 under the line, and reserve 2 in mind for the two tens, then I say 6 times 8 is 48, unto which if I add 2 kept in mind, the whole is 50, wherefore subscribing 0 in the next rank under the line (0 because there is no excess of 50 above 5 tens) I reserve 5 in mind for the 5 tens; again, I say 6 times nothing

```
   3084
     36
  -----
  18504
  9252
 ------
 111024
```

nothing is nothing, to which adding 5 that I kept in mind, the whole will be but 5, which I likewise subscribe under the line in the next rank; again, 6 times 3 is 18, which (in regard 3 is the last figure of the multiplicand) I write wholly down; so that the particular product arising from the multiplying figure 6 is 18504; in like manner proceeding with the multiplying figure 3, the particular product arising will be 9252. Lastly, these several products being placed in due order, and added together (after the manner of the 8th. Rule of this Chapter) will give 111024, which is the total product arising from the multiplication of 3084 by 36. and the operation will stand as in the Margent. After the same manner if 5073 be given to be multiplied by 256, the product will be found to be 1298688, & the operation will stand as you see in the Example.

```
    5073
     256
 -------
   30438
  25365
 10146
 -------
 1298688
```

X. When the two numbers given to be multiplyed, do one or both of them end with a Cypher or Cyphers towards the right hand, multiply the significant figures in both numbers, one by the other, neglecting such Cyphers, and when the multiplication of the significant figures is finished, annex on the right hand of the number produced by the multiplication, the Cypher or Cyphers with which one or both of the numbers first given did end, so will the whole give you the true product demanded: Example, 43100 being given to be multiplied by 15000 the product will be found to be 646500000, for omitting the Cyphers which stand in

```
   43100
   15000
 -------
  2155
  431
 -------
646500000
```

in the laſt places towards the right hand as well in the multiplicand as the multiplicator, I multiply the ſignificant figures 431, by the figures 15 (according to former rules) ſo there will ariſe 6465, to which annexing on the right hand all the Cyphers before omitted, the true product will be 646500000: more Examples hereof are theſe following.

43125	5108000
1500	125
215625	25540
43125	10216
64687500	5108
	638500000

XI. When in the multiplicator, Cyphers are included between ſignificant figures, multiply by the ſaid ſignificant figures, neglecting ſuch Cyphers or Cypher, but obſerve diligently to ſet the particular products of the ſignificant figures in their due places, according to the 8th. rule of this Chapter. So if 56324 be given to be multiplyed by 20006, I firſt multiply the whole multiplicand 56324 by 6, and place the product orderly underneath the line, then paſſing over the three Cyphers, I multiply 56324 by 2, and place 8, (which is the firſt exceſs of this particular product) directly under the multiplying figure 2, and the reſt in their order, ſo at laſt the true product will be found to be 1126817944, and the work will ſtand as you ſee in the Example.

56324
20006
337944
112648
1126817944

More

More Examples hereof are these that follow.

3094	23765
104	10302
12376	47530
3094	71295
321776	23765
	244827030

Note, That one of the principal *cautions* to be observed in Multiplication, is the due placing of the particular products arising by each multiplying figure, and that may be performed either by taking care to place the first figure or Cypher which ariseth in each product under the respective multiplying figure; or at least the first place arising in the second product must stand under the second place of the first product, and the first place of the third particular product under the third place of the first, &c.

XII. When a number is given to be multiplyed by a number that consists of 1 (or an unit) in the first place towards the left hand, and a Cypher or Cyphers on the right hand of such unit (such are 10, 100, 1000, 10000, &c. the multiplication is performed by annexing the Cypher or Cyphers of the multiplicator at the end (to wit on the right hand) of the multiplicand; so if 326 be given to be multiplyed by 10, the product is 3260; if by 100, the product is 32600; if by 1000, the product is 326000, in like manner if 170 be multiplyed by 10 the product is 1700, if by 100, 17000, &c.

Continual Multiplication.

XIII. When more numbers than two are given to be multiplyed one by the other, that kind of Multiplication is

is called *Continual*, and is thus performed, *Viz.* first multiply any two of the numbers given one by the other, then multiply the product by another of the numbers given, and this product by the fourth number given (if there be so many) and in that order till every one of the given numbers hath been made a multiplicator, so the last product is the true product required. Example, If 4, 18, and 22 were given to be multiplyed continually, first 18 multiplyed by 4, produceth 72, which multiplyed by 22 (the third number) produceth 1584, the last product or number required; see the work in the Margent. The proof of Multiplication is by Division as will appear by the next Chapter.

```
  18
   4
  --
  72 product 1.
  22
 ---
 144
144
----
1584 Prod. 2.
```

CHAP. VI.

Division by whole Numbers.

I. DIvision is that by which we discover, how often one number is contained in another, or (which is the same) it sheweth how to divide a number propounded into as many equal parts as you please.

II. In Division there are always three remarkable numbers which are commonly called by these names, the *Dividend*, the *Divisor*, and the *Quotient*

III. The *Dividend* is the number given to be divided into equal parts.

IV. The Divisor is the number by which the dividend is to be divided; that is, it is the number which declareth into how many equal parts the dividend must be divided.

V. The Quotient is the number arising from the division, and sheweth one of the equal parts required: so if 15 were given to be divided by 5, or into 5 equal parts, the number arising, or one of the equal parts will be 3, for 5 is found three times in 15: And here 15 is the *Dividend*, 5 the *Divisor*, and 3 the *Quotient*.

Division by a single Figure.

VI. Division being the hardest lesson in Arithmetick, must be heedfully intended by the Learner, for whose ease I shall use my utmost endeavours to make the way smooth by Rules and Examples, beginning with the easiest first, which will be in that case when the Divisor consists of one figure only; for example, let it be required to divide 192 by 8, or 192 pounds into 8 equal parts or shares; here 192 is the *Dividend*, 8 is the *Divisor* and the *Quotient* or one of the equal parts is sought.

VII. Place a crooked line at each end of the *Dividend*, that on the left hand serving for the place of the *Divisor*, and that on the right for the *Quotient*; then if the *Divisor* be a single figure, subscribe a point under the first figure of the *Dividend* towards the left hand, if such first figure be either equal unto or greater than the *Divisor*, but if such first figure be less than the *Divisor*, put a point under the next place of the *Dividend*; which number so distinguished by the point may be called the *Dividual*; so in the example given

What the Dividual is.

8) 192 (
 ˙

given in the 6 Rule, 192 being the *Dividend*, and 8 the *Divisor*, I subscribe a point under 9, not under 1, because it is less than the *Divisor*. This done the *Dividual*, or number whereof the question must be asked, is 19.

VIII. Having thus prepared the numbers, ask how often the *Divisor* is contained in the *Dividual*, and write the number which answers the question in the *Quotient*, then multiply the *Divisor* by the number placed in the *Quotient*, and subscribe the product underneath the *Dividual*; Lastly, having drawn a line under the product, subtract it from the *Dividual* and subscribe the remainder orderly underneath the line: So demanding how many times the *Divisor* 8 is found in the *Dividual* 19, the answer is two times, wherefore I write 2 in the *Quotient*, then multiplying the *Divisor* 8 by 2 (the number placed in the *Quotient*) the product is 16, which I subscribe orderly under the *Dividual* 19, and after a line is drawn underneath the product 16, I subtract it from the *Dividual* 19, and place the remainder 3 underneath the line.

```
8) 192 (2
   16
   --
    3
```

IX. Put another point under the next place of the *Dividend* towards the right hand, and bring down the Figure or Cypher standing in that place to the remainder; that is, set it next after it, so the whole will be a new *Dividual*: Thus a point being placed under 2 which stands in the next place of the *Dividend*, I write 2 next after (to wit, on the right hand of) the remainder 3, so is 32 a new *Dividual*, or number whereof the second question must be asked, & the work will stand as you see in the example.

```
8) 192 (2
   16
   --
    32
```

X. A new *Dividual* being set apart, renew the question and proceed according to the 8th Rule of this Chapter. Thus demanding how often the *Divisor* 8 is found in the *Dividual* 32, the answer is four times, wherefore I write 4 in the *Quotient*, then multiplying the *Divisor* 8 by 4 (the figure last placed in the *Quotient*) the product is 32, which I subscribe under the *Dividual* 32, and after a line is drawn underneath, I subtract the product 32 from the *Dividual* 32, and there being no remainder I subscribe 0 under the line, so the whole work being finisht, the *Quotient* is found to be 24, and the operation stands as you see in the Example; wherefore I conclude, if 192 pounds be equally divided amongst 8 persons, the share of each person will be 24 pounds.

```
8) 192 (24
    ..
   16
   ---
    32
    32
   ---
     0
```

A second Example, Let it be required to divide 936 pounds into 9 equal parts, having distinguished the first *Dividual* by a point (according to the 7th. Rule of this Chapter) I demand how often the *Divisor* 9 is found in the *Dividual* 9, and finding it once contained in it, I write 1 in the *Quotient*, then multiplying the *Divisor* 9 by 1, the product is 9, which I subscribe under the *Dividual* 9, after this, a line being drawn under the product 9, I subtract it from the *Dividual* 9, and there being no remainder, I place 0 underneath the line, as you see in the Example.

```
9) 936 (1
   .
   9
   -
   0
```

Again, placing a point under 3 which stands in the next place of the *Dividend*, I transcribe the said 3 next after the remainder 0, for a new *Dividual*, then asking how

```
9) 936 (10
   ..
   9
   --
   03
```

how often the *Divisor* 9 is contained in the *Dividual* 3, and not finding it once contained therein, I write 0 in the *Quotient*, and now because the product which ought to arise from the Multiplication of the *Divisor* by 0 (the Cypher last placed in the *Quotient*) amounts to 0, the *Dividual* 3, out of which that product should have been subtracted, remains the same without alteration; wherefore after a point is subscribed under 6 the next place of the *Dividend*, I annex 6 to the *Dividual* 3, so there will be a new *Dividual*, to wit, 36; then demanding how often the *Divisor* 9 is found in the *Dividual* 36, the answer will be 4 times, wherefore I place 4 in the *Quotient*, and multiplying the *Divisor* 9 by 4, the product is 36, which I subscribe under, and subtract from the *Dividual* 36, so the remainder is 0, thus the whole work being finisht, the *Quotient* is found to be 104, as you see in the Example; wherefore I conclude if 936 *l.* be divided equally amongst 9 persons, the share of each will be 104 *l.* In like manner if 296163 be divided by 7 the *Quotient* will be 42309.

```
9) 936 (104
   9
  ----
   036
    36
  ----
     0
```

The substance of division by what method soever.

The whole work of Division is briefly contained in this following Verse,

Dic quot, multiplica, subduc, transferque secundum.

Or thus,

First you must ask how oft, in Quotient answer make;
Then multiply, subtract, a new Dividual take.

A compendious way of dividing by a single figure.

XI. When in the Division the *Divisor* consists of a single Figure onely, the *Quotient* may be written

 down,

down, and all the operation performed in mind, without writing down any part thereof; so 82506 being given to be halfed or divided into two equal parts, the work will be thus, The *Divisor* 2 is found

2) 82506 (41253

in 8, four times; in 2, once; in 5, twice; and there will remain 1, which 1 being supposed to stand before the Cypher makes 10, then I say 2 is found in 10 five times; and last of all in 6 three times; so that the true *Quotient* or one half of the given number 82506 is found to be 41253.

In like manner if 82506 be given to be divided by 3 or into 3 equal parts, the work will be thus, the *divisor* 3

3) 82506 (27502

is found in 8 twice & there will remain 2, which 2 being supposed to stand before (to wit, on the left hand of) the following 2 makes 22, then I say 3 is found in 22, 7 times; in 15, 5 times, in 0 not at all; and lastly in 6, twice; so that the true quotient or one of the 3 equal parts required is 27502. After the same manner may division be wrought by any single figure, without much charge to the memory.

A Note, concerning the remainder after the Division is ended, if any happen.

Note here the *Learner* may ask what shall be done with the last remainder, if any happen, when the *Division* is finished? For a full answer to this, I refer the *Reader* to the Note in the fifth *Rule* of the seventh *Chapter*; yet I shall here propound an example where the said case happens, *viz*, let it be required to divide 351 by 8, or 351 *pounds* equally amongst 8 *persons*; now if the operation be prosecuted according to the former rules, the *Quotient* will be found to be 43, and after the *Division*

7
8) 351 (43

is

is finisht there will remain 7, that is, each person must have 43 pounds and there will be an overplus of 7 pounds, which must be also divided equally among the 8 persons, but that cannot be done till the 7 pounds be reduced into shillings, and then those shillings must be divided by 8 to give every person his due share of the shillings contained in the said 7 pounds; again, if there yet remain any surplusage of shillings, they must be reduced to pence, which must also be divided by 8 to give every person his due share of pence, so that when this question is fully answered each persons share will appear to be 43 *l.*—17 *s.*—6 *d.* But how the before mentioned *Reduction* is performed will be made manifest in the fifth rule of the next Chapter.

Division by two or more figures, the first and easyest method.

XII. When the divisor consists of two, three, or how many places soever the operation is more difficult than the former, but depends upon the same grounds, and therefore the learner being well vers'd in the preceding method of dividing by a single figure, will the more readily understand these that follow, which are two, whereof the first is the easier, but the latter more expeditious, and that which indeed is principally to be aimed at: For an example of the former, let it be required to divide 4112772 by 708, or (which is the same) to divide 4112772 into 708 equal parts.

First, a *Table* is to be made to shew at first sight any *Multiple* or product of the *Divisor*, it being taken twice, thrice, or any number of times under ten, so having first written down the *Divisor* it self 708, and drawn a line on the right hand thereof, I place 1 on the right hand of the line directly

The Divisor.	708	1
Multiples of the divisor.	1416	2
	2124	3
	2832	4
	3540	5
	4248	6
	4956	7
	5664	8
	6372	9

against the *Divisor*; then underneath the *Divisor* 708 I subscribe the double thereof which is 1416, and place the figure 2 directly against the said double, to wit, on the other side of the line. Again adding 1416 (to wit the double of the *Divisor*) to the *Divisor* it self 708, the sum is 2124 for the triple of the *Divisor*, this triple I subscribe under the double and place 3 on the other side of the line right against the triple; Again adding 2124 (the triple of the *Divisor*) to the *Divisor* 708 I find 2832, for the quadruple of the *Divisor*, which quadruple I subscribe under the triple, and proceeding in like manner, at last the Table is finisht, which readily shews the *Divisor*, with the *duple*, *triple*, *quadruple*, *quintuple*, *sextuple*, *septuple*, *octuple*, *and noncuple* of the *Divisor*.

Now for a proofe of the said Table, adding the last number thereof to wit 6372 (which was found to be nine times the *Divisor*) to the *Divisor* 708 I find the sum to be 7080 which (by the 12th Rule of the fifth chap.) is evidently ten times the *Divisor*, wherefore I conclude that the Table is true, in regard that the last number thereof is derived from all the superiour numbers.

The Table of Multiples or Products of the *Divisor* being thus prepared, write down the *dividend* on the right hand of the *Divisor*, then distinguish by a point so many of the formost places of the *dividend* towards the left hand as are either equal in value (being considered apart) to the *Divisor*, or

which

which being greater et come neareſt to the alue thereof, thus I ubſcribe a point under ,thereby ſetting apart .112 being the feweſt f the formoſt places which will contain the *Diviſor* 708 ſo is 4112 he *dividual*, (or num-

708	1	) 4112772 (5809
1416	2	3540
2124	3	
2832	4	5727
3540	5	
4248	6	5664
4956	7	6372
5664	8	6372
6372	9	0

ber whereof the firſt queſtion muſt be asked) then demanding how often the *Diviſor* 708 is contained n the *dividual* 4112, the anſwer will be found by the *Table* to be five times, for looking in the *Table* I cannot find the *dividual* exactly, but I ſee that 6 times the *Diviſor* is the next greater than the *dividual* 4112, and five times is the next leſſer; wherefore I write 5 in the quotient and the number in the *Table* which ſtands againſt 5, to wit, 3540 I ſubſcribe under the *dividual* 4112, then having drawn a line underneath, I ſubtract 3540 (which is five times the *Diviſor*) from the *dividual* 4112, and ſubſcribe the remainder 572 underneath the line; that done, I put a point under the next place of the *dividend* towards the right hand, and becauſe the figure 7 ſtands in that place, I tranſcribe 7 next after the remainder 572, ſo there is 5727 for a new *dividual*.

Then demanding how often the *Diviſor* 708 is contained in the *dividual* 5727, the anſwer will be found by the *Table* to be 8 times, for looking in the *Table* I find that 9 times the *Diviſor* is the next greater, but 8 times is the next leſſer than the *dividual*, wherefore I write 8 in the quotient, and the

the number in the *Table* which stands against 8, to wit, 5664 I subscribe under, and subtract from the *dividual* 5727, placing the remainder 63 underneath the line.

Again, I put a point under the next place of the *dividend*, where I find the figure 7, and therefore transcribing 7 next after the remainder 63, the new *dividual* will be 637; then demanding how often the *Divisor* 708 is contain'd in the *dividual* 637 and not finding it once contain'd therein, I write o in the quotient, and since in this case (that is, when a cypher answers the question) the *dividual* remains the same without alteration, the figure or cypher standing in the next place of the *dividend* is to be transcribed after the *dividual* for a new *dividual*, so writing 2 next after 637, the new *dividual* is 6372, wherefore demanding how often the *Divisor* 708 is contain'd in 6372, I find by the *Table* it is contain'd in it 9 times, wherefore writing 9 in the *Quotient*, and placing the number which stands against 9 in the *Table*, to wit, 6372 under the *dividual* 6372, and subtracting it from the *dividual* there will remain o. Wherefore I conclude if 4112772 be divided by 708, or into 708 equal parts, the true *Quotient* or one of the equal parts required is 5809.

Divisor.	188	1
Multiples of the Divisor.	376	2
	564	3
	752	4
	940	5
	1128	6
	1316	7
	1504	8
	1692	9

```
)20304(108.
 188
 ----
  1504
  1504
  ----
     o
```

In like manner if 20304 be divided by 188, that is into 188 equal parts, the *quotient* arising or one of those equal parts will be 108, and the operation will stand as you see. The

The preceding method of *Division* by the help of a *Table* of the *Multiples* or Products of the *Divisor*, as it is most easie, so in some Cases, (namely, where the *Divisor* is great, and a *Quotient* of many places is required; as in calculating *Tables* of *Interest*, *Astronomical Tables*, and such like) it excells all other waies of *Division*, both in respect of certainty and expedition, but for common practice it is too tedious, and therefore I shall proceed to the choicest practical method.

The latter and choicest practical method of Division, when the Divisor consists of many places.

XIII. *I now come to the last and principal method of Division, when the Divisor consists of many places, which to such as have the Table of Multiplication by heart will not be difficult*; for example, let 56304 be a number given to be divided by 184, that is, into 184 equal parts, and the *Quotient* or one of the equal parts is required.

First, distinguish by a point (as before) so many of the formost places of the *dividend* towards the left hand as are either equal in value (when they are consider'd apart) to the *Divisor*, or else which being greater, yet come nearest unto it, thus I subscribe a point under the figure 3, thereby setting apart 563 being the fewest of the formost places which will contain the *Divisor*, so is 563 the *dividual*, or number whereof the first question must be asked. Having thus prepar'd the numbers, I demand how often the *Divisor* 184 is contained in the *dividual* 563

184) 56304(

and since to answer this question and such like, there is a necessity of tryal, it will be requisite to shew how this tryal may fitly be made: first, therefore

fore compare the number of places, in the *dividual*. with the number of places in the *Divisor*, and when the number of places is the same in both, let it be asked how often the first or extream figure of the *Divisor* towards the left hand is contained in the first figure of the *dividual* towards the same hand; so here demanding how often 1 is contained in 5, the answer is 5 times, whence I infer that the *Divisor* 184 is not contained oftner than 5 times in the *dividual* 563 (for 6 times 184 is manifestly greater than 563) but whether it be contained 5 times in it or not, examination must be made either by multiplying (in some by-place) the *Divisor* 184 by the said 5, and comparing the product with the *dividual* 563, or else thus, saying 5 times 1 (to wit the 1 in the *Divisor*, is contained in 5, to wit, the first figure of the *dividual* 563, 5 times, but then 8, the following figure of the *Divisor*, cannot be found 5 times in 6, the following figure of the *dividend*, and consequently the *Divisor* 184 is not contained 5 times in the *dividual* 563, wherefore I make another tryal to see whether it may be contained 4 times in it or not; saying 4 times 1 is 4, which is found in 5, and there will remain 1, but then 4 times 8 which is 32 cannot be had in 16, (for the 1 before remaining being supposed to stand before 6 maketh 16) hence I conclude again, that the *Divisor* 184 is not contained 4 times in the *dividual* 563, wherefore I make another tryal to see whether it may be contained 3 times in it or not, saying 3 times 1 is 3, which is found in 5, and there will remain 2, again, 3 times 8 is 24, which is found in 26, (for the 2 before remaining being supposed to stand before the 6 in the

the *dividual* makes 26) and there will remain 2: lastly, 3 times 4 is 12, which is likewise found in 23 (for the 2 remaining being supposed to stand before the 3 in the *dividual* makes 23) whereby I see that the *Divisor* 184 is contained 3 times in the *dividual* 563, wherefore I write 3 in the *Quotient*, and proceeding according to the 8th *Rule* of this *Chapter*. I multiply the *Divisor* 184 by 3 (the figure placed in the *Quotient*) so the *Product* is 552, which I subscribe orderly underneath the *dividual* 563, then having drawn a line underneath the said *Product*, I subtract it from the *dividual*, and subscribe the remainder which is 11 under the line.

```
184) 563̣04 (3
     552
     ───
      11
```

Again, according to the 9th *Rule* of this *Chapter*, I bring down 0 which stands in the next place of the *dividend*, to the remainder 11, so there is 110 for a new *dividual*, then demanding how often the *Divisor* 184 is found in the *dividual* 110, and not finding it once contained in it, I write 0 in the *Quotient*, (which is to be done as often as the question is answered by nothing) now because the *Product* arising from the multiplication of the *Divisor* by 0, (the Cypher last placed in the *Quotient*) amounts to 0, the *dividual* 110 out of which that *Product* should be subtracted, remains the same without alteration; wherefore after a point is subscribed under 4 the following place of the *dividend*, I annex 4 to the last *dividual* 110, so there will be a new *dividual*, to wit, 1104; and here the question at large is to know how often 184 is found in 1104, but to lessen the

```
184) 563̣0̣4̣ (306
     552
     ────
     1104
     1104
     ────
        0
```

the tryal, because the *dividual* consists of one place more than is in the *Divisor*, it must be asked how often the first figure of the *Divisor* on the left hand is contained in the two formost places of the *dividual* towards the left hand, *viz.* I demand how often 1 is contained in 11, and although it may be had 11 times, yet I need never begin the tryal above 9 times, therefore I make tryal with 9, saying 9 times 1 is 9, which is found in 11 and there will remain 2; but then 9 times 8 which is 72 cannot be found in 20, (20 because the 2 remaining being supposed to stand before 0 in the *dividual* makes 20) therefore I make tryal with 8, saying 8 times 1 is 8, which is found in 11, and there will remain 3, but then 8 times 8 cannot be had in 30, (30 because the 3 remaining being supposed to stand before the 0 or Cypher makes 30) therefore I make tryal with 7, saying 7 times 1 is 7 which is found in 11 and there will remain 4; but then 7 times 8 cannot be had in 40, therefore I make tryal with 6, saying 6 times 1 is 6, which is found in 11 and there will remain 5; also 6 times 8 is 48, which is found in 50 and there will remain 2; lastly, 6 times 4 is 24, which is found in 24, whereby at length I see that the *Divisor* 184 is contained 6 times in the *dividual* 1104; wherefore I write 6 in the *Quotient*, and proceeding according to the 8th *Rule* of this *Chapter*, I multiply the *Divisor* 184 by 6 (the figure last placed in the *Quotient*) so the *Product* is 1104, which being subscribed under and subtracted from the *dividual* 1104, the Remainder is 0, so at last I conclude that the *Quotient* sought is 306.

Note, if the figure assumed for the *Quotient*, holds

holds good upon tryal as aforesaid, by two or three of the formost places of the *dividual*. it will for the most part hold throughout the *dividual*, but this must be a perpetual Rule, that whensoever the *Product* of the multiplication of the *Divisor* by the figure placed in the *Quotient*, happens to be greater than the *dividual* from which it ought to be subtracted, such *Product* must be struck out of the work, and a lesser figure is to be placed in the Quotient.

For a second *Example*, let it be required to divide 15114220 by 2987, or into 2987 equal parts.

First, the *Divisor* 2987 being greater than 1511 (to wit, the four formost places of the Dividend) I set a point under 4, thereby setting apart 15114 for a *Dividual*, then because the *Dividual* consists of one place more than the *Divisor*, I ask how often 2 (the first figure of the *Divisor* towards the left hand) is contained in 15 (the two formost places of the *Dividual*) and finding the answer to be 7 times, I infer thence that the *Divisor* 2987 cannot be contained more than 7 times in the *Dividual* 15114, but whether it will be contained 7 times in it or not, examination must be made, either by multiplying 2987 by 7, and comparing the Product with the *Dividual* 15114, or else by the manner of tryal before delivered in the last Example, so at length it will be discovered, that the *Divisor* 2987 will not be found above 5 times in the *Dividual* 15114, wherefore (according to the 8th *Rule* of this *Chapter*) writing 5 in the Quotient, and multiplying 2987 by 5, I subscribe the Product

2987) 15114220 (5
14935
――――
179

duct

duct of that multiplication which is 14935, under the *Dividual* 15114, then drawing a line underneath the said Product, and subtracting it from the *Dividual* 15114, I subscribe the Remainder 179 under the line.

Again (according to the 9th *Rule* of this *Chapter*) I bring down 2, the next place of the Dividend, to the said Remainder 179, so the new Dividual will be 1792, that done, asking how often the *Divisor* 2987 is contained in the *Dividual* 1792, and not finding it once contained in it, I write 0 in the Quotient, and here because the question is answered by 0, the next place of the *Dividend*, to wit 2, is to be brought down to the *Dividual* 1792, so the new *Dividual* is 17922. Then renewing the question, and proceeding as before, at length the Division being finisht, the Quotient will be found 5060 exactly, without any Remainder; but if any Remainder had hapned after the subtraction of the last Product, it must have been prosecuted according to the directions before given in the example at the latter end of the 11th *Rule* of this *Chapter*.

```
2987) 15114220 (50
        14935
        -----
         1792
```

```
2987) 15114220 (5060
        14935
        -----
         17922
         17922
         -----
            00
```

In like manner if 120893550 be divided by 19999, or into 19999 equal parts, the *quotient*, or one of those equal parts will be found 60450, and the Work will stand as here you see.

This

This latter method of Division is to be prefer'd before any of the common wayes of dividing by dashing out of Figures, where the steps of the Division are so confounded (besides the burden upon the memory by a promiscuous Multiplication and Division) that if any errour happen, it can scarce be corrected without beginning the work anew; But in the way before explained, the particular Multiplications, Subtractions, and Remainders, which belong to every figure of the Quotient, are so distinctly and clearly exprest, that if an errour happen, the work may easily be reformed.

```
19999)1208939550(60450
      119994
      ______
        89995
        79996
        _____
         99995
         99995
         _____
            00
```

How the number of places in the Quotient may be discovered.

XIV. So often as the Question is repeated in Division, so many places there must be in the Quotient (which may be discovered by the number of Points placed under the *Dividend*) and so many times is one and the same kind of operation repeated, the substance whereof is contained in the Verse before mentioned at the end of the 10th *Rule* of this *Chapter*.

A compendious way of dividing by 10. 100, 1000, &c.

XV. When the *Divisor* consists of 1 or an unit in the extream place towards the left hand, and nothing but Cyphers towards the right, the division is performed by cutting off with a line so many places of the *Dividend* towards the right hand as the *Divisor* hath Cyphers; so the figures which

which stand on the left hand of the line give the Quotient, and those cut off to the right (if they be significant figures) are to be proceeded with as a surplusage or overplus remaining, according to the Note at the end of the eleventh *Rule* of this *Chapter*. So if 4720 *l.* were given to be divided equally amongst 10 persons, the share of each would be 472 *l.* also if the said 4720 *l.* were to be divided equally amongst 100 persons, the share of each would be 47 *l.* and there would be a surplusage or remainder of 20 *l.* to be also subdivided amongst them, after the said 20 *l.* are converted into shillings, according to the fifth *Rule* of the next *Chapter*. Lastly, if the said 4720 *l.* were to be divided amongst 1000 persons, the share of each would be 4 *l.* and there would be a remainder of 720 *l.* to be also subdivided as aforesaid. See the form of the Work in the *Margent*.

10) 472 | 0 (472
100) 47 | 20 (47
1000) 4 | 720 (4

XVI. When the *Divisor* consists of any significant figure or figures in the first or formost place or places towards the left hand, and nothing but a Cypher or Cyphers towards the right, cut off by a line so many places of the *Dividend* towards the right hand as the *Divisor* hath cyphers towards the right; then divide the figures of the *Dividend* which stand on the left hand of the line, by the figures in the *Divisor* which remain when the said Cypher or Cyphers are omitted, remembring after the division is finished, to write down next after the last remainder the places of the *Dividend* which were first cut off: So if 36732 were given to be

Another Compendium in Division.

divided

divided by 20, the Quotient will be 1836, and there will remain 12, *viz.* if you cut off one place from the *Dividend* towards the right hand (becauſe the *Diviſor* ends with one Cypher) and then divide the reſt, to wit, 3673 by 2 (according to the 11th *Rule* of this *Chapter*) there will ariſe in the Quotient 1836, and the laſt remainder after ſuch diviſion is finiſht will be 1, unto which if 2 (the figure firſt cut off from the *Dividend*) be annexed, the total remainder is 12.

1
2|0) 3673|2 (1836

In like manner if 7456787 were given to be divided by 304000, the Quotient will be 24, and there will remain 160787; *viz.* If you cut off 3 places from the *Dividend* towards the right hand (3 places becauſe the *Diviſor* ends with 3 Cyphers) and then divide 7456 by 304, there will ariſe in the Quotient 24, and the laſt remainder after ſuch diviſion is finiſht, will be 160, unto which if 787 (the places firſt cut off from the *Dividend)* be annexed, the total remainder or ſurpluſage is 160787, which is to be proceeded with as is directed in the Note at the latter end of the eleventh *Rule* of this *Chapter.*

304|000) 7456|787 (24
608
1376
1216
160787

XVII. Diviſion and Multiplication do interchangeably prove one another; for in Diviſion if you multiply the *Diviſor* by the Quotient, the Product will be equal to the *Dividend*: So in the *Example* of the 13th *Rule* of this *Chapter*; if 184 the

The proof of Multiplication and Diviſion.

the *Divisor* be multiplyed by 306 the *Quotient*, the Product is 56304, which is the same with the *Dividend*; but when after the whole Division is finished any figures remain of the last Subtraction, add them likewise to the Product: So in the last *Example* of the 16th *Rule* of this *Chapter*, the *Divisor* 304000 being multiplyed by the *Quotient* 24, produceth 7296000, unto which if you add the number remaining, to wit, 160787, the sum is 7456787, which is the same with the *Dividend*. Again, in Multiplication, if the Product be divided by the Multiplicator, the *Quotient* will give you the Multiplicand, or if the Product be divided by the Multiplicand, the *Quotient* will give you the Multiplicator: So in the first Example of the 9th *Rule* of the last *Chapter*, if the Product 111024 be divided by the Multiplicand 3084, the *Quotient* gives the Multiplicator 36.

There is also of Multiplication a *Common proof* argued from the Multiplicand, the Multiplicator and the Product by casting away nines, but by that way of proof (though rightly used) a *false* Product will be affirmed to be true: Example, if 3462 be multiplyed by 786, the true Product is 2721132, but if I say 4953132 or 5153132 is the Product (or many others which may be given) the proof by nines will confirm them to be true Products, though they are false as will be evident to such as know the *Rule*, which I mention here only to set a brand upon it, that it may be avoided by all lovers of Truth.

CHAP.

CHAP. VII.

Reduction.

I. FOrasmuch as in *Money*, there are diversities of kinds, *viz.* in *England*, *Pounds*, *Shillings*, *Pence*, and *Farthings*; also divers kinds of *Weights*, *Measures*, &c. as hath been fully declared in the second Chapter; and because it is oftentimes required to find how many pieces of one kind of *Money* are equal in value to a given number of another (and so likewise of *Weights*, *Measures*, &c.) it will be convenient in this place to shew how that is performed, since thereby the Rules of *Multiplication* and *Division* before delivered will be exercis'd; this kind of operation is called *Reduction*.

II. Reduction is either descending or ascending.

III. Reduction descending is when a number of a greater denomination being given, it is required to find how many of a lesser denomination, are equal in value to that given number of the greater: As when it is required to find how many *shillings* are contained in 30 *l.* Likewise how meny *pence* in 320 *s.* or how many *hours* in 365 *days*, &c.

IV. Reduction ascending is, when a number of a lesser denomination being given, it is required to find how many of a greater denomination, are equal in value to that given number of the lesser: As when it is required to find how many *pence* are contained in 500 *farthings?* likewise how many *shillings* in 348 *pence?* or how many *dayes* in 364 *houres?* &c.

V. Reduction descending is performed by *Multiplication*; for if the given number of Integers of a greater denomination, be multiplyed by such number of Integers of the lesser as are equal to one of the Integers given; the Product is the number of Integers of the lesser denomination required.

Reduction descending is performed by Multiplication.

So 230 *l.* of English Money will be reduced into 4600 *s.* for if 230 be multiplyed by 20 (the number of *shillings* which are equal to 1 *pound*) the product is 4600; in like manner 4600 *s.* will be reduced into 55200 *d.* for if 4600 be multiplied by 12(the number of *pence* contained in 1 *shilling*) the product is 55200. Also 55200 *pence* being multiplyed by 4 (because 4 *farthings* make a *penny*) are reduced into 220800 *Farthings* as by the operation in the *Margent* is evident.

```
   230 Pounds.
    20
  4600 Shillings
    12
   92
   46
 55200 Pence.
     4
220800 Farthings.
```

The like method is to be observed in *Weights Measures*, &c. So 345 *Ounces Troy* are reduced into 6900 *Penny Weights*, and 6900 *Penny Weights* to 165600 *Grains*, as by the operation in the *Margent* you may see.

```
   345 Ounces.
    20
  6900 Penny weights.
    24
  276
 138
165600 Grains.
```

Note, By this *Rule* the Learner is furnished with skill to resolve that case in Division when the *Dividend* is *less* than the *Divisor*: *Example*,

Compare this with the Note upon the last Example of the 11th Rule of the 6th Chapter.

Example, Let it be required to divide 7 *pounds* of English Money equally amongst 8 Persons; here it is evident that the *Dividend* 7 is less than the *Divisor* 8; that is, the number of *Pounds* is less than the number of Persons; and consequently each share must be less than a Pound; so that in effect it is required to find how many *Shillings* and *Pence* belong to each Person for his share: First, therefore reduce the 7 *Pounds* into *Shillings*, which will be 140, these divided by 8 give 17 *Shillings* to each Person, and there will yet be a remainder of 4 *Shillings* to be also equally divided into 8 parts, but these 4 *Shillings* must be first reduced into *Pence*, which will be 48, then dividing 48 by 8, the *Quotient* will give 6 *Pence* more to every Person, so at last it appears that if 7 *Pounds* of English Money be equally divided into 8 parts, the entire *Quotient* (or one of the equal shares) will be 17 *Shillings* and 6 *Pence*.

In like manner, if 354 *Pounds* of English Money be given to be divided equally amongst 125 Persons, the share of each will be found to be 2 *Pounds*, 16 *Shillings*, 7 *Pence*, 2 *Farthings*, and somewhat more, but the parts of a *Farthing* being of no moment (and not properly to be handled in this place) are neglected.

Compare these two Examples with the last Example of the eleventh Rule of the sixth Chapter.

In *Reduction* descending, the Learner may receive farther help by the subsequent *Tables*.

1. *Of English Money.*

Pounds	*Multiplied by*	20	*Produce*	*Shillings.*
Shillings		12		*Pence.*
Pence		4		*Farthings.*

2. *Of Troy Weight.*

Pounds	*Multiplied by*	12	*Produce*	*Ounces.*
Ounces		20		*Penny Weights.*
Penny W.		24		*Grains.*

Also in *Apothecaries Weights.*

Ounces Troy	*Multiplied by*	8	*Produce*	*Drams.*
Drams		3		*Scruples.*
Scruples		20		*Grains.*

3. *Of Averdupois Weights.*

Hundred W.	*Multip. by*	4	*Produce*	*Quarters.*
Quarters		28		*Pounds.*
Pounds		16		*Ounces.*
Ounces		16		*Drams.*

4. *Of Liquid Measures.*

Hogsheads	*Multip. by*	63	*Produce*	*Gallons.*
Gallons		2		*Pottles.*
Pottles		2		*Quarts.*
Quarts		2		*Pints.*

5. Of

5. *Of Dry Measures.*

Quarters	*Multiplied by*	8	*Produce*	*Bushels.*
Bushels		4		*Pecks.*
Pecks		2		*Gallons.*
Gallons		2		*Pottles.*
Pottles		2		*Quarts.*
Quarts		2		*Pints,*

6. *Of Long Measures.*

English miles	*Multiplied by*	8	*Produce*	*Furlongs.*
Furlongs		220		*Yards.*
Yards		3		*Feet.*
Feet		12		*Inches.*
Inches		3		*Barley Corns.*

Also,

Yards or *Ells*	*Mult. by*	4	*Produce*	*Quarters.*
Quarters		4		*Nailes.*

7. *Of Superficial Measures of Land*

Acres	*Mult. by*	4	*Produce*	*Roods*
Roods		40		*Perches* or *Poles.*

8. *Of Time.*

Weeks	*Mult. by*	7	*Produce*	*Dayes.*
Dayes		24		*Houres.*
Hours		60		*Minutes.*

To reduce Integers of divers denominations into the lowest of those denominations.

VI. Integers of divers denominations are reduced into the least of those denominations according to the fifth *Rule* aforegoing, by descending orderly to the next inferiour denomination,

nation, and adding to each Product such Integers (if there be any) which are of the same name.

So 12 *Pounds*, 13 *Shillings*, and 10 *Pence*, are reduced into 3046 *Pence* in this manner, *viz.* 12 *l.* multiplied by 20 (because 20 *s.* make one *l.*) produce 240 *Shillings*, unto which adding 13 *s.* the sum is 253 *Shillings*: Again, 253 *s.* multiplyed by 12 (because 1 *Shilling* is equal to 12 *Pence*) produce 3036 *Pence*, unto which if 10 *Pence* be added, the sum is 3046 *Pence*, as by the operation in the *Margent* is manifest.

l. *s.* *d.*
12——13——10
20
240
add 13
253 *Shillings.*
12
506
253
3036
add 10
3046 *Pence.*

So 35 *Ounces*, 16 *Penny Weights*, and 12 *Grains Troy* will be reduced into 17196 *Grains.*

Reduction ascending is performed by Division.

VII. Reduction ascending is performed by Division, for if the number of Integers given, be divided by such a number of the same Integers as are equal to one of the Integers required, the *Quotient* is the number of Integers sought.

So 220800 *Farthings* being divided by 4 (the number of *Farthings* in a *Penny*) give 55200 *Pence* in the *Quotient*; In like manner if 55200 *Pence* be divided by 12 (the number of *Pence* in a *Shilling*) the *Quotient* is 4600 *Shillings*. Lastly, 4600 *Shillings* being divided by 20 (because 20 *s.* make a *Pound sterling*) the *Quotient* is 230 *Pounds sterling*) which are eqnal to 220800 *Farthings* first given. The operation is as followeth.

```
         12)      20)
4) 220800 (55200 (460|0 (230 l.
    20       48
    --       --
     20       72
     20       72
     --       --
      08      000
      08
      --
      000
```

In like manner, 34268 *Grains Troy* will be reduced to 5 *l.* 11 *Ounces*, 7 *Penny Weight*, and 20 *Grains*. This kind of *Reduction* may be made the easier to the Learner by the following *Tables*.

1. *Of English Money.*

Farthings	*Divi. by*	4	*give*	*Pence.*
Pence		12		*Shillings.*
Shillings.		20		*Pounds.*

2. *Of Troy Weight.*

Grains	*Divi. by*	24	*give*	*Penny Weights.*
Penny W.		20		*Ounces.*
Ounces		12		*Pounds Troy.*

Also in *Apothecaries Weights.*

Grains	*Divi. by*	20	*give*	*Scruples.*
Scruples		3		*Drams.*
Drams		8		*Ounces Troy.*

3. *Of Averdupois Weight.*

Drams	*Divided by*	16	*give*	*Ounces.*
Ounces		16		*Pounds.*
Pounds		28		*Quarters of* C.
Quarters.		4		*Hundred Weight.*

4. Of

4. *Of Liquid Measures.*

Pints	Divided by	2	give	*Quarts.*
Quarts		2		*Pottles.*
Pottles		2		*Gallons.*
Gallons		63		*Hogsheads.*

5. *Of Dry Measures.*

Pints	Divided by	2	give	*Quarts.*
Quarts		2		*Pottles.*
Pottles		2		*Gallons.*
Gallons		2		*Pecks.*
Pecks		4		*Bushels.*
Bushels		8		*Quarters.*

6. *Of Long Measures.*

Barley C.	Divided by	3	give	*Inches.*
Inches		12		*Feet.*
Feet		3		*Yards.*
Yards		220		*Furlongs.*
Furlongs		8		*English Miles.*

Also,

Nails	Divi. by	4	give	*Quarters of Yards, also of Ells.*
Quarters		4		*Yards, also Ells.*

7. *Of Superficial Measures of Land.*

Perches or Poles	Divi. by	40	give	*Roods or Quarters of Acres.*
Roods		4		*Acres.*

8. *Of Time.*

Minutes	Divi. by	60	give	*Houres.*
Houres		24		*Dayes.*
Dayes		7		*Weeks.*

Note;

Note, that if after Division is finisht in Reduction ascending there be any remainder, it is of the same denomination with the Dividend.

Note also that Reduction descending and ascending do mutually prove one another, by inverting the question.

Questions to exercise Reduction.

1. In 257 *l.* how many shillings? *Answer,* 5140.
2. In 3076 *l.* how many shillings? *Answ.* 61520.
3. In 902 shillings how many pence? *An.* 10824.
4. In 2179 shillings how many farthings? *Answer.* 104592.
5. In 49 *l.*—13 *s.*—7 *d.* how many pences? *Answer.* 11923.
6. In 2053 *l.*—14 *s.*—9 *d.*—2 *f.* how many farthings? *Answ,* 1971590.
7. In 354 *lb.* of Troy weight, how many grains? (of Gold-smiths weight?) *Answ.* 2039040.
8. In 300 English miles, how many yards? *Answer* 528000.
9. In 1 English mile, how many barly cornes length? *Answ.* 190080.
10. In 560 Acres, how many Perches? *Answer.* 89600.
11. In 225 Acres, 3 Roods, and 30 Perches, how many Perches? *Answ.* 36150.
12. 30565 pence how many pounds? *Answer,* 127 *l.*—7 *s.*—1 *d.*
13. In 5764684 farthings, how many pounds? *Answ.* 6004 *l.*—17 *s.*—7 *d.*
14. In 234678 Perches, how many Acres? *Answer,* 1466 Acres, 2 Roods, and 38 Perches.
15. In 525960 minutes of an hour, how many dayes

dayes? *Answ.* 365 daies and 6 houres, (or 1 year very near.)

16. In 10080 Pintes, how many Hogsheads? *Answ.* 20.

17. In 34678 grains of Apothecaries weight, how many ounces Troy? *Answ.* 72 Ounces, 1 Dram, 2 Scruples, and 18 grains.

18. In 106735 Pintes of wheat, how many Quarters? *Answ.* 208 Quarters, 3 Bushels, 2 Pecks 1 Gallon, 1 Pottle, 1 quart, 1 Pinte.

19. In 3969301 Barley corns length, how many Miles? *Answ.* 20 Miles, 7 Furlongs, 12 Yards, 2 Feet, 4 Inches, and 1 Barley corns length.

20. In 1900800 Barley corns length, how many Miles? *Answ.* 10.

CHAP. VIII.

Of the Rule of Three Direct.

I. THE Rule of Three is so called, because by three numbers known or given, it teacheth to find a fourth unknown, it is also called the Golden Rule for the excellency thereof; Lastly, It is called the Rule of Proportion for the reason hereafter declared.

II. The Rule of Three is either single or compound.

III. The single Rule is, when three terms or numbers are propounded, and a fourth proportional unto them is demanded. *The Rule of Three*

IV. Four numbers are said to be proportionals, when the first containeth the second, or is contained

ned by the second in the same manner as the third containeth the fourth, or is contained by the fourth: so these 4 numbers are said to be Proportionals, 8, 4, 12, 6, for as 8 containeth 4 twice; so doth 12 contain 6 twice, and therefore 8 is said to have such proportion to 4 as 12 hath to 6; likewise these are Proportionals, 4, 8, 6, 12. For as 4 is the half of 8, so is 6 the halfe of 12; and therefore 4 is said to have such proportion to 8 as 6 hath to 12.

The divers denominations of the terms in the Rule of Three.

V. The terms or numbers of the Rule of Three, (to wit, the three numbers given, and the fourth sought) consist of two different denominations, *viz.* two of the three given terms have one name, and the other given term with the term required have another: so this question being demanded, if four Students spend 19 pounds in certain months, how much money will serve 8 Students for the same time, and at the same rate of expense? Here Students and pounds are the two denominations of the terms in the question, *viz.* 4, and 8 (being two of the terms propounded) have the denomination of Students, and 19 the other term given, together with the term required, have the denomination of pounds.

VI. In the Rule of Three, two of the three given terms imply a supposition, and the third moves a question: so in the aforementioned question a supposition is made, that 4 Students spend 19 pounds, and a question is moved with the number 8, to wit, how many pounds will 8 Students spend?

VII. In

VII. In the Rule of Three, the numbers given *The right ordering of the terms given.* must be so ranked, that the known number or term upon which the question is moved, must possess the third place in the Rule, also of the other two that which hath the same denomination with the third, must be in the first place: lastly, the other known term which is of the same denomination with the fourth term sought (or answer of the question) must possess the second place: so in the question before mentioned, the terms 4, 19, and 8, are to be thus placed, *viz.* 8 is the term upon which the question is moved, and therefore to possess the third place in the Rule, 4 is of the same denomination with 8, *viz.* of Students, and therefore to be in the first place. Lastly, 19 being of the same denomination with the term sought, *viz.* of money is to be in the second place, and so they will be placed in the rule thus,

Students.	*Pounds.*	*Students.*
If 4 ———	19 ———	8

That is to say, if 4 Students spend 19 pounds, what will 8 Students spend? And here for the better discerning of the term or number upon which the question is moved, you may observe, that for the most part it is the known number in the question which immediately followeth these or such like words; *viz.* *How many? How much? What will? How long? How far?* &c.

VIII. The Rule of Three is either Direct or Inverse.

IX. The Rule of Three direct is, when the sense *The Rule of Three Direct.* or tenour of the question requireth that the fourth number sought must

have

have such proportion to the second, as the third number hath to the first; so in the aforementioned question, if 4 Students spend 19 pounds, how many pounds will 8 Students spend at the same rate of expence? It is evident that the thing required is to find a number which may have such proportion to 19, as 8 hath to 4; that is, as 8 is the double of 4, so ought the fourth number to be the double of 19; for if 19 pounds be required to maintain 4 Students a certain time, as much more must needs be required for the maintenance of 8 Students the same time; and therefore in this case we may say in a direct proportion, as 4 is to 8, so is 19 to a number which ought to be as much more as 19.

How to work the Rule of Three Direct, the three given terms being single numbers.

X. In the direct Rule of Three, if you multiply the second term by the third, or (which is all one) the third term by the second, and then divide the Product by the first, the *Quotient* will give the fourth term or fourth proportional required: so in the question before propounded, if you multiply 19 by 8, the product is 152, which if you divide by 4, the quotient will give you 38 the fourth term demanded, and the work will stand thus;

```
 Stud. l.  Stud.   l.
If 4—19—8—(38
         8
 ─────────────
    4)152 (38 pounds.
      12
 ─────────────
       32
       32
 ─────────────
        0
```

A second Example may be this, if 8 yards cost 9 pounds, how much will 3 yards cost? *Answer*, 3 *l.*——7 *s.*—— 6 *d.*

This question being stated according to the seventh Rule of this Chapter, will stand as here you see, then multiplying (as before) the second term 9 by the third term 3, the Product is 27, which being divided by the first term 8 the quotient is 3 pounds, and there is a remainder of three pounds which must be reduced into 60 shillings, and after those shillings are divided by 8, and the rest of the work prosecuted according to the Note at the latter end of the 11th Rule of the sixth Chapter, at length the entire quotient or answer of the question is 3 *l.*—7 *s.*—6 *d.*

y. l. y. l. s. d.
8—9—3—(3 : 7 : 6
3
8)27(3 *pounds*
24
3 *the remainder.*
20
8)60(7 *shillings.*
56
4 *the remainder*
12
8)48(6 *pence.*

A third Example. if 51 ounces of silver plate be sold for 13 pounds sterling, what is the price of 1 ounce of that plate? *Answ.* 5 *s.*—1 *d.* and somewhat more. The operation is thus, After the three known terms of this question are rightly ordered, they will stand as here you see in the Example then multiplying the second term 13 by the third term 1, the product will be also 13, (for multiplication by 1 makes no alteration) which 13 being divided by 51, after the manner of operation delivered

oz. l. oz.
51—13—1
1
13
20
51)260(5 *shillings.*
255
5
12
51) 60 (1 *penny.*
51
9

livered in the note upon the 5th Rule of the 7th Chapter, the entire Quotient or answer of the question will at length be found to be 5 *s*.—1 *d*. and somewhat more, but the surplusage being less than a farthing is omitted as uselesss.

Example 4. What must be paid to a labourer for his wages for 27 weeks at the rate of 4 *s*. for 1 week? *Answer*, 5 *l*.—8 *s*.

After the 3 given terms are rightly placed in the Rule, they will stand as you see in the Example; then multiplying the third term 27 by the second term 4 the product is 108, which I should divide by the first term 1, but in regard Division by 1 makes no alteration, the Quotient is also 108; so that the fourth term sought is 108 shillings, which being reduced to pounds (according to the seventh Rule of the seventh Chapter,) give 5 *l*.—8 *s*. for the answer of the question.

Week	*Shil:*	*Week.*
1	4	27
		4
		108

To prepare the terms of the Rule of Three when they are compounded of divers denominations.

XI. In the Rule of Three, if after the question is stated according to the seventh Rule of this Chapter, any of the 3 given terms be a compound term consisting of divers denominations, as pounds, shillings, and pence, or weeks dayes, hours, &c. such compound term must first be reduced into the lowest of those denominations (by the sixth Rule of the seventh Chapter) to the end that the three given terms may be three single numbers; also of these three single numbers the first and third must alwais be of one and the same denomination: for if it happen that they express things of different

names, ſuch of the two which hath the greater name (or denomination) is to be reduced into the ſame name with the leſſer (by the fifth Rule of the ſeventh Chapter) theſe preparations being obſerved, the reſt of the work is to be proſecuted according to the tenth Rule of this Chapter. Example, what will 48 ounces, 17 penny weight, and 20 grains of ſilver plate amount unto at the rate of 5 *s.*—6 *d.* the ounce? *Anſwer*, 13 *l.*—8 *s.*—10 *d.*—3 *f.* very near.

oz.	*s.*	*d.*	*oz. p.w. gr.*
1	5	6	48—17—20
20	12		20
20	60		960
24	6		17
480	66		977
			24
			3908
			1954
			23448
			20
			23468 *grains.*

This queſtion being ſtated according to the ſeventh Rule of this Chapter, will ſtand in the Rule as you ſee in the Example, to wit, if 1 ounce coſt 5 *s.*—6 *d.* what will 48 *oz.*—17 *p. w.*—20 *gr.* coſt? Here becauſe the third term is compounded of divers denominations, it muſt be reduced into the loweſt of thoſe denominations to wit, grains, ſo by the ſixth Rule of the ſeventh Chapter there will be found 23468 grains for the third term; likewiſe, becauſe the ſecond term 5 *s.*—6 *d.* is a compound term, whoſe loweſt name is pence, it muſt be reduced into pence (by the aforeſaid rule) ſo there will be found 66 pence for the ſecond term; moreover becauſe the firſt term hath the name ounce, and the third term the name, grain,

grain, the firſt term 1 ounce muſt be converted into 480 grains (which are equal to 1 ounce) then will the three terms or ſingle numbers ſtand in the rule, as here you ſee, *viz.* if 480 grains coſt 66 pence, how many pence will 23468 grains coſt? Now proceeding according to the tenth Rule of this Chapter, there will ariſe in the quotient 3226 pence, beſides a remainder af 408 pence, which being reduced to farthings, and thoſe divided by the firſt term 480 the quotient will be 3 farthings, ſo that the entire quotient is 3226 pence, 3 farthings, and ſomewhat more (but the parts of a farthing being of no moment, may be neglected.) Laſtly, the ſaid 3226 pence being reduced according to the ſeventh Rule of the ſeventh Chapter, give 13 *l.*——8 *s.*——10 *d.*——3 *f.* ſo that 13 *l.*——8 *s.*——10 *d.*——3 *f.* and ſomewhat more, will be the Anſwer of the Queſtion.

gr.	*pence*	*gr.*
480——	66 ——	23468

The proof of the Rule of Three direct.

XII. For the proof of the Direct Rule of Three multiply the fourth term by the firſt, which done, if that Product be equal to the Product of the ſecond term multiplyed by the third, the work is right, otherwiſe it is erroneous: ſo in the firſt Example, 38 the fourth term, being multiplyed by the firſt term 4, the Product is 152 which is alſo the Product of 19 multiplyed by 8. But if it happen that after the fourth term, or anſwer of the queſtion is found in the ſame denomination with the ſecond term, there is yet a remainder, ſuch remainder muſt be added to the Product of the firſt term, multiplyed by ſuch fourth term, and then the ſum muſt be equal to the Product of the ſecond and

third terms: (the second term consisting of the same denomination with the fourth) so in the last Example the fourth term is 3226, and there happens to be a remainder of 408, which being added to the Product of the multiplication of the said 3226 by the first term 480, gives 1548888, which is the same with the Product of the third term 23468 multiplyed by the second term 66 as will appear by the work.

A compendious operation in the Rule of Three direct, when the first term is 1 or unity

XIII. When the first of the three given numbers in the Rule of three Direct, is 1 or unity the question may oftentimes be answered more speedily than by the Rule of Three, even by those who have but little skill in Arithmetick, as will partly appear by the following Examples, *viz.*

1. At 17 *s.* —— 9 *d.* the yard, what will 84 yards cost? *Answer*, 74 *l* —— 11 *s.* For reason sheweth that 84 yards must (at the said rate) cost 84 Angels, 84 Crowns, 84 half Crowns, and 84 Three-pences, all which being computed and added together, will give the full value of 84 yards, *Viz.*

	l.	*s.*	*d.*
84 Angels make	42	00	00
84 Crowns	21	00	00
84 Half Crowns	10	10	00
84 Three-Pences	1	01	00
Sum	74	11	00

2. At

2. At the rate of 9 *s.* the Bushel of Wheat, what will 51 Quarters amount unto? *Answer*, 183 *l.* —— 12 *s.* —— 0 *d.*

It is evident that the price of 1 Quarter (which consists of 8 Bushels) will be 8 Angels wanting 8 Shillings; therefore,

	l.	*s.*	*d.*
from 8 Angels, to wit ——————	4 ——	00 ——	00
subtract ——————————————	0 ——	08 ——	00
remains the price of 1 Quarter —	3 ——	12 ——	00

Then the value of 51 Quarters at the rate of 3 *l*, — 12 *s.* — 0 *d.* the Quarter may be found in manner following, *Viz.*

	l.	*s.*	*d.*
	51 ——	00 ——	00
51 times 3 *l.* or 3 times 51 *l.* is	51 ——	00 ——	00
	51 ——	00 ——	00
51 Angels make ——————————	25 ——	10 ——	00
51 Shillings doubled make ——	5 ——	02 ——	00
the price of 51 Quarters ———	183 ——	12 ——	00

3. What is a Chest of Sugar worth, that weigheth neat weight (the Tare being subtracted) 7 *C.* 3 *q.* 7 *lb.* at the rate of 6 *l.* —— 3 *s.* —— 4 *d.* for 1 *C*? *Answer*, 48 *l.* —— 3 *s.* 6 *d.* —— 2 *f.*

Tare is that wherein any thing is put as a Bag for Pepper, a Chest for Sugar.

	l.	s.	d.	f.
7 times 6 pounds make	42	00	00	
7 times 3 Shillings	1	01	00	
7 Groats	0	02	04	
The half of 6 *l.*—3 *s.*—4 *d.* for 2 *qu.* is	3	01	08	
The half of 3 *l.*—1 *s.*—8 *d.* for 1 *qu.* is	1	10	10	
The fourth part of 1 *l.*—10 *s.*—10 *d.* (because 7 *l.* is a fourth part of 28 *l* or of 1 *qu.*) is	0	07	08	2
	48	03	06	2

Practical rules of this nature cannot be compleatly understood without some skil in fractions, as will hereafter appear in the second Chapter of the Appendix, and therefore I shall conclude this Chapter with the following Questions, whose Answers are annexed to them, and may be found out by the preceding Rules; but the operations are purposely omitted, and left as an exercise for the Learner.

Questions to exercise the Rule of Three direct.

1. If 17 yards of Cloth cost 19 *l.* 2 *s.* 6 *d.* what will 35 yards cost at that rate? *Answer*, 39 *l.* 7 *s.* 6 *d.*

2. If 35 yards cost 39 *l.* 7 *s.* 6 *d.* how many yards may be bought at that rate for 19 *l.* 2 *s.* 6 *d.* *Answer*, 17 yards.

3. If 35 yards cost 39 *l.* 7 *s.* 6 *d.* what are 17 yards worth at that rate? *Answer*, 19 *l.* 2 *s.* 6 *d.*

4. If 17 yards be sold for 19 *l.* 2 *s.* 6 *d.* how many yards will 39 *l.* 7 *s.* 6 *d.* buy at that rate? *Answ.* 35 yards.

5. What

5. What must I pay for the carriage of 17 hundred weight, 3 quarters, and 11 pounds *Averdupois*, at the rate of 7 shillings the hundred weight, *Answ.* 6*l.*—4 *s.* — 11 *d.* —1 *farth.*

6, If 6 *l.* — 4 *s.* — 11 *d.* — 1 *f.* be pay'd for the carriage of 17 hundred weight, 3 quarters, and 11 pounds, what was payd for the carriage of 1 pound weight? *Answ.* 3 Farthings.

7. What must I pay for 39 ounces, 7 penny weight, and 18 grains of white Plate at the rate of 5 *s*, and 5 *d.* the ounce? *Answ.* 10 *l.* —13 *s.* —4 *d.* and three quarters of a farthing.

8. What must 1 *l.* (or 20 *s.*) pay towards a Tax, when 326 *l.* — 6 *s.* — 8 *d.* is assessed at 41 *l.* —16 *s.* — 2 *d.* — 3 *f.* *Answ.* 2 *s.* — 6 *d.* — 3 *f.*

9. What will the Interest of 876 *l.* — 17 *s.* — 6 *d.* amount unto for 1 year at the rate of 6 *l.* for 100*l.* for the same time? *Answ.* 52 *l.* —12 *s.* —3 *d.*

CHAP. IX.

Of the Inverse Rule of Three.

I. THE Rule of Three Inverse is, when the fourth term required ought to proceed from the second term, according to the same rate or proportion that the first proceeds from the third, so this question being propounded, if 8 Horses will be maintained 12 dayes with a certain quantity of Provender, how many dayes will the same quantity maintain 16 Horses? Here as 8 is half 16, so ought the fourth term required to be half

half 12; for if certain bushels of Provender serve 8 Horses 12 dayes, 16 Horses will eat up as much Provender in half that time; and therefore you cannot say here in a direct proportion (as before in the Rule of Three direct) as 8 to 16, so is 12 to another number which ought to be in that case as great again as 12, but contrariwise by an *inverted Proportion*, beginning with the last term first, as 16 is to 8, so is 12 to another number which ought to be in this case half 12, And by the due observation of this definition, together with that of the Rule of Three direct (propounded in the ninth Rule of the eight Chapter) when any question belonging to the single Rule of Three is propounded, you may readily discern by which of those Rules it ought to be resolved; for if the three terms given look for a fourth in a direct proportion as they stand ranked in the Rule, you must resolve the question by the direct Rule; contrariwise when the proportion is inverted or turned backwards, it ought to be resolved by the Inverse Rule of Three, which here followeth.

horses	*dayes*	*horses*
8 ——	12 ——	16

How to work the Inverse rule of Three.

II. In the Inverse Rule of Three, after the three given terms are rightly placed in the Rule, and reduced (if there be need) according to the eleventh Rule of the eighth Chapter, multiply the first term by the second, or (which is the same) the second term by the first, and then divide the Product by the third term, so the *Quotient* will give you the fourth term required, or answer of the question; thus in the question premised

mised in the last Rule, if you multiply 12 by 8; the Product is 96, which if you divide by 16, the *Quotient* gives you 6, the fourth term required, as by the subsequent operation is manifest.

horses	*dayes*	*horses*	*dayes*
8 ———	12 ———	16 ———	(6
	8		
	16) 96 (6		
	96		
	0		

III. For the more ready discovering, whether a question propounded belongs to the Rule of Three direct, or to the Rule Inverse, observe the directions following, *Viz.* 1 By the sense and tenour of the question, consider whether more be required or less; that is, whether the number sought must be greater or less than the second term: Secondly, esteeming the first and third terms as extreams in respect of the second, this will be a general rule; namely, When more is required, the lesser extreme is the *Divisor*; but when less is required, the greater extreme is the *Divisor*. Lastly, the *Divisor* being found out, it will be apparent whether the Rule be Direct or Inverse, for when the *Divisor* is the first term it is a rule direct; but when the *Divisor* is the third term, the Rule is Inverse. Another Example of the Rule Inverse may be this; If 12 Mowers do mow certain Acres in

How to discern whether a question in the Rule of Three is to be resolved by the Rule Direct, or by the Rule inverse.

in 4 dayes, in what time will 23 Mowers perform the ſame work? *Anſwer*, 2 dayes, 2 hours, and ſomewhat more.

M. *D.* *M.*
12 —— 4 —— 23
4
———
23) 48 (2 *dayes.*
46
———
2
24
———
23) 48 (2 *hours.*
46
———
2

Here, the 3 known terms being rightly placed in the Rule, will ſtand as you ſee in the Example; and ſince it is evident that 23 men will require leſs time than 12 men to finiſh the ſame work, therefore (by the Rule aforegoing) the greater of the two extream numbers 23 and 12 muſt be the *Diviſor*, and becauſe the *Diviſor* 23 ſtands in the third place; this queſtion is to be wrought by the Rule Inverſe, wherefore multiplying the firſt term 12 by the ſecond term 4, the product is 48, which being divided by the firſt term 23, the *Quotient* gives 2 dayes, and there is a remainder of 2 dayes, which being reduced to hours, and thoſe divided by 23, the *Quotient* will be 2 hours, and there is yet a remainder of 2 hours to be ſubdivided into 23 parts if you pleaſe; ſo that the fourth term ſought, or anſwer of the queſtion is 2 dayes, 2 hours,, and ſomewhat more.

Again, take this for a third Example, If I lend my Friend 356 pounds for one year and 35 days (the year being ſuppoſed to conſiſt of 365 days) how long time ought he to lend me 500 pounds to requite my courteſie? *Anſwer*, 284 dayes and ſomewhat more, as by the ſubſequent operation is manifeſt.

l. *y.* *D.* *l.*
356 —— 1 —— 35 —— 500

365
add 35

multiply {400 / 356}

5|00) 1424|00 (284 *days.*

```
5|00) 1424|00 ( 284 days.
      10
      --
       42
       40
       --
        24
        20
        --
        400
```

IV. The proof of the Inverse Rule of Three is this, multiply the third term by the fourth, then if this Product be equal to the Product of the first term multiplyed by the second, the work is true, otherwise erroneous; so in the Example of the second Rule, the Product of 16 and 6 is equal to the Product of 8 and 12. But if it happen that after the fourth term or answer of the question, is found in the same denomination with the second term, there is yet a remainder, such remainder must be added to the Product of the third term multiplyed by the fourth, and then the sum must be equal to the Product of the first and second terms (such second term being of the same particular denomination with the fourth) so in the last Example, the fourth term is 284 days and there is a remainder of 400 after the division was finisht, this 400 being added to the Product

The proof of the Rule of Three Inverse.

of

in 4 dayes, in what time will 23 Mowers perform the same work? *Answer*, 2 dayes, 2 hours, and somewhat more. Here, the 3 known terms being rightly placed in the Rule, will stand as you see in the Example; and since it is evident that 23 men will require less time than 12 men to finish the same work, therefore (by the Rule aforegoing) the greater of the two extream numbers 23 and 12 must be the *Divisor*, and because the *Divisor* 23 stands in the third place; this question is to be wrought by the Rule Inverse, wherefore multiplying the first term 12 by the second term 4, the product is 48, which being divided by the first term 23, the *Quotient* gives 2 dayes, and there is a remainder of 2 dayes, which being reduced to hours, and those divided by 23, the *Quotient* will be 2 hours, and there is yet a remainder of 2 hours to be subdivided into 23 parts if you please; so that the fourth term sought, or answer of the question is 2 dayes, 2 hours,, and somewhat more.

```
M.     D.     M.
12 —— 4 —— 23
 4
————
23 ) 48 (2 dayes.
     46
————
      2
     24
————
23 ) 48 (2 hours.
     46
————
      2
```

Again, take this for a third Example, If I lend my Friend 356 pounds for one year and 35 days (the year being supposed to consist of 365 days) how long time ought he to lend me 500 pounds to requite my courtesie? *Answer*, 284 dayes and somewhat more, as by the subsequent operation is manifest.

356

l. *y.* *D.* *l.*

356 —— 1 —— 35 —— 500

365

add 35

400

multiply {400
356}

5|00) 1424|00 (284 *days.*

10

42

40

24

20

400

IV. The proof of the Inverse Rule of Three is this, multiply the third term by the fourth, then if this Product be equal to the Product of the first term multiplyed by the second, the work is true, otherwise erroneous; so in the Example of the second Rule, the Product of 16 and 6 is equal to the Product of 8 and 12. But if it happen that after the fourth term or answer of the question, is found in the same denomination with the second term, there is yet a remainder, such remainder must be added to the Product of the third term multiplyed by the fourth, and then the sum must be equal to the Product of the first and second terms (such second term being of the same particular denomination with the fourth) so in the last Example, the fourth term is 284 days and there is a remainder of 400 after the division was finisht, this 400 being added to the Product

The proof of the Rule of Three Inverse.

of

of the Multiplication of the third term 500 by the fourth term 284 gives 142400, which is equal to to the Product of the first term 356 multiplyed by the second term 400 dayes.

CHAP. X.

The double Golden Rule Direct, performed by two single Rules.

I. THE Compound Golden Rule is, when more than three terms are propounded.

II Under the Compound Golden Rule is comprehended the double Golden Rule, and divers Rules of plural proportion.

The double Golden Rule.

III. The double Golden Rule is, when five terms being propounded, a sixth proportional unto them is demanded: as in this question; If 4 Students spend 19 pounds in 3 moneths, how much will serve 8 Students 9 moneths? Or this, if 9 Bushels of Provender serve 8 Horses 12 dayes, how many dayes will 24 Bushels last 16 Horses?

The parts into which the terms of the same rule are distributed.

IV. The five terms given in this Rule consist of two parts, *Viz.* A supposition expressed in the three first terms; and a demand propounded in the two last: So in the first Example of the last Rule, this clause (If 4 Students spend 19 pounds in 3 moneths) is the supposition, and this (how much will serve 8 Students nine moneths)

moneths) is the demand : likewise in the other Example of the same Rule, this clause (If nine Bushels of Provender serve 8 Horses 12 dayes) is the supposition, and this (How long, or how many dayes will 24 Bushels last 16 Horses) is the demand propounded.

The right ordering of the terms.

V. Here for ranking the terms propounded in their due order, first observe amongst the terms of supposition, which of them hath the same denomination with the terme required, then reserving that term for the second place, write the other two terms of supposition one above another in the first place; and lastly the terms of demand likewise one above another in the third place of the Rule, in such sort that the uppermost may have the same denomination with the uppermost of those in the first place: Example, if 4 Students spend 19 pounds in 3 moneths, how much will serve 8 Students 9 moneths? Here the three terms of supposition are 4, 19, and 3, and of these terms 19 hath the same denomination with the term required, *viz.* of Pounds, for you are to enquire how much Money is requisite for the maintenance of 8 Students 9 moneths, wherefore reserving 19 for the second place, I write 4 and 3 one above another thus; then drawing a line upon the right hand of 4, I write 19 in the second place; this done, the Work will stand as in the Margent. Last of all, the terms of demand being 8 and 9, and 8 having the denomination of Students, I place it in the same line with 4 and 19, and write 9 under

4
3

4——19
3

under it ; all this performed, the terms in this question rank themselves as followeth :

Viz. *thus*,

4	19	8
3		9

Or thus,

3	19	9
4		8

The right ordering of the terms.

In like manner, if the second question of the third Rule of this Chapter were propounded, the terms thereof ought to be disposed

Thus,

8	12	16
9		24

Or thus,

9	12	24
8		16

VI. Questions belonging to the double Golden Rule may be resolved by two single Rules of Three, or by the Golden Rule Compound of five Numbers.

The Proportions of the double Golden Rule, when it is performed by two single Rules.

VII. When Questions of this nature are resolved by two single rules, the proportions are as followeth :

I. As the uppermost term of the first place, is to the middle term ; So is the uppermost term of the last place to a fourth Number.

II. As

VI. As the lower term of the first place is to that fourth Number; so is the lower term of the last place to the term required.

So in this Example before recited: using tacitly the lower term of the first place as a common number in the first proportion, say *thus*,

4	19	8
3		9

I. If 4 Students spend 19 pounds (in three moneths) what will serve 8 Students (the same time?)

Or thus, If 4 Students spend 19 pounds, what will 8 spend?

Which Rule of Three will be discovered to be direct (by the third Rule of the ninth Chapter) therefore the fourth proportional proceeding from the said three given numbers 4, 19, & 8 is 38 (by the 10th Rule of the 8th Chapter aforegoing.) Again, to find the term required, using tacitly the uppermost term of the third place as a common Number in this last proportion, *say as followeth*,

II. If in three moneth 38 pounds are spent (by 8 Students) how much will serve them for 9 moneths?

Or thus, If 3 give 38, what will 9 yield you?

Which rule of Three will likewise be discovered to be direct (by the third rule of the ninth Chapter) therefore the fourth proportional proceeding from the said 3 numbers, 3, 38, and 9, you shall likewise find (by the 10th Rule of the 8th Chapter before recited) to be 114, for 38 being multiplyed by 9, the Product is 342, which divided by 3 yields you in the *Quotient* 114: So that I conclude, If four Students spend nineteen pounds in three moneths, 114 pounds will serve 8 Students

G

dents 9 moneths; as you may further observe by the Work following:

```
        4————19————8
        3              9——(114
4—19—8—( 38          | 3—38—9—( 114
      8              |      9
4) 152(38            | 3 ) 342 (114
   12                |     3
   32                |     04
   32                |     3
   0                 |     12
                     |     12
                     |     0
```

In like manner if two single Rules of Three be formed (according to the preceding 7th Rule) out of the five numbers given in the last mentioned question, the same being ranked according to the latter manner of ordering the said numbers in the fifth Rule, each of the said two Rules of Three will be a Rule direct, and the same answer of the question, to wit, 114 pounds will be discovered as you may see by the subsequent operation.

```
        3————19————9
        4              8—(114
3—19—9—( 57          | 4—57—8—( 114
     9               |      8
3) 171 (57           | 4) 456 ( 114
   15                |    4
   21                |    05
   21                |    4
   0                 |    16
                     |    16
                     |    0
```

The

VIII. The double Golden Rule is either Direct or Inverse.

IX. The double Golden Rule Direct is, when both the single rules do each of them look for a fourth term in a direct proportion: As in the Example of the seventh Rule, where each of the two single rules of Three is a Rule direct. *The double Golden Rule direct.*

For another Example take this, if the carriage of 8 C. weight 128 miles, cost 48 shillings, for how much may I have 4 C. weight carried 32 miles after the same rate? The terms of this question according to the fifth rule of this Chapter, rank themselves in *this order*:

128———48———32
8 4

Now taking tacitly the lower term of the first place, as a common number, I form the first Rule of Three according to the seventh rule, *saying*.

I. If the carriage of a certain weight (to wit, 8 C.) 128 miles will cost 48 shillings, what will the carriage of the same weight 32 miles cost?

Here it is easie to discern that the fewer miles any weight is carried, the less money will pay for the carriage of that weight, therefore the fourth number sought by the said rule of three must be less than the second number 48: And forasmuch as by the third rule of the ninth Chapter, when less is required, the greater extreme (whether it be the first or third number) must be the Divisor, therefore the first number 128 is the Divisor, and consequently the rule of three above propounded is a rule direct, wherefore finding out the fourth num-

ber by the tenth rule of the eighth Chapter, to be 12 shillings, I proceed to the second proportion, and say:

II. If the carriage of 8 C. (32 miles) cost 12 shillings, how much must I give to have 4 C. carried the same distance?

And here likewise finding a fourth number to be looked for in a direct proportion, I discover that fourth, by the said tenth rule of the eighth Chapter, to be 6 *s.* which is the term demanded, and the answer to the question propounded: so that at last I conclude, If the carriage of 8 C. 128 miles cost 48 *s.* the carriage of 4 C. 32 miles will cost 6 *s.* according to the same rate; see the whole work.

128 —— 48 —— 32
8 4 —— (6

---

128 —— 48 —— 32 —— (12
32
96
144
128) 1536 (12
128
256
256
0

---

8 —— 12 —— 4 —— (6
4
8) 48 (6
48
0

CHAP.

CHAP. XI.

The Double Golden Rule Inverse, performed by two single Rules.

THE Double Golden Rule Inverse is, when one of the single rules looks for a fourth term in an inverted proportion: As in the last Example propounded in the fifth rule of the last Chapter. For if you rank the the terms of that question, according to the said fifth rule, *thus.* *The double Golden Rule Inverse.*

8 —— 12 —— 16
9   24

And then work by two single rules of Three, formed according to the seventh rule of the last Chapter, you shall find by the third rule of the ninth Chapter, that the first of the said two rules of Three will be Inverse, and the latter direct; for saying first, if 8 horses be maintained 12 daies (by 9 bushels of provender) how many daies will 16 horses be kept by so much provender? Here the answer 6 daies, will be found out by the rule of three Inverse: Secondly, saying, if 9 bushels of provender be eaten up (by 16 horses) in 6 daies, in how many daies will 24 bushels be spent: here the answer 16 daies will be found out by the rule of Three direct.

But if you order the given terms of the same question, *thus*,

9————12————24
8   16

And then work by two single rules of Three formed according to the seventh Rule of the last Chapter you shall find by the third rule of the ninth Chapter, that the first of the said two rules of Three will be direct, and the latter inverse, for saying first, If 9 bushels of provender will last 12 daies (to maintain 8 horses) how many daies, will 24 bushels serve the same number of horses, The answer 32 daies will be found out by the rule of Three direct; secondly, saying, If 8 horses will be maintained 32 daies (by 24 bushels of provender) how long will 16 horses be kept by the same quantity of provender? Here the answer 16 daies will be found out by the Rule of Three direct.

Wherefore, whensoever a question belonging to the double rule of Three is severed into two single rules of Three, (according to the preceding rules) if one of them happens to be a rule inverse, that double rule is called the double Rule inverse.

Now the Resolution of the Question propounded being ranked after the first manner, is as followeth.

8——12——16
9 24—(16

8——12——16—(6
8

16) 96 (6
96

0

9——6——24—(16
6

9) 144 (16
9

54
54

0

Again, The Resolution of the same Question, being ranked after the last manner, *is this* ;

```
9 ——— 12 ——— 24
8         16 —— (16

9 ——— 12 ——— 24 —— (32
           12
        8
           48
           24
           96
      9)  288  (32
           27
           18
           18
            0

8 ——— 32 ——— 16 —— (16
       8
  16) 256  (16
       16
        0
       96
       96
```

So that at last I say, If 9 Bushels of Provender serve 8 Horses 12 dayes, 24 Bushels will last 16 Horses 16 dayes, which is the resolution of the Question propounded.

CHAP.

CHAP. XII.

The Golden Rule compounded of five Numbers.

I. THE Golden rule compound of five numbers is, when the terms being ranked, as before, instead of the double terms we use their products, and then proceed to find the term required by one single Rule of Three.

The Golden Rule compound of five numbers performed by one single Rule direct.

II. Here when the Question propounded ought to be performed by the double rule direct, multiplying the terms of the first place, the one by the other, take their product for the first term, the middle number for the second, and the product of the two last terms for the third term; this done, having found by the rule of three direct, a fourth proportional unto those three, that fourth term so found is the number you look for: so this question being again propounded, if 4 Students spend 19 *l.* in 3 moneths, how much will serve 8 Students 9 moneths: and the terms thereof being ranked as before, *viz. thus*

$$\begin{array}{ccc} 4 & \text{———}\ 19\ \text{———} & 8 \\ 3 & & 9 \end{array}$$

The product of 4 multiplied by 3 is 12, and the product of 8 multiplied by 9 is 72; wherefore *I say*, As 12 to 19, so 72 to the term required, which I find by the single rule of Three direct to be 114.

So

So that if 4 Students ſpend 19 *l.* in three moneths, 114 *l.* will be requiſite for the maintenance of 8 Students 9 moneths, ſee the whole operation, *as followeth,*

```
    4———19————8
    3          9—(114
  ————      ————
   12        72
             19
           ————
            648
             72
           ————
       12) 1368 (114
             12
           ————
             16
             12
           ————
             48
             48
           ————
              0
```

In like manner this being the Queſtion as before (in the laſt rule of the tenth Chapter) if the carriage of 8 *C.* 128 miles, coſt 48 *s.* what will the carriage of 4 *C.* 32 miles ſtand me in ? the Anſwer thereunto will be 6 *s.* as appeares by the work,

128—

128	48	32
8	128	4
1024	384	128
	96	
	48	
1024)	6144 (6 *Shillings*	
	6144	
	0	

III. When the Question propounded ought to be resolved by the double rule Inverse, having multiplyed the double terms a cross, that is, the uppermost term of the first place by the lower of the last, and the uppermost of the last place by the lower of the first, write each product under the lower term by which it is produced, and then if the inverse proportion be found in the uppermost line, using those products as single terms, proceed to find the term required by the single Rule of Three direct: But in case you find the Inverse proportion in the lower line, perform the work by the single rule of Three Inverse.

The Golden Rule compound of five Numbers performed by one single Rule direct or inverse.

So in the Example above mentioned, if 9 bushels of provender serve 8 horses 12 daies, how long will 24 bushels last 16 horses? Here if you rank the terms *thus*, you shall find the Inverse proportion in the first line, as is observed in the last Chapter: And therefore having subscribed the

8	12	16
9		24

products

products according to the direction given you in this rule, I proceed to satisfie the demand of this question by the single rule of Three direct, as appears by the work following.

```
8———12———16
9          24—(16
———        ————
144        192
            12
          ————
           384
           192
          ————
    144) 2304 (16
          144
          ———
           864
           864
          ————
             0
```

But the terms of this Question being ranked *thus*, the Inverse proportion is found in the lower line, as you may observe likewise by the last Chapter: whereupon in this case to resolve the Question, I proceed by the single rule of Three Inverse, as appeares by the work hereunto annexed: howsoever therefore you work the Question, you shall find the term required to be 16; so that at last I conclude, as before in the last Chapter: If 9 bushels of provender serve 8 horses 12 daies, 24 bushels will last 16 horses 16 dayes.

9———12———24
8   16

```
9 —— 12 —— 24
8          16 —— (16
———        ———
192        144
12
———
384
192
———
144) 2304 (16
     144
     ———
     864
     864
     ———
       0
```

CHAP. XIII.

The Rule of Fellowship.

THE rules of plural proportion are those, by which we resolve Questions, that are discoverable by more golden rules than one, and yet cannot be performed by the double golden rule mentioned before in the three last Chapters. Of these rules there are divers kinds and varieties according to the nature of the question propounded, for here the terms given are sometimes four, sometimes five, sometimes more, & the terms required sometimes more than one, &c.

Rules of plural proportion.

II.

II. Two particular rules of plural proportion are these, the Rule of Fellowship, and the Rule of Alligation.

III. The Rule of Fellowship is that, by which in accompts amongst divers men (their several stocks together with the whole gain or loss being propounded) the gain or loss of each particular man may be discovered: As in this Example, *A* and *B* were sharers in a parcel of Merchandize, in the purchase of which *A* layd out 7 *l.* and *B* 11 *l.* and they having sold this Commodity, find that their clear gains amounts to 54 *s.* Now here the Question to be resolved by this Rule is, what part of that 54 *s.* accrews to *A*, and what to *B*, according to the rate of the several sums or stocks which they adventured? Again, *A*, *B*, and *C*, fraight a Ship from the *Canaries* for *England*, with 108 Tuns of Wine, of which *A* had 48, *B* 36, and *C* 24, the Mariners meeting with a storm at Sea, were constrained for the safety of their lives, to cast 45 Tun thereof over-board; here the Question to be resolved is, How many of the 45 Tun each particular Merchant hath lost, according to the rate of his Adventure?

The Rule of Fellowship.

IV. The Rule of Fellowship is either single or double.

V. The single Rule is, when the stocks propounded do continue in the Adventure (or common Bank) equal times, to wit, one stock as long time as another.

How to work the single Rule.

VI. In the single Rule of Fellowship, take the total of all the stocks for the first term, the whole gain or loss, for

for the ſecond, and the particular ſtocks for the third terms; this done, repeating the Rule of Three ſo often, as there are particular ſtocks in the Queſtion, the fourth terms produced upon thoſe ſeveral operations, are the reſpective gains or loſſes of thoſe particular ſtocks propounded: So in the firſt Example above mentioned 7 *l.* and 11 *l.* are the ſtocks propounded, whoſe total is 18 *l.* which I take for the firſt term: Again, 54 *s.* the common gain, is the ſecond term, and 7 *l.* the firſt particular ſtock, is the third term of the firſt proportion; whereupon I ſay, as 18 *l.* to 54 *s.* ſo 7 *l.* to another number, which by the direct Rule of Three I find to be 21 *s.* *viz.* the part of the gain due to *A*, that expended the 7 *l.* ſtock. Then for the ſecond proportion, I ſay, as 18 *l.* to 54 *s.* ſo 11 *l.* to another number, which I likewiſe find by the Rule of Three direct to be 33 *s.* *viz.* the part of the gain due to *B*, for his 11 *l.* ſtock,

$$\left.\begin{matrix}7\\11\end{matrix}\right\}18\text{———}54\text{———}\left\{\begin{matrix}7\text{—}21\\11\text{—}33\end{matrix}\right.$$

Again, in the other premiſed Example, the particular loſs that happens to *A*, is 20 Tun, to *B* 15 and to *C* 10 Tun,

$$\left.\begin{matrix}48\\36\\24\end{matrix}\right\}108\text{———}45\text{———}\left\{\begin{matrix}48\text{—}20\\36\text{—}15\\24\text{—}10\end{matrix}\right.$$

VII. The double Rule of Fellowſhip is, when the ſtocks propounded are double numbers, *viz.* when each ſtock   2. *Double.*

hath

hath relation to a particular time; Example, *A*, *B*, and *C*, hold a pasture in common, for which they pay 45 *l. per annum*. In this pasture *A* had 24 Oxen went 32 dayes, *B* had 12 there 48 dayes, and *C* fed 16 Oxen there 24 dayes; now the Question to be resolved by this Rule is, what part each of these Tennants ought to pay of the 45 *li.* rent? and here you may *observe*, that the stocks propounded are *double* numbers, and each stock of Oxen hath reference to a *particular* time, for the respective stock of *A* is 24 Oxen, and its *particular* time is 32 dayes; again, the stock of *B* is 12 Oxen, and the *respective* time is 48 dayes; and lastly, the stock of *C* is 16 Oxen, and its *peculiar* time is 24 dayes, which as you see are *double* numbers.

VIII. In the double Rule of Fellowship, multiply each particular stock by its respective time, and take the total of their Products for the first term, the whole gain or loss for the second, and the said particular Products of the double numbers for the third term: This done, repeating, as before, the Rule of Three, so often as there are Products of the double numbers; the fourth terms produced upon those several operations, are the numbers you look for: So in the *Example* of the last Rule, the Product of 24 and 32, is 768, the Product of 12 and 48, is 576, and the Product of 16 and 24, is 384, the sum of these Products is 1728, which is the first term in the Question, then 45 *l.* the rent, is the second term, and 768 the first Product, is the third term of the first proportion. Wherefore I say, as 1728 to 45 *l.* so 768 to another number, which I find by the di-

How to work the double Rule.

rect

rect rule of *Three* to be 20 *l. viz.* the part of the rent that *A* ought to pay: Then for the second proportion *I say*, as 1728 to 45 *l.* so 576 to 15 *l.* which is the part that *B* ought to *pay*: And lastly, as 1728 to 45 *l.* so 384 to 10 *l. viz.* the part that *C* must pay.

$$\left.\begin{matrix}768\\576\\384\end{matrix}\right\} 1728 \text{——} 45 \text{——} \left\{\begin{matrix}768 \text{——} 20\\576 \text{——} 15\\384 \text{——} 10\end{matrix}\right.$$

A second Example of the eight Rule, Three Merchants, *A*, *B*, and *C* enter Partnership, and agree to continue in a joynt Adventure 16 moneths; *A* puts into the common stock at the beginning of the said term 100 pounds, at 8 moneths end he takes out 40 pounds, and 4 moneths after such taking out he puts in 140 pounds. *B*. puts in at first 200 pounds, at 6 moneths end he puts in 50 pounds more, and 4 moneths after the putting in of the 50 pounds, he takes out 100 pounds. *C* puts in at first 150 pounds, at four moneths end he takes out 50 pounds, and 8 moneths after such taking out puts in 100 pounds. Now at the end of the said 16 moneths they had gained 357 pounds, the Question is how much of the said gain belongs to each Merchant for his share.

In Questions of this nature, two things are principally to be observed. 1. The whole time of partnership. 2 The respective time belonging to each mans particular stock; so here, it is evident that the whole time is 16 moneths, and the particular stocks and times belonging to each Merchant will be as followeth, *viz.*

A had 100 *l.* in the common ſtock for 8 moneths, therefore 100 multiplied by 8 produceth	800
Alſo 60 *l.* for 4 moneths, therefore 60 multiplied by 4 produceth	240
Alſo 200 *l.* For 4 moneths, therefore 200 multiplyed by 4 produceth	800
The total of the products of money and time for *A*, is	1840
B had 200 *l.* in the common ſtock for 6 moneths, therefore 200 multiplied by 6 produceth	1200
Alſo 250*l* for 4 moneths, therefore 250 multiplied by 4 produceth	1000
Alſo 150*l.* for 6 moneths, therefore 150 multiplied by 6 produceth	900
The total of the products of money and time for *B*, is	3100
C had 150 *l.* in the common ſtock for 4 moneths, therefore 150 multiplied by 4 produceth	600
Alſo 100 *l.* for 8 moneths, therefore 100 multiplied by 8 produceth	800
Alſo 200 *l.* for 4 moneths, therefore 200 multiplied by 4 produceth	800
The total of the products of money and time for *C*, is	2200

Then adding the ſaid three totals together, to wit, 1840, 3100 & 2200, the ſum is 7140, wherefore proceeding as in the laſt Example, I ſay by the rule of three direct, as 7140 is to the total gain 357 pounds;

pounds; so is 1840 to 92 pounds the gain of *A*: again, As 7140 is to 357; so is 3100 to 155 the gain of *B*: Lastly, as 7140 is to 357; so is 2200, to 110 the gain of *C*.

IX. The rule of fellowship is proved by Addition of the terms required, whose sum ought to be equal to the second term in the Question, otherwise the whole work is erroneous: so in the first *example* of the sixth Rule aforegoing, 21 *s.* and 33 *s.* being added together are equal to 54 *s.* the *second term* in that Question: likewise in the last *Example* of the same rule, as also in the first *Example* of the last rule, the *sum* of 20, 15, and 10 the terms required are equal to 45, the *second term* propounded. *The proof.*

CHAP. XIV

The Rule of Alligation.

I. THE Rule of Alligation is that, by which we resolve Questions, that concern the mixing of divers simples together.

II. Alligation is either Medial or Alternate.

III. Alligation Medial is when having the several quantities and rates of divers simples propounded, we discover the mean rate of a mixture compounded of those simples. *Alligation Medial.* So 10 bushels of wheat at 4 *s.* or (which is all one) 48 *d.* the bushel; 40 bushels of rye at 3 *s.* or 36 *d.* the bushel; and 50 bushels of barley at 2 *s.* or 24 *d.* the bushel; being mixed

 with

with 20 bushels of oats at 12 *d.* the bushel, the rule of *Alligation medial* sheweth you the mean price of that mistling.

The operations and proportions of the same Rule.

IV. In Alligation medial, first sum the given quantities, then find the total value of all the simples: this done, the proportion will be as followeth.

As the sum of the quantities is to the total value of the simples:

So is any part of the mixture propounded to the required mean rate or price of that part.

Repeating again the premised example of the third rule; I demand how much one bushel of that mistling is worth? Now the sum of 10, 40, 50, 20, (the given quantities) is 120 bushels. and the value of the 10 bushels of wheat at 48 *d.* the bushel, amounts to 480 *d.* for 48 being multiplied by 10, the product is 480: again the value of the 40 bushels of rye at 36 *d,* the bushel, is 1440 *d.* The value of the 50 bushels of barley at 24 *d.* the bushel, is 1200*d.* And the value of 20 bushels of oats at 12*d* the bushel is 240 *d.* All these values being added together, their total is 3360 *d.* I say then by the rule of *Three Direct,* if 120 bushels give 3360 *d.* what will 1 bushel yeild? The Rule presently answers me 28 *d.* whereupon I conclude, that a bushel of that mistling may be afforded for 28 *d,* that is, 2 *s.* 4 *d.* which is the resolution of the *question propounded.*

120 —— 3360 —— 1 —— 28

In

In like manner if it be demanded what 8 Bushels or a Quarter of that Mistling is worth? The *Answer* will be 224 *d.* which being divided by 12, and by that means reduced into *shillings*, is 18 *s.* 8 *d.*

120 —— 3360 —— 8 —— 224 .

V. In *Alligation Medial*, the tryal of the work is by comparing the total value of the several simples with the value of the whole mixture: For when those sums accord, the operation is perfect; so in the first Example of the last Rule.

The Proof.

The Value of		l.	s.	d.
	10 Bushels of Wheat at 4 *s.* the Bushel is	2	0	0
	40 Bushels of Rye at 3 *s.* the Bushel is	6	0	0
	50 Bushels of Barley at 2 *s.* the Bushel is	5	0	0
	And 20 Bushels of Oats at 12 *d.* the Bushel is	1	0	0
	All which amount to	14	0	0

which is likewise the value of 120 Bushels at 28 *d.* or 2 *s.* 4 *d.* the Bushel, for that also amounts to 14 *l.*

VI. Alligation Alternate is, when having the several rates of divers Simples given, we discover such quantities of them, as are necessary to make a mixture, which may bear a certain rate propounded.

Alligation Alternate.

Example: A man being determined to mix 10 Bushels of Wheat at 4 *s.* or 48 *d.* the Bushel, with

Rye of 3 *s.* or 36 *d.* the Buſhel, with Barley of 2 *s.* or 24 *d.* the Buſhel, and with Oats of 1 *s.* or 12 *d.* the Buſhel, the Rule of *Alligation Alternate* will diſcover unto you how much Rye, how much Barley, and how much Oats he ought to add unto the 10 Buſhels of Wheat; in ſuch ſort that the mixture of them altogether may bear a certain rate or price propounded.

The right ordering of the Terms.

VII. In Queſtions of *Alligation Alternate*, you muſt rank the terms in ſuch ſort, that the given rate of the mixture may repreſent the root, and the ſeveral rates of the Simples may ſtand as branches iſſuing from that root: So the Example of the laſt Rule being propounded, I demand how much Rye, Barley, and Oats, ought to be added to the 10 Buſhels of Wheat, that the mixture of all together may bear the rate or price of 28 *d.* or 2 *s.* 4 *d.* the Buſhel: And therefore drawing a line of connexion, I place 28 *d.* the given rate of the mixture, upon the left hand thereof by it ſelf repreſenting the *Root*, and likewiſe write the other rates propounded, *viz.* 48 *d.* 36 *d.* 24 *d.* and 12 *d.* one above another upon the right hand of that line of Connexion, which rates are conceived to iſſue from 28 *d.* as branches from the *Root*, the fabrick hereof appears plainly in the Margent.

$$28 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right.$$

How to couple the Terms.

VIII. Having ranked the terms in their due order, link the branches together by certain Arks, in ſuch ſort, that one that is greater than the Root or rate of the mixture, may alwayes be coupled with another

ther that is leſs than the ſame : So in the premiſed Example, 48 may be linked with 12, and 36 with 24, or otherwiſe 48 may be coupled with 24, and 36 with 12, and then the Work will ſtand

$$\textit{Thus, } 28\begin{cases}48\\36\\24\\12\end{cases} \qquad \textit{Or thus, } 28\begin{cases}48\\36\\24\\12\end{cases}$$

How to order the differences.

IX. Having alligated the branches, and found the differences betwixt them and the Root, write the differences of each branch juſt againſt his reſpective yoke-fellow. So the branches of the Example aforegoing being linked after the firſt manner, and the difference between 28 and 48 (by the third or fourth Rule of the fourth Chapter of this Book) being 20, I place 20 juſt againſt 12, the reſpective yoke-fellow of 48. Again, 16 being the *difference* betwixt 28 and 12, I write it juſt againſt 48. In like manner 8 being the *difference* between 28 and 36, I place it right againſt 24. And laſtly, 4 the *difference* betwixt 28 and 24, I write juſt againſt 36 : In the end the whole *Fabrick* of the Work (as the branches are thus linked) will ſtand as in the Example.

$$28\begin{cases}48\\36\\24\\12\end{cases}\left|\begin{array}{r}16\\4\\8\\20\end{array}\right.$$

 But

But the branches being linked after the other manner, the Work will be thus diſpoſed ;

28	48	4
	36	16
	24	20
	12	8

For in this caſe 48 hath 24 for his yoke-fellow, and the reſpective *Comerado* of 36 is 12 ; and here the interchangeable placing of the *differences* (as in the premiſed Examples) is that which is more particularly termed *Alternation.*

X. When one branch is linked to divers other branches, and not to one alone, the differences ought to be as often tranſcribed, as it is ſo diverſly linked. So in the premiſed Example, you may (if you pleaſe) conceive 12 to be coupled both with 48 and 36 ; likewiſe 24 may be conceived to be linked with the ſame 48, and 36 ; wherefore the *difference* betwixt 28 and 12 being 16, I write it both juſt againſt 48 and 36 : In like manner the *difference* between 28 and 24, being 4, I write it likewiſe over againſt the ſame numbers 48 and 36. Again, 20 being the *difference* betwixt 28 and 48, I place it juſt againſt 24 and 12 ; and 8 being the *difference* between 28 and 36, I write it likewiſe over againſt the ſame numbers 24 and 12 : All this performed, the whole frame of the Work will ſtand as in the *Margent.*

28	48	16. 4
	36	16. 4
	24	20. 8
	12	20. 8

2. Take this for another Example : It is requi-

red

red to mix 10 Bushels of Wheat at 48 *d.* the Bushel with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, and with Oats of 12 *d.* the Bushel, and the Question now is, How much Rye, Barley, and Oats ought to be added to the 10 Bushels of Wheat, that the entire mixture may be afforded at 16 *d.* the Bushel? Here the branches of this Question (according to the eighth Rule of this Chapter) ought to be linked *thus*,

$$16\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right.$$

And as for the *Alternation* of the differences, it is evident (by the present Rule) that the difference between 16 and 12 being 4, ought to be thrice transcribed, *viz.* first just against 48, then against 36, and last of all against 24. Again, 32 the difference betwixt 16 and 48, as also 20 the difference between 16 and 36; and lastly, 8 the difference betwixt 16 and 24, ought all to be placed just against 12.

$$16\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right. \left|\begin{array}{l}4\\4\\4\\32.\ 20.\ 8.\end{array}\right.$$

3. I determining to mix 10 Bushels of Wheat at 48 *d.* the Bushel, with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, and with Oats of

of 12 *d.* the Bushel, desire to know how much of each I ought to take, that I might afford the whole mixture at 40 *d.* the Bushel: Here the whole work being ordered according to the Rules aforegoing, it will stand as *followeth.*

40	48	4. 16. 28.
	36	8.
	24	8.
	12	8.

4. A man intending to mix 10 Bushels of Wheat at 48 *d.* the Bushel, with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, with Pease of 16 *d.* the Bushel, and with Oats of 12 *d.* the Bushel, desires to know how much Rye, Barley, Pease, and Oats he ought to add to the 10 Bushels of Wheat, that the whole mass of Corn so mixed might be afforded at 20 *d.* the Bushel? This *Question* being thus propounded, the terms thereof (by the Rules aforegoing) may be *Alligated*, and the differences of the terms *Alternated*, as *followeth.*

20	48	4.
	36	4.
	24	4. 8.
	16	28. 16. 4.
	12	4.

5. Lastly. A Goldsmith hath some Gold of 24 *Carects*, other of 21 *Carects*, and other some of 19 *Carects fine*, which he would so mix with *Alloy*, that 192 Ounces of the intire *mixture* might bear

17

17 *Carects fine*; now the *Question* is how much of each sort, as also how much *Alloy* he must take to accomplish his desire? Before you can well understand this *Question*, it will be necessary to explain what a *Carect fine*, and what *Alloy* is: the Mint-Masters and Goldsmiths to distinguish the different *fineness* of Gold, esteem an intire ounce to contain 24 *Carects* and one ounce of Gold that being tried in the fire loseth nothing of the weight, is said to be 24 *Carects fine*: again, the ounce that being tried loseth one four and twentieth part of the weight, is said to be 23 *Carects fine*: In like manner that which loseth two four and twentieth parts of the ounce, is esteemed to be 22 *Carects fine*, and so consequently of the rest: And as for *Alloy* it is silver, copper, or some other baser mettal, with which the Goldsmiths use to mix their Gold, to the intent they may moderate, or abate the *fineness* thereof. Here you may also observe, that as the *fineness* of *Gold* is measured by *Carects*, so is the *fineness* of *silver* estimated by *Ounces*: In such sort that a *pound* of *silver*, which being tried a certain time in the fire, loseth nothing of the *weight*, is said to be 12 *Ounces fine*. But a *pound* that being tried loseth somewhat of the *weight*, is said to be the *remainder* of the *weight fine*. *Example*; a *pound* of *Silver* that loseth in the fire one *ounce* 8 *p.* is estimated to be 10 ounces 12 *p. fine*, and that which loseth 2 *ounces* 8 *p.* 10 *Grains*, is said to be 9 *ounces*, 11 *p.* 14 *grains fine*, &c. Now to rank the terms of the last mentioned *Question*, as also the differences of the terms in their due order, because the three given branches (*viz.* 24

What a Carect fine, and what Alloy is.

Carects

Carects, 21 *Carects*, and 19 *Carects*) are all greater than 17 *Carects* the *root* or rate of the *mixture*. I adde o as another branch which I conceive to be less than the *root*, and then proceed as in the former operations; the whole *frame* of the work is expressed here, as followeth:

17	24	17
	21	17
	19	17
	0	7. 4. 2.

XI. When in one and the same line there are found more differences than one, add them together, and write the sum just against the same differences before a straight line drawn towards the right hand of the Work.

How to adde the differences.

So the first Example of the last Rule being propounded, the sum of 16 and 4 (the differences placed just against the first branch) being 20, I write it over against the same differences before the new line drawn upon the right hand of the Work, and so consequently the rest in their due order, as appears by the Example hereunto annexed.

28	48	16. 4.	20
	36	16. 4.	20
	24	20. 8.	28
	12	20. 8.	28

In

In like manner the last Example of the last Rule being offered, the whole Fabrick of the Work will stand, as followeth:

17	24	17	17
	21	17	17
	19	17	17
	0	7.4.2.	13

XII. Alligation Alternate is, either Partial, or Total.

XIII. Alternation Partial is, when having the several rates of divers Simples, and the quantity of one of them given, we discover the several quantities of the rest, in such sort that a mixture of those Simples being made according to the quantity given, and the quantities so found, that mixture may bear a certain rate propounded: Of this kind is the Example of the sixth Rule, as also all the Examples of the tenth Rule, except the last.

Alternation Partial.

XIV. In Questions of Alternation Partial, the proportion is as followeth.

The proportions used in this Rule.

As the difference annexed to the first branch is to the several differences of the rest:

So is the quantity propounded to the several quantities required.

So the Example of the sixth and seventh Rules of this Chapter being again repeated, and the terms thereof, as also the differences of the terms being ordered after the first manner (shewed you in the ninth Rule aforegoing) it is evident that

for

The first Case.

$$28\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right. \quad \left|\begin{array}{l}16\\4\\8\\20\end{array}\right.$$

for every 16 Bushels of Wheat that I take in the mixture, I ought to take 4 Bushels of Rye, 8 Bushels of Barley, and 20 Bushels of Oats; and therefore *I say*,

I. As 16 the difference annexed to the first branch (being the rate of the Wheat) is to 4 the difference annexed to the next, being the rate of the Rye; so is 10 the given quantity of the Wheat to another number, which being found by the Rule of *Three direct*, to be two Bushels and an half (or two pecks) is the quantity of Rie necessary in the *mixture*.

II. As 16 to 8, so is 10 to another Number, which being likewise found by the Rule of *Three* to be five Bushels, is the quantity of Barley, necessary in the *mixture*.

III. As 16 to 20, so is 10 to another number, which being in like sort found by the Rule of *Three* to be 12 Bushels, and half of a Bushel is the quantity of Oats requisite in the *mixture*.

So that at last I conclude, a heap of Corn being composed of 10 Bushels of Wheat, 2 Bushels and a half of Rye, 5 Bushels of Barley, and 12 Bushels and an halfe of Oats (when those several Grains bear the *prices* aforesaid) may be afforded at 2 *s.* 4 *d.* the Bushel.

2. Case. The same Example being ordered after the second manner (expressed likewise in the 9th rule of this present Chapter (*I say*;

1. As

I. As 4 the *difference* annexed to the *rate* of the wheat, is to 16 the *difference* annexed to the *rate* of the rye; ſo is 10 the *given* quantity of the wheat, to 40 buſhels the *required* quantity of the rye.

II. As 4 to 20, ſo is 10 to 50 buſhels, the *requiſite* quantity of the barley.

III. As 4 to 8, ſo is 10 to 20 buſhels, the quan of the oats *neceſſary* in the mixture.

$$28\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right. \qquad \begin{array}{|l}4\\16\\20\\8\end{array}$$

So that I conclude *again*, a maſs of Corn being compounded of 10 buſhels of wheat, 40 buſhels of rye, 50 buſhels of barley, and 20 buſhels of oats, (when thoſe grains bear the prices propounded in this example) may be afforded at 2 *s.* 4 *d.* the buſhel as before.

3. That example being diſpoſed after the third manner (expreſſed in the tenth and eleventh rules of this Chapter) I ſay, 3. *Caſe.*

I. As 20 the *ſum* of the differences annexed to the *rate* of the wheat, is to 20 the ſum of the differences annexed to the rate of the rye; ſo is 10 the *given* quantity of the wheat, to 10 buſhels the *required* quantity of the rye.

II. As 20 to 28, ſo is 10 to 14 buſhels the *requiſite* quantity of the barley.

III. As 20 to 28, ſo is 10 to 14 buſhels, the quantity of oats *demanded* in the mixture.

28	48	16.4	20
	36	16.4	20
	24	20.8	28
	12	20.8	28

Whereupon this *third time* likewise I conclude, that (those grains still retaining the given rates) 10 bushels of wheat, 10 bushels of rye, 14 bushels of barley, and 14 bushels of oats being all mixed together, will constitute a *mass* of Corn, that may be afforded at 28 *d.* or 2 *s.* 4 *d.* the bushel.

By this example thus *diversified* it plainly appears, that the quantities required may be altered as often as the question given will admit divers *Alligations*, and yet the *mixture* produced will still hold the *rate* propounded; but when the *question* propounded will admit but one only way of *Alligation*, the quantities required to make the *mixture*, cannot be varied; so the second *example* of the tenth rule of this Chapter, being again produced, and ordered according to the direction of the eleventh rule aforegoing, *I say*,

I. As 4 to 4, so 10 to 10 bushels of rye.
II. As 4 to 4, so 10 to 10 bushels of barley.
III. As 4 to 60, so 10 to 150 bushels of oats.

16	48	4	4
	36	4	4
	24	4	4
	12	32,20,8.	60

So that for this Question I conclude, to 10 bushels of wheat you ought to add 10 bushels of rye, 10 bushels of barley, and 150 of oats, to the end that a *mixture* of Corn might be made, which may be sold at 16 *d.* the bushel: And here the quantities found (*viz.* 10, 10, and 150) cannot be *altered*, because the terms of this Question will not admit any other variety of *Alligation*.

The Proof.

XV. In Alternation Partial, the proof is likewise by comparing the total value of the several simples, with the value of the whole mixture: So in the second example of the last Rule, the total *value* of the 10 bushels of wheat, 40 bushels of rye, 50 bushels of barley, and 20 bushels of oats amounts to 14 *l.* which is also the value of the whole mixture at 2 *s.* 4 *d.* the bushel, as appears by the example of the fifth rule of this present Chapter.

Alternation total.

XVI. Alternation total is, when having the total quantity of all the simples together with their several rates, we produce their several quantities in such sort, that a mixture of them being made according to the quantities so found, that mixture may bear a certain rate propounded: Of this sort is the last example of the tenth Rule aforegoing, as also *this*, a Goldsmith having divers sorts of Gold, *viz.* some of 24 Carects, other of 22 Carects, some of 18 Carects, and other some of 16 Carects *fine*, is desirous to melt of all these sorts so much together as may make a *mass* containing 60 ounces of 21 Carects *fine*: Now this rule of *Alternation total* sheweth you how much you are to take of each sort, to the end the whole mass

may

may contain just 60 ounces of 21 Carects, the *fineness* propounded.

Alternation total proportion. XVII. In Questions of Alternation total the proportion is, as followeth:

As the sum of all the differences is to the total quantity of all the simples: So is the correspondent difference of each rate to the respective quantity of the same rate.

So the last example of the last Rule being propounded, *I say,*

I. As 12 the sum of the differences is to 60 ounces the *total* quantity of all the simples; so is 5 the correspondent *difference* of 24 Carects the first rate, to 25 ounces, *viz.* the required quantity of the Gold of the same rate, which may be taken to make the mixture propounded.

II. As 12 to 60, so is 3 the correspondent *difference* of 22 Carects the second rate, to 15 ounces, *viz.* the quantity of the Gold of 22 Carects, that ought to be used in the mixture.

III. As 12 to 60, so is 1 to 5 ounces of the Gold of 18 Carects *fine*.

IV. As 12 to 60, so is 3 to 15 ounces of the Gold of 16 carects *fine*, which are requisite to be taken for the mixture propounded.

Where-

Whereupon *I conclude*, that 25 ounces of 24 Carects *fine*, 15 ounces of 22 Carects, 5 ounces of 18 Carects, and 15 ounces of 16 Carects *fine*, being all melted together will produce a *mass* of Gold containing 60 ounces of 21 carects *fine*, which is the resolution of the *Question* propounded.

Again, the last Example of the tenth Rule being here repeated, and ordered according to the direction of the eleventh Rule, *I say*,

I. As 64 to 192, so is 17 to 51 ounces of 24 Carects *fine*.

~~II. As 64 to 192, so is 17 to 51 ounces of 21~~ Carects *fine*.

III. As 64 to 192, so is 17 to 51 ounces of 19 Carects *fine*.

IV. As 64 to 192, so is 13 to 39 ounces of *Alloy*.

17	42	17	17
	21	17	17
	19	17	17
	0	7 4 2	13
			64

And therefore for *confirmation* I say, that 51 ounces of Gold, 24 Carects fine, 51 ounces of 21 Carects fine, 51 ounces of 19 Carects fine, and 39 ounces of *Alloy* being all mixed together, will produce a *mass* containing 192 ounces of Gold, 17 Carects fine, which is the *satisfaction* of the *question* premised.

And here *observe* (as before in the Exposition of the fourteenth Rule of this Chapter) that the operations of the first of these Examples may be *varied* according to the diversity of the *Alligations*

which it will admit; whereas the last Example is not subject to any *Variety*, the *Alligations* thereof remaining alwayes the same.

XVIII. Here the operation is perfect, when the sum of the quantities found agrees with the total quantity propounded: So in the first Example of the last Rule, 25, 15, 5, and 15 (the quantities found) being all added together amount to 60, which is the total quantity propounded.

The Proof.

CHAP. XV.

The Rule of False.

I. THE Rule of False, is alwayes performed by false & supposititial numbers taken at pleasure after the Proposition is made, and the Question propounded; for things are said to be found out by the *Rule of False*, when by false terms *supposed*, we *discover* the true terms *required*.

II. The *Rule of False*, is either of single or double position.

III. The Rule of single position is, when at once, *viz.* by one false position we have means to discover the true resolution of the Question propounded.

The Rule of single Position.

For Example; *A*, *B*, and *C*, determining to buy together a certain quantity of Timber, that should cost them 36 *l.* agree amongst themselves that *B* shall pay of that sum a *third* part more than *A*, and that *C* shall pay a *fourth* more than *B*. Now the Question is, What particular sum each of these

parties

parties ought to pay of the 36 *l*. To resolve this Question; first, put the case that *A* ought to pay 6 *l*. of the 36 *l*. and then *B* must pay 8 *l*. because he pays one third part more then *A*. And lastly, *C* ought to pay 10 *l*. because he is to lay out one fourth part more then *B*. This done, although by addition of these three sums, *viz*. 6, 8, and 10. I find that I have made a wrong *Position* (their total amounting onely to 24 *l*. which ought to have been 36 *l*.) neverthelesss by those *suppositital* Numbers, I have means to discover the true sums which the several parties ought to pay: for I say by the Rule of *Three Direct*.

I. As 24 to 36, so is 6 to 9 *l*. the part that *A* must pay.

II. As 24 to 36, so 8 to 12 *l*. the part that *B* ought to pay.

III. As 24 to 36, so is 10 to 15 *l*. the part of the 36 *l*. that *C* must pay.

IV. Here for tryal of this Rule the total of the sums found ought to accord with the sum given: So in the Example of the last Rule, 9, 12, and 15 being all added together amount to 36, the sum propounded. *The Proof.*

V. The Rule of double *Position* is, when two false *Positions* are supposed for the resolution of the Question propounded As in *this*, A workman having thresht out 40 Quarters of Grain (part thereof being Wheat, and the rest Barley) received for his labour 28 *s*. being payd after the rate of 12 *d*. for every Quarter of Wheat, and 6 *d* for each Quarter of Barley: Now here the question is, how many of those 40 Quarters were Wheat and how

The Rule of double Position.

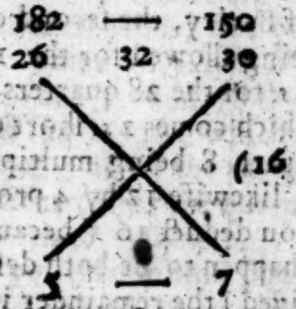
how many Barley? Here therefore I first suppose at random, that there was 26 Quarters of Wheat, and 14 of Barley, and then to discover whether I have guessed right or wrong, I find how much money is due unto the Workman at the rate of 12 *d.* the Quarter of Wheat, and 6 *d.* the Quarter of Barley, which I find to be 33 *s.* (*viz.* 26 *s.* for the 26 Quarters of Wheat, and 7 s. for the 14 Quarters of Barley) which he ought to have received, if my *supposition* had been right; but because it differs from 28 *s.* the true sum that he received, I perceive I have mist the mark, and therefore discovering how much I have err'd by finding the *difference* betwixt 28 *s.* and 33 *s.* I keep in minde 5 their *difference*, which is called the *first error*, or the *error of the first Position*; Again, I propound for the *second Position*, that there was 30 Quarters of Wheat, and 10 Quarters of Barley; and then the *second error* I find to be 7; for there is then due to the Workman for the 30 Quarters of Wheat 30 *s.* and for the 10 Quarters of Barley 5 *s.* in all 35 *s.* which differs from 28 *s.* the true sum that he received, by 7 *s.* and here by these two *false Positions*, together with their *errors*, you may discover how many Quarters of Wheat, and how many of Barley the Workman thresht, as shall be farther explained by the *Rule following*.

The Operation. *V. I.* In the Rule of double *Position* having drawn two lines across, and placed the terms of the false *Position* (*viz.* those that have the same Denomination) at the uppermost end of that Cross, as also each *error* under his respective *Position* at the lower end of the same Cross, multiply each *error* by the contrary

Position;

Position, that is, the second *error* by the first *Position*, and the first *error* by the second *Position*; this done, when both the *errors* are of one and the same kind (*viz.* both excesses or both defects) subtract the less Product out of the greater, and then the remainder is your Dividend; but if the *errors* be of differing kinds (*viz.* one of them an excess, and the other a defect) add those Products together, and then the sum will be your Dividend, which if you divide by the difference of the *errors* (when they are of one and the same kind) or by their sum (when they are of different kinds) the *Quotient* will give you a number you look for, having the same Denomination with the false *Positions* placed at the upper end of the Cross.

1. *Example*, The Question of the last Rule being again propounded, I place these terms, *viz.* 26 (having the Denomination of the Quarters of Wheat in the *first Position*) and 30 (having the same Denomination in the *second Position*) at the upper end of the Cross: As also 5 and 7 the two *errors* respectively under them at the lower end of the same Cross, as you may see it exemplified by the Pattern *following*.



Note that this Character — signifies that the lesser of the two Numbers, betwixt which it is found, ought to be subtracted from the greater.

This done, having multiplyed 26 by 7, the *product* is 182, and likewise 30 by 5, the *Product* is 150, which being deducted out of 182 (because the *errours* here are both of the same kinde, that is, are each of them an excesse above 28 *s*, the summe that the workman received) the *remainder* is 32, which being divided by 2 (the difference betwixt 5 and 7 the two errours) leaves in the *Quotient* 16, for the quarters of Wheat that the workman thresht, whose complement to 40 *viz.* 24 are the quarters of Barley, that he likewise thresht; so at last I conclude, the Workman receiving 28 *s.* for his wages in threshing out 40 Quarters of Grain (being part Wheat, part Barley) at 12 *d.* the Quarter of Wheat, and 6 *d.* the Quarter of Barley, threshed in all 16 Quarters of Wheat, and 24 Quarters of Barley.

2. *Example.* The same Question being again propounded, I suppose for my *first Position* that there are 8 quarters of Wheat, and 32 quarters of Barley, and then the first errour will be 4 *s.* for 8 *s.* being accompted for the 8 quarters of Wheat, and 16 *s.* for the 32 quarters of Barly, make in all 24 *s.* which wants 4 *s.* of 28 *s.* the sum received: Again, *supposing* that there are 12 quarters of Wheat, and 28 quarters of Barley, the second errour will be 2 *s.* for 12 *s.* being allowed for the 12 quarters of Wheat, and 14 *s.* for the 28 quarters of Barley, the sum is 26 *s.* which comes 2 *s.* short of 28 *s*, the right sum: now then 8 being multiplyed by 2, the Product is 16; likewise 12 by 4 produceth 48, out of which if you deduct 16 (because the errours in this case happen to be both defects under 28 *s.* the sum received) the remainder is 32, which being

being divided by 2, (the difference of the errours) gives you in the quotient 16, *viz.* the quarters of Wheat, *as before.*

3. Example. The ſame demand being the third time produced, I take for my *firſt Poſition* 10 quarters of Wheat, and 30 quarters of Barley, and then proceeding as before, the firſt errour will prove 3 *s.* which upon that *Poſition*, I want of 28 *s.* the right ſum: Again here for the *ſecond Poſition* I take 26 quarters of Wheat, and 14 quarters of Barley, and then the ſecond errour will be 5 *s.* which upon that *Poſition* I have exceeded 28 *s.* the true ſum: now then multiplying 10 by 5, the Product is 50, and 26 by 3, the Product is 78: And here (becauſe the errours are of different kinds, one of them being a *defect*, and the other an *exceſs* of 28 *s.* the true ſum) you are to add 50 and 78 the two Products together, whoſe ſum is 128, which being divided by 8, the ſum of 3 and 5 the two errours, gives you in the quotient 16 for the quarters of Wheat, *as before* in the former reſolutions. So that what *Poſitions* ſoever you take in this *Queſtion*, you ſhall alwayes find, that the workman threſhed 16 quarters

ters of wheat, and 24 quarters of Barley, which is the resolution of the Question propounded.

Note that this Character + intimates that the Numbers, betwixt which it is found, ought to be added together.

50	+	78
10	128	26
		)16
3	+	5
	8	

VII. Here the trial is the same with that which is used in finding out the errours: So in the Example premised 16 and 24 being the numbers found, and 16 *s*, being allowed for the 16 quarters of Wheat, likewise 12 *s*. for the 24 quarters of Barley, their summe is 28 *s*. which was the summe received by the *workman*.

4 Example. A certain man being demanded what was the age of each of his 4 Sons? Answered, that his eldest son was 4 years elder then the second; his second son was 4 years elder then the third; his third son was 4 years elder then the fourth or youngest; and his fourth or youngest, was half the age of the eldest, the Question is, what was the age of each son? Here I guesse the age of the eldest son to be 16, then it may be inferr'd from the Question, that the age of the second son was 12, the age of the third 8, and the age of the fourth or youngest 4, this 4 should be half 16 (for the Question saith, that the age of the youngest was half the age of the eldest) but it wants 4 of what it ought

ought to be ; wherefore I make a ſecond Poſition, and take 20 for the age of the eldeſt, then the age of the ſecond muſt neceſſarily be 16, the age of the third 12, and the age of the fourth 8, which ſhould be half 20, but it wants 2 : now (according to the Rule) multiplying 16 (the firſt Poſition) by 2 (the ſecond errour) the product is 32, alſo multiplying 20 (the ſecond Poſition) by 4 (the firſt errour) the Product is 80, and becauſe the errours are both of one kind, to wit, both defective ; I ſubtract the leſſer Product from the greater, ſo the remainder is 48 for a Dividend, alſo ſubtracting the leſſer errour from the greater, the remainder is 2 for a Diviſor : Laſty, dividing 48 by 2, the quotient is 24, and ſuch was the age of the eldeſt Son, therefore the age of the ſecond was 20 ; the age of the third 16, and the age of the fourth 12, which is half the age of the eldeſt, as was declared by the Queſtion.

32	—	80
16	48	20
4	—	2
	2	
	2)48(24	

The Doctrine of Vulgar Fractions.

CHAP. XVI.

Notation of Vulgar Fractions.

I. THus far of *Arithmetick* in *whole numbers* only, the doctrine of *Fractions* ensueth, which depends upon this supposition, that Unity, or at least one whole thing, whatsoever it be, may in mind be conceived divisible into any number of equal parts: some will not allow 1 or unity to be a number, when it is consider'd in the Abstract, and separated from matter, but forasmuch as that Prince of Arithmeticians *Diophantus* of *Alexandria*, in divers of his subtil Problemes doth mention unity as a number, and propounds it to be divided into numbers, I shall take the like liberty to esteem 1 or unity as a number, and likewise suppose it divisible into any number of equal parts.

A Fraction. II. A broken number, otherwise called a *Fraction*, is only part of an Integer or whole thing, as if you would express in figures the length of a piece of cloath, that contains *three fourths*, or (which is all one) *three quarters* of a yard, you are to write it thus $\frac{3}{4}$ that is, an intire yard being supposed to be divided into four equal parts, the length of the piece propounded

pounded is three of those four parts: In like manner (a Foot being divided into 12 inches) you must write six inches thus $\frac{6}{12}$, that is, *six twelfth parts* of a foot; or if the foot be divided into one hundred equal parts, to express five and twenty of those parts, set them down thus $\frac{25}{100}$, that is five and twenty hundredth parts of a foot.

III. A Fraction consists of two parts, the *Numerator* and the *Denominator*, which are placed one above the other, and separated by a little line.

IV. The *Numerator* is the number placed above the line, and the *Denominator* is the number placed underneath: so in $\frac{3}{4}$ *Numerator*, the aforementioned *Fraction* $\frac{3}{4}$, the 4 *Denominator*, number 3 placed above the line is the *Numerator*, and the number 4 placed underneath is the *Denominator*. Also in this Fraction $\frac{6}{12}$ the *Numerator* is 6 and the *Denominator* is 12. The *Denominator* is so called, because it denominates or declares into how many equal parts the Integer or whole thing is supposed to be divided; and the *Numerator* is so called, because it numbreth or expresseth how many of those equal parts of the Integer are signified by the Fraction.

V. A Fraction is either proper or improper.

VI. A proper fraction is that whose Numerator is less than the Denominator, such are the *fractions* before mentioned $\frac{3}{4}$, $\frac{6}{12}$, and the like. *A proper Fraction.*

VII. A proper fraction is either single or compound.

VIII. A single fraction is that which consists of one Numerator, and one Denomi- *A single Fraction.*

Denominator; such are $\frac{3}{4}$, $\frac{5}{13}$, $\frac{185}{1000}$, and the like.

IX. A single fraction doth often arise in Division of whole numbers, for when Division is finisht, if any number remain, it is to be esteemed as the Numerator of a Fraction, which hath the Divisor for a Denominator, and is to be annexed to the Integer or Integers in the quotient as part of the quotient, which fraction doth always express certain parts, or at least a part of an Integer or intire unity, which hath the same denomination with one of the Integers in the quotient; so if 17 pounds be given to be divided equally amongst 5 persons, there will arise 3 intire pounds

$5)\ 17\ (3\frac{2}{5}$ in the quotient, and there will be remainder or surplusage of 2 pounds

which 2 is to be placed as the *Numerator* of a *fraction* over the Divisor 5 as a *Denominator*, so will the *fraction be* $\frac{2}{5}$ and the compleat quotient will be $3\frac{2}{5}$, that is three pounds and two fifth parts of a pound for each persons share.

A single Fraction doth likewise arise, when a lesser whole number is given to be divided by a greater, for in such case the *Dividend* is to be made the *Numerator* of a Fraction, and the *Divisor* the *Denominator*, which fraction is the true quotient, and doth always express certain parts (or at least a part) of an Integer, which hath the same name with the *Dividend*; so if 3 pounds sterling be given to be divided equally amongst 4 Persons; the share of each, that is, the quotient will be $\frac{3}{4}$, to wit, three fourth parts of a pound; in like manner, if 5 be given to be divided by 8, the quotient is $\frac{5}{8}$, so that the *Numerator* of a Fraction is always a *Dividend*, the *Denominator* is a *Divisor*; and the *fraction* it self is the *quotient*.

X. A Compound Fraction (otherwise called a Fraction of a Fraction) is that which hath more *Numerators* and *Denominators* than one, and may be discovered by the word [*of*] which is interpos'd between the parts of such compound Fraction. so $\frac{2}{3}$ of $\frac{3}{4}$ is a fraction of a fraction, or *compound fraction*, and expresseth two thirds of three fourths of an *Integer*, viz. a pound sterling being supposed the *Integer* and first divided into four parts three of those four parts are equal to 15 *s*. Again, if the said 15 *s*. be divided into three parts, two of those three parts are equal to 10 *s*. therefore the *compound fraction* $\frac{2}{3}$ of $\frac{3}{4}$ of a pound sterling doth express 10 *s*. In like manner the compound fraction $\frac{1}{4}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a pound sterling, that is, one fourth of three fourths of four fifths of a pound sterling doth express 3 *s*. as will be farther manifest by the sixteenth and ninth rules of the seventeenth Chapter.

A Compound Fraction.

XI. An improper Fraction is that, whose Numerator is either greater, or at least equal unto the Denominator: so this Fraction $\frac{16}{4}$ that is sixteen fourths, is called an *Improper Fraction*, and so is this $\frac{4}{4}$ for indeed a Fraction of this kind may well be surnamed *Improper*, because it will not admit the definition of a true Fraction, since it is alwayes greater than an intire unity, or at least equal unto it, so sixteen Farthings, or $\frac{16}{4}$ of a penny are equal to 4 intire pence, and 4 Farthings, or $\frac{4}{4}$ of a penny are equal to 1 penny; therefore when the Numerator is greater than the Denominator, such *improper* Fraction signifieth more then 1 or an Integer; but when the Numerator is equal to the Denominator

An improper Fraction.

(be

(be it what number soever) such *improper* Fraction is alwayes equal to unity, or 1 Integer.

A mixt Number.

XXI. A mixt number is that, which besides the Integers or entire unities, of which it consists, hath also a Fraction annexed: So if you would express in Figures a length of a piece of Timber, that contains 12 foot and half a foot, you are to write it thus $12\frac{1}{2}$ foot. In like manner 7 Miles and three quarters or four[illegible] of a Mile are to be written thus $7\frac{3}{4}$ Miles.

XXII. A mixt number hath two parts, the whole or integral part, and the broken or fractional part; so in this mixt number $12\frac{1}{2}$, the number of Integers 12 is the *integral part*, and the Fraction $\frac{1}{2}$ is the *Fractional part*.

CHAP. XVII.

Reduction of Vulgar Fractions.

I. THe same parts of *Numeration*, as have been wrought in *whole Numbers* in the preceding Chapters are likewise to be performed in *Fractions*; but first of all *Reduction* of *Fractions* in divers kinds must be known, which being the principal skill in the doctrine of Fractions must be diligently observed by the Learner.

II. A number is said to be a common measure or Divisor unto two or more numbers given, when it will measure or divide every one of the numbers given, and leave no remainder; so 4 is a common measure unto the numbers 12 and 20, for if they be divided by 4, the *Quotient* will be exactly 3, without

without any remainder or surplusage; also if 20 be divided by the same Divisor 4, the quotient will be precisely 5 without any remainder; in like manner 5 is a *common measure* unto these three numbers 10, 25 and 40.

To finde the greatest common measure unto any two numbers.

III. Two numbers being given, their greatest common measure (that is, the greatest number which will measure or divide each of the numbers given without leaving any remainder) may be found in this manner, *viz.* Divide the greater number by the less, then divide the last Divisor by the remainder, (if there be any) and so continue dividing the last Divisors by the remainders until there be no remainder, (neglecting the quotients) so is the last Divisor the greatest common measure unto the numbers given.

Thus, if the greatest *common measure* unto the numbers 91 and 117 be sought, divide the greater number 117 by 91, so the remainder is 26, by which dividing 91, the remainder is 13, by which dividing 26, the remainder is 0; so is 13 the greatest *common measure* unto the numbers 117 and 91, as is manifest in dividing each of them by 13; for 13 is found in 91 precisely 7 times, and in 117 precisely 9 times.

```
91) 117 (1
     91
    ----
26) 91 (3
    78
   ----
13) 26 (2
    26
   ----
     0
```

To reduce a Fraction into the least terms, viz. 1. By a general Rule.

IV. A single fraction may be reduced into the least terms, by dividing the Numerator and Denomi-

nator

nator by their greatest common measure, for the quotients will be the Numerator and Denominator of a fraction equal to the former, and in the least terms.

So if the fraction $\frac{91}{117}$ be given to be reduced into the *least terms*, find the greatest common measure unto 91 and 117 by the last *Rule*, which will be found 13, and then dividing 91 by 13, the quotient will be 7 for a new Numerator; also dividing 117 by 13, the quotient will be 9 for a new Denominator, so is the fraction $\frac{91}{117}$, reduced into the *least terms*, *viz.* into the fraction $\frac{7}{9}$: but here you are to observe, that if the greatest *common measure* unto the Numerator and Denominator be 1, such Fraction is in its *least terms* already, so the fraction $\frac{19}{451}$ cannot be reduced into lower terms, because the greatest *common measure* will be found 1, (by the third *Rule* of this Chapter) the like may happen of infinite others: and although the last be a general *Rule* for the Reduction of Fractinos into their *least terms*, yet there are other practical *Rules*, which in some cases will be more ready; (especially unto beginners) *viz.*

2. By particular Rules.

V. When the Numerator and Denominator are even numbers, they may be measured or divided by 2. Therefore in such case you may (as is taught in the Rules of the 6th Chapter) take the half of the Numerator for a new Numerator, also the half of the Denominator for a new Denominator. So if $\frac{16}{64}$ be given, draw at length the line which separates the Numerator from the Denominator, and cross

16	8	4	2	1
64	32	16	8	4

cross the same with a downright stroke. near the Fraction, as you may see in the *Margent*, then take the half of 16, which is 8, for a new Numerator, also the half of 64, which is 32, for a new Denominator; Again, the half of 8 is 4, for a new Numerator, also the half of 32 is 16, for a new Denominator, and proceeding in like manner, there will be found $\frac{1}{4}$, equivalent unto $\frac{16}{64}$.

VI. When the Nominator and Denominator do each of them end with 5, or one of them ending with 5, and the other with a Cypher, they may be both measured or divided by 5. So $\frac{225}{475}$, will be reduced into $\frac{9}{19}$; and $\frac{50}{425}$ into $\frac{2}{17}$, as by the operation in the *Margent* is manifest.

$$\frac{225 | 45 | 9}{475 | 95 | 19}$$

$$\frac{50 | 10 | 2}{425 | 85 | 17}$$

VII. Whensoever you can espy any other number, which will exactly measure the Numerator and Denominator, (although it be not the greatest common measure) you may divide the Numerator and Denominator by such number as before: So $\frac{28}{84}$, may be first reduced into $\frac{7}{21}$ by 4, and $\frac{7}{21}$ may be reduced into $\frac{1}{3}$ by 7 as by the operation is manifest.

$$\frac{28 | 7 | 1}{84 | 21 | 3}$$

VIII. When the Numerator and Denominator do each of them end with a Cypher or Cyphers, cut off equal Cyphers in both, so will the fraction be reduced into lesser terms: So $\frac{400}{500}$ is reduced into $\frac{4}{5}$, and $\frac{700}{9000}$ into $\frac{7}{90}$.

$$\frac{4 | 00}{5 | 00}$$

$$\frac{7 | 00}{90 | 00}$$

IX. The value of a single fraction in the known parts of

To find the value of a single fraction in the known parts of the Integer.

of the Integer, may be found in this manner, *viz.* multiply the Numerator of the fraction propounded, by the number of known parts of the next inferiour denomination which are equal to the Integer, and divide that product by the Denominator, so is the quotient the value of the fraction in that inferiour denomination, and if there happen to be any fraction in the quotient, you may find the value thereof in the next inferiour denomination, by the same Rule, and so proceed till you come to the least known parts.

```
      20
       9
   ------
16) 180 (11 16/4
     16
   ------
      20
      16.
   ------
       4
      12
   ------
16)  48 (3
     48
   ------
      0
```

So the value of $\frac{9}{16}$ of a pound *sterling* will be found 11 *s*. 3 *d. viz.* multiply the Numerator 9, by 20 (the number of shillings which are equal to a pound *sterling*) the product is 180, which being divided by the Denominator 16, the Quotient is 11 $\frac{4}{16}$ shillings. In like manner, the value of $\frac{4}{16}$ of a shilling will be found 3 pence, for multiplying the Numerator 4 by 12, (the number of pence in a shilling) the product is 48, which being divided by the Denominator 16, the quotient is 3 pence. Also the value of $\frac{7}{13}$ of a pound *sterling*, will be found 10 *s*. 9 $\frac{3}{13}$ *d*. And $\frac{31}{96}$ of a pound *Troy* will be found equivalent unto 3 ounces 17 penny weight and 12 grains.

To reduce a mixt number into an improper Fraction.

X. A mixt number may be reduced into an improper fraction equivalent unto the mixt number, in this manner, *viz.* Multiply the Integral part of the mixt number, by the Denominator of the fraction annexed to the Integers, and unto the Product add the Numerator of the said fraction, so is the sum the Numerator of an improper fraction, whose Denominator is the same with that of the said fraction annexed.

So $4\frac{11}{12}$ will be reduced into the improper fraction $\frac{59}{12}$ for 4 being multiplied by 12, the Product is 48 unto which adding the Numerator 11, the sum is 59 for a new Numerator, which being placed over the denominator 12, gives the improper fraction $\frac{59}{12}$, which is equivalent unto $4\frac{11}{12}$, (as will appear by the 13th *Rule* of this Chapter.) In like manner $7\frac{1}{2}$ will be reduced into $\frac{15}{2}$.

To reduce a whole number into an improper fraction.

XI. A whole number is reduced into an improper fraction, by placing the whole number given, as a Numerator, and 1, as a Denominator.

So 14 Integers will be reduced into the *improper fraction* $\frac{14}{1}$, and one Integer into the *improper fraction* $\frac{1}{1}$.

XII. A whole number is reduced into an improper fraction which shall have any Denominator assigned, in multiplying the whole number given, by the denominator assigned, and placing the Product as a Numerator, over the said denominator.

So if 13 be given to be reduced into an *improper fraction* whose denominator shall be 4, multiply 13

by 4, so is the Product 52, which being placed over 4, gives the *improper fraction* $\frac{52}{4}$, equivalent unto 13, (as will appear by the next *Rule*) in like manner 13 may be reduced into $\frac{91}{7}$.

To reduce an improper fraction into its equivalent whole or mixt number.

XIII. An improper fraction may be reduced into its equivalent whole number or mixt number, in this manner, *viz.* divide the Numerator by the Denominator, so is the quotient the whole number or mixt number sought; So the improper fraction $\frac{59}{12}$ will be reduced into the *mixt-number* $4\frac{11}{12}$, for if 59 be divided by 12, the quotient is $4\frac{11}{12}$; Also the improper fraction $\frac{52}{4}$ will be reduced into the whole number 13.

To reduce fractions to a common denominator, viz. 1. When two fractions are propounded.

XIV. Fractions having unequal Denominators, may be reduced into fractions of the same value which shall have equal Denominators, by this Rule and the next following, *viz.* when two fractions having unequal denominators are propounded, to be reduced into two other fractions of the same value which shall have a common denominator, multiply the Numerator of the first fraction, (that is, either of them) by the denominator of the second, so is the product a new Numerator (correspondent unto the Numerator of that first fraction,) also multiply the Numerator of the second fraction by the Denominator of the first, so is the Product a new Numerator (correspondent unto the Numerator of the second fraction) lastly multiply the Denominators one by the other, so is the Product

Product a common denominator to both the new Numerators.

Thus if the fractions $\frac{2}{3}$ and $\frac{4}{5}$ be propounded, multiply 2 by 5, so is the product 10 for a new Numerator correspondent unto 2: also multiply 4 by 3, so is the product 12, which is a new Numerator correspondent unto 4: lastly, multiply 3 by 5, so is the product 15, which shall be a common *denominator* unto the new Numerators, so the fractions $\frac{10}{15}$ and $\frac{12}{15}$ are found which have *equal denominators* and each of these new fractions is equal unto its correspondent fraction first given, *viz.* $\frac{10}{15}$ is equal unto $\frac{2}{3}$, and $\frac{12}{15}$ is equal unto $\frac{4}{5}$; (as will be manifest by the 4th *Rule* of this Chapter.)

$$\frac{2}{3} \times \frac{4}{5} \qquad \frac{10}{15} \quad \frac{12}{15}$$

2ly When 3, or more fractions are propounded. See continual Multiplication in the 13 rule of the 5. Chapter.

XV. When three or more fractions which have unequal denominators, are given to be reduced as in the last Rule, multiply the Numerator of each fraction and all the denominators excepting its own continually, so are the several products arising from such continual multiplication, new Numerators; Lastly, multiply all the denominators continually, so is the Product a common denominator to all the new Numerators.

So if the fractions $\frac{3}{8}$ $\frac{2}{5}$ and $\frac{5}{7}$, having unequal denominators, are given to be reduced into three other fractions of the same value, which shall have *equal denominators*, multiply the Numerator 3, into the denominators 5 & 7 continually (according to the 13th Rule of the 5th Chapter;) so is the product

105; also multiply the Numerator 2, into the denominators 8 and 7 continually, so is the product 112; in like manner multiplying the Numerator 5, into the denominators 8 and 5 continually, the product is 200, which 3 products are 3 new Numerators; Lastly, multiply all the denominators 8, 5, and 7 continually, so is the product 280, which is a *common denominator*, to all the new numerators; thus the fractions $\frac{105}{280}$, $\frac{112}{280}$, and $\frac{200}{280}$ are found, which have equal Denominators, and each of these new fractions is equal unto its correspondent fraction first given, *viz.* $\frac{105}{280}$ is equal unto $\frac{3}{8}$, $\frac{112}{280}$ is equal unto $\frac{2}{5}$, and $\frac{200}{280}$ is equal unto $\frac{5}{7}$, as will be manifest by the fourth rule of this Chapter.

$\frac{3}{8}$ $\frac{2}{5}$ $\frac{5}{7}$

$\frac{105}{280}$ $\frac{112}{280}$ $\frac{200}{280}$

Note, Sometimes the work of the two last mentioned Rules may be lessened, *viz.*

Compendious Reductions.

1. When you can discover that the lesser denominator of two fractions can be multiplied by some number as a Multiplicator, which will cause the Product to be equal to the greater Denominator, you may in such case multiply the Numerator of such lesser Denominator by the said Multiplicator, and place the Product over the said greater Denominator; so in these fractions $\frac{2}{3}$ and $\frac{5}{18}$ because 3 the lesser Denominator being multiplied by 6 produceth 18, which is equal to the greater Denominator, I multiply 2 (the Numerator belonging to the lesser Denominator) by the said 6, and place the Product which is 12 over the Denominator 18; now I say that $\frac{12}{18}$ hath

$\frac{2}{3}$ $\frac{5}{18}$

$\frac{12}{18}$ $\frac{5}{18}$

hath the same value with $\frac{2}{3}$, and hath the same denominator with $\frac{5}{18}$ which is required by the *Rule*.

2. When you can discover that the unequal denominators of two fractions can be multiplied severally by two unequal numbers or multiplicators, which will produce one and the same common Product, you may in such case multiply the Numerators by the said unequal Multiplicators respectively, and place the Products over the said common Product or Denominator: so in these fractions $\frac{5}{12}$ and $\frac{7}{18}$, because the Denominator 12 being multiplied by 3 produceth 36, and the Denominator 18 being multiplied by 2, doth likewise produce 36, I multiply the Numerator 5 by the aforesaid 3, and place the Product which is 15 over the said 36; again, I multiply the Numerator 7 by the aforesaid 2, and place the Product which is 14 over 36, now I say, that these two fractions $\frac{15}{36}$ and $\frac{14}{36}$ have equal Denominators, & are equal respectively to $\frac{5}{12}$ and $\frac{7}{18}$ which were first propounded to be reduced to a *common denominator*. This latter rule may easily be extended to three, four, or more fractions that are capable of the operation before mentioned.

$$\frac{5}{12} \quad \frac{7}{18}$$
$$\frac{15}{36} \quad \frac{14}{36}$$

To reduce a compound fraction to a single fraction. See continual multiplication in the last rule of the 5th Chapter.

XVI. A compound fraction (otherwise called a *fraction of a fraction*) may be reduced into a single fraction in this manner, *viz.* Multiply all the Numerators continually, so is the Product a new Numerator, also multiply all the Denominators continually, so is the Product a new Denominator.

Thus

Thus if the *compound fraction* $\frac{2}{3}$ of $\frac{3}{4}$ be given to be reduced into a single fraction, multiply the Numerators 2 and 3, one by the other, so is the Product 6 for a new Numerator. Also multiplying the Denominators 3 and 4 one by the other, the product is 12 for a new Denominator, so is $\frac{6}{12}$ (or $\frac{1}{2}$) the single fraction sought, being equivalent unto $\frac{2}{3}$ of $\frac{3}{4}$ the compound fraction given to be reduced.

$\frac{2}{3}$ of $\frac{3}{4}$
$\frac{6}{12}$ or $\frac{1}{2}$

In like manner the compound fraction $\frac{1}{4}$ of $\frac{3}{4}$ of $\frac{4}{5}$ will be reduced into the single fraction $\frac{12}{80}$ or (in its least terms) $\frac{3}{20}$.

By this Rule a fraction or mixt number of a lesser name may be reduced to a fraction of a greater name: so if $3\frac{1}{2}$ pence be propounded to be reduced into an improper fraction of a pound sterling, the operation will be in this manner, *viz.* $3\frac{1}{2}$ or $\frac{7}{2}$ of a penny is $\frac{7}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound sterling, which compound fraction will (by the aforesaid Rule) be reduced to $\frac{7}{480}$ *l.* In like manner $42\frac{3}{16}$ minutes of an hour are equal to $\frac{45}{64}$ of an hour, for $\frac{675}{16}$ (that is $42\frac{3}{16}$) of $\frac{1}{60}$ are equal to $\frac{675}{960}$ (or in its least terms) $\frac{45}{64}$.

Here you may also observe, that when a compound fraction is one of the given terms in any question, it is first of all to be reduced to a single fraction by the aforesaid sixteenth Rule.

To find whole numbers, which shall have the same reason as any fractions or mixt numbers given.

XVII. Two or more fractions being given, there may be whole numbers found, which shall have the same reason or proportion as the fractions given, *viz.* When the fractions

ctions given have unequal denominators, reduce them into equivalent fractions which shall have a common denominator, (by the 14th or 15th rule of this Chapter) then rejecting the common denominator, the Numerators shall have the same reason or proportion as the fractions first given.

So $\frac{3}{5}$ and $\frac{5}{8}$ being given, will first of all be reduced into their equivalent fractions $\frac{24}{40}$ and $\frac{25}{40}$, then rejecting the common denominator 40, the Numerators 24 & 25 will have the same reason with $\frac{3}{5}$ and $\frac{5}{8}$, *viz.* As $\frac{3}{5}$ is to $\frac{5}{8}$ so is 24 to 25: also if the fractions $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ were given, there will be found 8, 16, and 32, which are in the same proportion one to the other as the fractions given: In like manner, if mixt numbers be given, there may be whole numbers found which shall have the same *reason* or *proportion*, as the mixt numbers, so $5\frac{2}{3}$ & $3\frac{5}{8}$ being given, will be first reduced into the improper fractions $\frac{17}{3}$ and $\frac{29}{8}$ (by the tenth Rule of this Chapter:) also the said $\frac{17}{3}$ and $\frac{29}{8}$ will be reduced into $\frac{136}{24}$ and $\frac{87}{24}$, then rejecting the common Denominator 24, the Numerators 136 and 87 will have the same *reason* as $5\frac{2}{3}$ and $3\frac{5}{8}$, *viz.* As 136 is to 87, so is $5\frac{2}{3}$ to $3\frac{5}{8}$: also $16\frac{1}{2}$ and 18 being given, there will be found 33 and 36, which being divided by their common measure 3 (found by the third rule of this Chapter) will give 11 and 12 which have the same *reason* as $16\frac{1}{2}$ and 18.

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CHAP. XVIII.

Addition of Vulgar Fractions and mixt Numbers.

I. WHen the numbers given to be added are single fractions and have equal Denominators, add all the Numerators together, so is the sum the Numerator of a fraction, whose denominator is the same with the common denominator, which new fraction is the sum of the fractions given to be added.

To add single fractions, viz. 1. When they have equal denominators.

So $\frac{3}{9}$ and $\frac{2}{9}$ being given to be added, their sum will be found $\frac{5}{9}$; *viz.* the sum of the numerators, 3 and 2, is 5, which being placed over the common denominator 9, gives $\frac{5}{9}$: In like manner the sum of these fractions $\frac{7}{8}$, $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{2}{8}$ will be found $\frac{17}{8}$, which (by the thirteenth *Rule* of the seventeenth Chapter) will be found equivalent unto $2\frac{1}{8}$, so that $2\frac{1}{8}$ is the sum of the fractions given to be added.

II. When the fractions given to be added have unequal denominators they are first to be reduced into fractions of the same value, which shall have a common Denominator (by the fourteenth or fifteenth rule of the seventeenth Chapter) and then they may be added by the first rule of this Chapter.

2. When they have unequal denominators.

So if $\frac{2}{3}$ and $\frac{3}{5}$ were given to be added, their sum will be found $1\frac{4}{15}$; for (by the fourteenth rule of the

the seventeenth Chapter) $\frac{2}{3}$ and $\frac{3}{5}$ will be reduced into their equivalent fractions $\frac{10}{15}$ and $\frac{9}{15}$ which having equal Denominators may be added according to the first rule of this Chapter, and so the sum will be found $1\frac{4}{15}$: In like manner the sum of these fractions $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{1}{4}$ will be found $1\frac{1}{8}$.

$$\frac{2}{3} \times \frac{3}{5}$$

10
9
$\frac{19}{15}$ that is $1\frac{4}{15}$

The Addition of compound fractions.

III. When any of the fractions given to be added is a compound Fraction, such compound fraction is first of all to be reduced into a single fraction (by the sixteenth rule of the seventeenth Chapter) and then you may proceed as before.

So $\frac{3}{5}$ and $\frac{2}{3}$ of $\frac{1}{4}$ being given to be added, their sum will be found $\frac{23}{30}$, for the compound fraction $\frac{2}{3}$ of $\frac{1}{4}$ will (by the sixteenth rule of the seventeenth Chapter) be reduced to $\frac{2}{12}$ (or in its least terms) $\frac{1}{6}$, which added to the single fraction $\frac{3}{5}$ (according to the second rule of this Chapter) gives $\frac{23}{30}$. Here you may observe, that the fractions given to be added in all the former cases, are supposed to be fractions of Integers which have one and the same *particular denomination, viz.* if one of the fractions given to be added, be a fraction of a pound *sterling*: all the rest are also to be fractions of a pound *sterling*, and the like is to be understood of other denominations.

By denomination is meant the name of any Integer or thing.

To add fractions of Integers which have different denominations.

IV. When fractions of Integers of different denominations are given to be added, they are first of all to be reduced into fractions of Integers

gers which shall have one and the same particular denomination (by the sixteenth rule of the seventeenth Chapter) and then they may be added by the first or second Rule of this Chapter.

So if $\frac{2}{9}$ of a pound *sterling*, $\frac{3}{5}$ of a shilling, and $\frac{5}{8}$ of a penny were given to be added, reduce the two latter into fractions of a pound *sterling* (by the sixteenth Rule of the seventeenth Chapter) *viz.* $\frac{3}{5}$ of a shilling is $\frac{3}{5}$ of $\frac{1}{20}$ of a pound *sterling*, which compound fraction being reduced into a single fraction, gives $\frac{3}{100}$ *li.* Likewise $\frac{5}{8}$ of a penny, is $\frac{5}{8}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound *sterling*, which compound fraction being reduced, gives $\frac{1}{384}$ *li.* Lastly, $\frac{2}{9}$ *li.* $\frac{3}{100}$ *li.* and $\frac{1}{384}$ *li.* being added according to the second Rule of this Chapter, their sum will be found $\frac{88068}{345600}$ or in its least terms $\frac{7339}{28800}$ *li.*

V. When mixt numbers are given to be added, finde first of all the sum of the fractions (by the first and second rule of this Chapter) then add the Integer or Integers (if there be any found) in the sum of the fractions, unto the whole numbers, and collect the sum of them as you were taught by the Rules of the third Chapter.

To add mixt numbers.

So if $3\frac{1}{2}$, $4\frac{1}{3}$ and $16\frac{5}{8}$ were given to be added, their sum will be found $24\frac{11}{24}$, *viz.* the sum of the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{8}$ will be found (by the second rule of this Chapter) to be $1\frac{11}{24}$, and the sum of the whole numbers 3, 4, and 16, is 23, unto which adding 1 (the Integer found in the sum of the fractions) the sum is 24; so that $12\frac{11}{24}$ is the sum of the mixt numbers given to be added.

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CHAP. XIX.

Subtraction of Vulgar Fractions and mixt Numbers.

I. WHen the numbers given are both single fractions and have equal denominators, subtract the lesser numerator from the greater, and place the remainder over the common denominator, so is such new fraction the difference between the fractions given.

The subtraction of single fractions, viz. 1. When they have a common denominator.

Thus the difference between the fractions $\frac{9}{11}$ and $\frac{7}{11}$ is $\frac{2}{11}$; which is found by subtracting the lesser numerator 7 from the greater denominator 9, and placing the remainder 2 over the common Denominator 11, also the difference between the fractions $\frac{11}{21}$ and $\frac{17}{21}$ is $\frac{6}{21}$, that is, the fraction $\frac{17}{21}$ exceeds $\frac{11}{21}$ by $\frac{6}{21}$.

II. When the numbers given are both single fractions and have not a common denominator, reduce them into fractions of the same value which shall have a common Denominator (by the fourteenth or fifteenth rule of the seventeenth Chapter) and then find their difference by the last Rule.

2. When they have unequal denominators.

So the difference between the fractions $\frac{6}{7}$ and $\frac{3}{4}$ will be found $\frac{3}{28}$, *viz.* reducing the fractions given into

into their equivalent fractions $\frac{48}{56}$ and $\frac{49}{56}$ which have a common denominator, the difference sought will be found $\frac{1}{56}$, by the first rule of this Chapter.

The subtraction of mixt numbers, viz. 1. *By a general Rule.*

III. When one of the numbers given is a whole number or a mixt number, also when both of them are mixt numbers, reduce such whole, or mixt numbers into an improper Fraction or Fractions by the tenth or eleventh rule of the seventeenth Chapter, and then the operation will be according to the first or second rule of this Chapter.

So $7\frac{3}{5}$ being given to be subtracted from 12, the remainder will be found $4\frac{2}{5}$, *viz.* First $7\frac{3}{5}$ will be reduced into the improper Fraction $\frac{38}{5}$, also 12 will be reduced to $\frac{12}{1}$, then these two improper fractions $\frac{38}{5}$ and $\frac{12}{1}$ will be reduced into their equivalent fractions $\frac{38}{5}$ and $\frac{60}{5}$ (which have a common Denominator.) Lastly, the difference between $\frac{38}{5}$ and $\frac{60}{5}$ is $\frac{22}{5}$ or $4\frac{2}{5}$. In like manner $9\frac{1}{2}$ being given to be subtracted from $12\frac{1}{5}$, the remainder will be found $2\frac{7}{10}$; as by the subsequent operation is manifest.

12	$7\frac{3}{5}$	$12\frac{1}{5}$	$9\frac{1}{2}$
$\frac{12}{1}$	$\frac{38}{5}$	$\frac{61}{5}$	$\frac{19}{2}$
60		122	
38		95	
$\frac{22}{5}$ that is $4\frac{2}{5}$		$\frac{27}{10}$ that is $2\frac{7}{10}$.	

Although the three last Rules be sufficient for all cases in *subtraction* of *Fractions*, mixt numbers, or whole

whole and mixt, neverthelesſs the following *Rules* will be more expeditious in the ſubtraction of mixt numbers, or whole and mixt, eſpecially when the integers conſiſt of many places, as will be manifeſt by the operation, *viz.*

IV. When a whole number is given to be ſubtracted from a mixt number, ſubtract the ſaid whole number from the whole part of the mixt number (as is taught by the rules of the fourth Chapter) and unto the remainder annex the fractional part of the mixt number given, ſo is the mixt number thus found, the remainder or difference ſought.

2. By particular rules, viz. 1. A whole number, from a mixt number.

As if 7 be given to be ſubtracted from $24\frac{3}{8}$, the remainder will be $17\frac{3}{8}$, as by the operation is manifeſt.

$$\begin{array}{r} 24\frac{3}{8} \\ 7 \\ \hline 17\frac{3}{8} \end{array}$$

V. When a fraction is given to be ſubtracted from an Integer, ſubtract the Numerator from the Denominator, and place that which remains over the Denominator, which new fraction thus found, is the remainder or difference ſought.

2. A Fraction from an Integer.

So $\frac{2}{5}$ being ſubtracted from an Integer, or 1, the remainder is $\frac{3}{5}$: Alſo $\frac{13}{19}$ being ſubtracted from 1, the remainder is $\frac{6}{19}$.

VI. When a fraction is given to be ſubtracted from a whole number greater than 1, ſubtract the ſaid fraction from one of the Integers given (by the laſt Rule) ſo the remaining fraction being annexed to the number of Integers leſſened by unity or 1; gives the remainder or difference ſought.

3. A Fraction from a whole number greater than 1.

L Thus

Thus $\frac{1}{7}$ being ſubtracted from 17, the remainder is $16\frac{6}{7}$: alſo $\frac{7}{12}$ being ſubtracted from 39, the remainder is $38\frac{5}{12}$.

4. A mixt number from a whole number.

VII. When a mixt number is given to be ſubtracted from a whole number, ſubtract firſt of all (by the fifth Rule of this Chapter) the fractional part of the mixt number, from an Integer borrowed from the whole number given, and ſet down the remaining fraction, then adding the Integer borrowed, unto the Integers of the mixt number, ſubtract the ſaid ſum from the whole number given, (as is taught in ſubtraction of whole numbers) ſo that which remains, together with the remaining fraction before found, is the remainder or difference ſought.

$$\begin{array}{r} 50 \\ 9\frac{7}{12} \\ \hline 40\frac{5}{12} \end{array}$$

So if $9\frac{7}{12}$ be ſubtracted from 50, the remainder is $40\frac{5}{12}$, as by the operation is manifeſt.

5. A fraction from a mixt number by this and the next rule.

VIII. When a fraction is given to be ſubtracted from a mixt number, and the ſaid fraction is leſs than the fractional part of the mixt number, ſubtract the leſſer fraction from the greater by the firſt or ſecond rule of this Chapter, ſo the remaining fraction being annexed to the whole part of the mixt number, gives the remainder or difference ſought.

$$\begin{array}{r} 12\frac{7}{8} \\ 0\frac{5}{9} \\ \hline 12\frac{23}{72} \end{array}$$

So $\frac{5}{9}$ being ſubtracted from $12\frac{7}{8}$ the remainder is $12\frac{23}{72}$, as by the operation is manifeſt.

IX. When a fraction is given to be ſubtracted

tracted from a mixt number, and the said fraction is greater than the fractional part of the mixt number, subtract the said greater fraction from an Integer borrowed from the mixt number, (by the fifth rule of this Chapter) and add the remaining fraction unto the fractional part of the mixt number (by the first or second rule of the eighteenth Chapter) so the fraction found by that addition, being annexed to the whole part of the mixt number lessened by an Integer, or 1, gives the remainder or difference sought.

Thus $\frac{1}{9}$ being subtracted from 13 $\frac{3}{8}$, the remainder is 12 $\frac{59}{72}$, *viz.* subtracting $\frac{1}{9}$ from 1, the remainder is $\frac{4}{9}$, which added to $\frac{3}{8}$ gives $\frac{59}{72}$, which being annexed to 12, (the number of Integers in the mixt number lessened by 1 or unity) gives 12 $\frac{59}{72}$ the remainder sought.

$$\begin{array}{r} 13\ \frac{3}{8} \\ 0\ \frac{1}{9} \\ \hline 12\ \frac{59}{72} \end{array}$$

X. When a mixt number is given to be subtracted from a mixt number, and the fractional part of the mixt number to be subtracted, is less than the fractional part of the mixt number from which you are to subtract, subtract the said lesser fraction from the greater, (by the first or second rule of this Chapter) and set down the remaining fraction: also subtract the Integers of the lesser mixt number from the Integers of the greater (as in Subtraction of whole numbers) so is the mixt number thus found, the remainder or difference sought.

6. A mixt number from a mixt number by this and the next Rule.

$20\frac{2}{7}$ $17\frac{3}{8}$ ——— $3\frac{19}{56}$	So if $17\frac{3}{8}$ be given to be ſubtracted from $20\frac{2}{7}$, the remainder will be found $3\frac{19}{56}$, *viz.* ſubtracting $\frac{3}{8}$ from $\frac{2}{7}$, the remainder is $\frac{19}{56}$; alſo ſubtracting 17 from 20, the remainder is 3.

XI. When a mixt number is given to be ſubtracted from a mixt number, and the fractional part of the mixt number to be ſubtracted, is greater than the fractional part of the mixt number from which you are to ſubtract, ſubtract the ſaid greater fraction from an Integer borrowed from the greater mixt number (by the fift rule of this Chapter) and add the remaining fraction unto the fractional part of the greater mixt number (by the firſt or ſecond rule of the 18th Chapter) ſo is the ſum to be reſerved as the fractional part of the remainder ſought; then add the Integer borrowed, unto the Integers of the leſſer mixt number, and ſubtract the ſum from the Integers of the greater mixt number, (as in ſubtraction of whole numbers) ſo that which remains, together with the fraction before reſerved, is the remainder or difference ſought.

	Thus if $20\frac{7}{8}$ be given to be ſubtracted from $35\frac{3}{5}$, the remainder will be found $14\frac{29}{40}$, *viz.*
$35\frac{3}{5}$ $20\frac{7}{8}$ ——— $14\frac{29}{40}$	ſubtracting $\frac{7}{8}$ from an Integer or 1, the remainder is $\frac{1}{8}$, which added to $\frac{3}{5}$ gives $\frac{29}{40}$, then adding the Integer borrowed, unto 20, it will be 21, which ſubtracted from

35, the remainder is 14, ſo that the remainder or difference ſought is $14\frac{29}{40}$.

When you cannot clearly diſcern which is the greater of two fractions, having unequal denominators,

nators, reduce them into fractions of the same value which shall have a common Denominator, (by the fourteenth rule of the seventeenth Chapter) and then it will be apparent which of the two fractions is the greater.

To discern the greater of two fractions.

CHAP. XX.

Multiplication of Vulgar Fractions and mixt numbers.

I. WHen the numbers given to be multiplied are both single fractions, multiply the Numerators one by the other, so is the product a new numerator: also multiply the denominators one by the other, so is the product a new denominator, which new fraction is the product sought:

To multiply single Fractions.

So $\frac{7}{12}$ and $\frac{5}{8}$ being given to be multiplied, the product will be found $\frac{35}{96}$, for 7 multiplied by 5 produceth 35 for a new numerator, and 12 multiplied by 8 produceth 96 for a new denominator: also $\frac{3}{7}$ and $\frac{5}{7}$ being multiplied one by the other, the product will be found $\frac{15}{49}$, Here you may observe that in the multiplication of proper *Fractions*, the product is always less than either of the terms given, for in multiplication such proportion as unity or 1 hath to either of the terms given, the same proportion hath the other term to the product.

II. When one of the numbers given is a whole number or a mixt number; also when both of them are mixt numbers, reduce such whole number or mixt number or numbers into an improper fraction or fractions by the tenth or eleventh Rule of the seventeenth Chapter, and then the operation will be the same as in the last rule.

To multiply mixt numbers.

So $8\frac{2}{3}$ being given to be multiplied by 5, the product will be found $43\frac{1}{3}$, *viz.* $8\frac{2}{3}$ being reduced into an improper fraction will be $\frac{26}{3}$: also 5 will be $\frac{5}{1}$, then multiplying 26 by 5, the product is 130 for a new numerator: also multiplying 3 by 1, the product is 3 for a new denominator, which new Fraction $\frac{130}{3}$ being reduced (according to the thirteenth rule of the seventeenth Chapter) will be $43\frac{1}{3}$ the product sought. In like manner $7\frac{1}{2}$ being multiplied by $5\frac{3}{5}$, the product will be found 42. Here observe, that when either of the terms given is a compound Fraction, it is first of all to be reduced into a single Fraction and then the operation is as before.

To illustrate the preceding rules of multiplication of fractions, I shall add four notes.

Note 1. When two fractions are to be multiplied one by the other, and the Numerator of the first is equal to the Denominator of the second; place the Numerator of the second fraction over the Denominator of the first, so shall this new fraction be the Product sought: for example, if $\frac{5}{7}$ be to be multiplied by $\frac{4}{5}$, I write 4 as a Numerator over 7 as a Denominator, so this new fraction $\frac{4}{7}$ shall

ſhall be the Product of the multiplication of $\frac{5}{7}$ by $\frac{4}{5}$: Likewiſe $\frac{14}{3}$ multiplied by $\frac{15}{14}$ produceth $\frac{15}{3}$, that is 5.

Note 2. To take any part or parts of a number propounded is nothing elſe but to multiply the ſaid number by the Fraction which declareth what part is to be taken: ſo if you deſire to know what is $\frac{5}{8}$ of 320, multiply 320 by $\frac{5}{8}$ (according to the ſecond rule of this Chapter) and the product will be $\frac{1600}{8}$, or 200. In like manner $\frac{2}{3}$ of $45\frac{3}{8}$ is $30\frac{1}{4}$. Alſo $\frac{1}{4}$ of 120 is $\frac{120}{4}$ or 30.

Uſeful notes in multiplication of fractions.

Note 3. Sometimes the work of multiplication in mixt numbers may be compendiouſly performed after the manner of theſe following examples, *viz.* if it be required to multiply 120 $\frac{1}{4}$ by 48 $\frac{1}{2}$, firſt multiply the whole numbers mutually, to wit, 120 by 48, and place the particular products orderly one under the other as in Multiplication of whole numbers; then multiply the ſaid whole numbers firſt given by the Fractions alternately, *viz.* take $\frac{1}{4}$ of 48 which is 12, alſo take $\frac{1}{2}$ of 120 which is 60, and place the ſaid 12 and 60 orderly to be added to the former particular products: Laſtly, add all together, and to the ſum annex the product of the two fractions, to wit in this example, the product of the multiplication of $\frac{1}{4}$ by $\frac{1}{2}$, which is $\frac{1}{8}$, ſo the total product required will be $5832\frac{1}{8}$, as you ſee by the example in the Margent. In like manner, if $18\frac{1}{2}$ be multiplied by $40\frac{1}{3}$, the product will be $746\frac{1}{6}$; and if $29\frac{1}{2}$ be multiplied by 50 the product will be 1475, as you ſee by the examples following.

$$\begin{array}{r} 120\frac{1}{4} \\ \underline{48\frac{1}{2}} \\ 960 \\ 480 \\ 12 \\ \underline{60} \\ 5832\frac{1}{8} \end{array}$$

$18\frac{1}{2}$	$29\frac{1}{2}$
$40\frac{1}{3}$	50
720	1450
20	25
6	1475
$746\frac{1}{6}$	

Note 4. When a fraction is to be multiplied by its Denominator, take the Numerator for the product; so if this fraction $\frac{3}{4}$ be propounded to be multiplied by the Denominator 4, the product will be 3, which is the same with the Numerator 3. In like manner if $\frac{5}{8}$ be multiplied by the denominator 8, the product is equal to 5 the Numerator of the said $\frac{5}{8}$.

CHAP. XXI.

Concerning Division by Vulgar Fractions and mixt numbers.

The division of single fractions.

I. WHen the numbers given are both single fractions, multiply the Denominator of the Divisor by the numerator of the Dividend, so is the product a new numerator: also multiply the numerator of the Divisor by the denominator of the Dividend, so is the product a new denominator, which new fraction is the quotient sought.

So if $\frac{4}{9}$ be given to be divided by $\frac{3}{5}$, the quotient will be found $\frac{20}{27}$; *viz.* multiplying 5 by 4 the product is 20 for a new numerator, also multiplying 3 by 9, the product is 27 for a new denominator, so is $\frac{20}{27}$ the quotient sought; in like manner if $\frac{5}{8}$ be given to be divided by $\frac{2}{7}$, the quotient will be found to be $\frac{35}{16}$ that is $2\frac{3}{16}$, as you see in the Example: here you may observe, that in Division by proper fractions, the quotient is always greater than either of the fractions given; for in Division, as the divisor is to 1 or unity, so is the dividend to the quotient.

$\frac{3}{5}$) $\frac{4}{9}$ ($\frac{20}{27}$

$\frac{2}{7}$) $\frac{5}{8}$ ($\frac{35}{16}$

II. When one of the numbers given is a whole number or a mixt number; also when both are mixt numbers, reduce such whole number or mixt number or numbers into an improper fraction or fractions, by the tenth or eleventh rule of the seventeenth Chapter, and then the operation will be the same as in the last rule.

So if 42 be divided by $7\frac{1}{2}$, the quotient will be found to be $5\frac{3}{5}$, for $7\frac{1}{2}$ and 42 will be reduced into these improper fractions $\frac{15}{2}$ and $\frac{42}{1}$, then multiplying 42 by 2, the propuct is 84 for a new Numerator, also multiplying 15 by 1, the product is 15 for a new denominator, so is $\frac{84}{15}$ the quotient sought, which is equal to $5\frac{3}{5}$ (as is evident by the thirteenth rule of the seventeenth Chapter.) In like manner, if $6\frac{1}{8}$ be divided by $3\frac{2}{5}$, the quotient will be $1\frac{31}{34}$. Also if $5\frac{1}{3}$ be divided by $12\frac{1}{2}$ the quotient will be $\frac{32}{75}$.

$7\frac{1}{2}$) 42 (

$\frac{15}{2}$) $\frac{42}{1}$ ($\frac{84}{15}$

15) 84 ($5\frac{3}{5}$

Note.

Note. When a fraction is to be divided by a fraction, and the Dividend and Divisor have equal Denominators, place the Numerator of the Dividend over the Numerator of the Divisor, so shall this new fraction be the Quotient sought; for example, if by $\frac{3}{8}$ it be required to divide $\frac{5}{8}$, I write 5 as a Numerator over 3 as a Denominator, so this new fraction $\frac{5}{3}$ shall be the Quotient saught. In like manner if by $\frac{3}{7}$ it be required to divide $\frac{12}{7}$, the Quotient will be found $\frac{12}{3}$, that is 4.

Questions to exercise the Rules of Vulgar Fractions before delivered.

Quest. 1. The difference of two numbers is $1\frac{13}{24}$, the lesser number is $2\frac{1}{8}$, what is the greater? *Answ.* $3\frac{2}{3}$, (found by *Addition.*)

Q. 2. What number is that which if added to $3\frac{5}{8}$ gives the sum $8\frac{23}{88}$? *Answ.* $4\frac{7}{11}$ (found by *Subtraction.*)

Quest. 3. There is in three bags the sum of $121\frac{9}{40}$ *l.* *viz.* in the first bag $50\frac{5}{8}$ *l.* in the second $40\frac{4}{15}$ *l.* what is in the third bag? *Answ.* $30\frac{1}{3}$ *l.* (found by *Addition* and *Subtraction.*)

Quest. 4. Two Merchants *A* and *B*, have certain shares in a Ship, the share of *A* is $\frac{7}{10}$ of the Ship, that of *B* $\frac{9}{13}$, what is the difference between their parts? *Answ.* the share of *A* exceeds the share of *B* by $\frac{1}{130}$ (found by *Subtraction.*)

Quest. 5. What is $\frac{5}{8}$ of $130\frac{2}{3}$? *Answ.* $81\frac{2}{3}$ (found by *Multiplication.*)

Quest. 6. What number is that which being multiplied by $\frac{3}{5}$ produceth $25\frac{2}{5}$? *Answ.* $42\frac{1}{3}$ (found by *Division.*)

Now followeth the doctrine of *Decimal Fractions.*

CHAP.

The Doctrine of Decimal Fractions.

CHAP. XXII.

Notation of Decimal Fractions.

1. IT is hard to determine, who was the first that brought *Decimal Arithmetick* to light, though it be a late Invention, but without doubt it hath received much improvement within the compass of a few years, by the industry of *Artists*, and now seems to be arrived at perfection. The excellency thereof is best known to such as can apply it to the practical part of the *Mathematicks*, and to the Construction of *Tables*, which depend upon standing or constant proportions, such are *Trigonometrical Canons*, *Tables* for computing of *Compound Interest*, &c. In which cases decimal operations do afford so great help, that (in my opinion) many ages have not produced a more useful invention; but it may be objected, that *Decimal Arithmetick* for the most part gives an imperfect solution to a question, this I grant, yet the Answer so given may be as useful as that which is exactly true; for in common affairs, the loss of $\frac{1}{1000}$ part of a *grain*, or of an *inch*, &c. to wit, any quantity which cannot

The proper use of Decimal Arithmetick.

cannot be seen is inconsiderable: but I would not be mistaken, for in extolling *Decimals* I do not cry down *Vulgar Fractions*, since experience sheweth that *Decimal Fractions* are commonly abused, by being applyed to all manner of questions about *money, weight*, &c. when indeed many questions may be resolved with much more facility by *Vulgar Arithmetick*, as may partly appear by this Example, *viz.* at 9 *l.*—6 *s.*—8 *d.* the hundred weight of Tobacco, what will 987 hundred weight cost? *Answ.* 9212 *l.* which by the common *Rule of Practice* by *Aliquot parts* is found out, in a quarter of the time that will necessarily be required to work it by *Decimals*, which at last will give an imperfect answer; I might instance the like inconvenience divers ways, were it not for loss of time; so that the right use of *Decimals* depends upon the discretion of the *Artist*.

Decimal Fractions sometimes abused.

II. When a single Fraction hath for its denominator a number consisting of 1 or unity in the extream place towards the left hand, and nothing but a Cypher or Cyphers towards the right, it is more particularly called a *Decimal*: of this kind are these that follow, $\frac{5}{10}$, that is five tenths, $\frac{5}{100}$, five hundredth parts; likewise these are decimal fractions, $\frac{34}{1000}$, $\frac{205}{10000}$, $\frac{1023}{10000}$, &c.

The definition of a Decimal Fraction.

III. A Decimal fraction may be exprest without the denominator, by prefixing a point or comma before (to wit, on the left hand of) the Numerator, so $\frac{5}{10}$ may be written thus, .5 or thus ,5 and $\frac{25}{100}$ thus, .25 or thus ,25.

IV. In

IV. In Decimals when the Numerator consists not of so many places as the Denominator hath Cyphers, fill up the void places in the Numerator with Cyphers prefixed on the left hand: so $\frac{5}{100}$ is written thus .05; likewise $\frac{50}{1000}$ thus, .050; and $\frac{205}{10000}$, thus, .0205, likewise $\frac{6}{1000}$, thus, .006.

V. In Decimals thus exprest, the Denominator is discoverable by the places of the Numerator: for if the Numerator consists of one place, the Denominator consists of 1 or unity with one Cypher, if of two places, the Denominator consists of 1 with two Cyphers annexed: if of three, the Denominator consists of 1 or unity with three Cyphers annexed: so the Denominator of .25 is 100, the Denominator of .050 is 1000, and the Denominator of .096 is 1000.

VI. Cyphers at the end of a Decimal do neither augment or diminish the value thereof: so .2, .20, .200, .2000 are *decimals*, which have one and the same value, for $\frac{20}{100}$ being abreviated by the eighth *rule* of the seventeenth *Chapter*, will be made $\frac{2}{10}$ and so will $\frac{200}{1000}$ or $\frac{2000}{10000}$.

VII. Wherefore Decimal fractions are easily reduced to a common Denominator, (which is a troublesome work in *Vulgar Fractions*) for if all the Numerators of as many decimal fractions as are given, be made to consist of the same number of places, by annexing a Cypher or Cyphers at the end (that is on the right hand) of such Numerators as are defective, they will all be reduced to a common Denominator, so these *Decimals* .2, .03, .027 (which signifie $\frac{2}{10}$, $\frac{3}{100}$, $\frac{27}{1000}$) may be reduced into these, .200, .030, .027, which have 1000 for a common Denominator.

VIII. The

VIII. The order of places in any Decimal proceedeth from the left hand to the right, contrary to the order of places in the Integers, which is from the right hand to the left: ſo in this *Decimal* .247, the figure 2 ſtandeth in the firſt place, (being the outermoſt towards the left hand, and next to the point,) the figure 4 ſtandeth in the ſecond place, and 7 in the *third*. Alſo in this *Decimal* .0245, a Cypher ſtands in the *firſt place*, 2 in the *ſecond*, 4 in the *third*, and 5 in the *fourth*.

IX. Every place in the Numerator of a Decimal Fraction hath a peculiar Denominator or proper value, *viz.* the Denominator of the firſt place is 10; of the ſecond, 100; of the third, 1000; *&c.* ſo that the firſt place of a Decimal ſignifies tenth parts of an unit or Integer; the ſecond place, hundredth parts of an Integer; the third place, thouſandth parts of an Integer, *&c.* Hence it is manifeſt, that this *Decimal* .3254 (every place thereof being conſidered apart by it ſelf) conſiſts of .3, .02, .005, .0004, (*viz.* $\frac{3}{10}$, $\frac{2}{100}$, $\frac{5}{1000}$, $\frac{4}{10000}$,) which being reduced to a common denominator (by the ſeventh *rule* of this *Chapter*) will give theſe, .3000, .0200, .0050, .0004, (to wit, $\frac{3000}{10000}$, $\frac{200}{10000}$, $\frac{50}{10000}$, $\frac{4}{10000}$) all which collectively make .3254 (or $\frac{3254}{10000}$.)

X. In whole numbers, the firſt place above (that is on the left hand of) the place of unities ſignifies Tens of unities; but the firſt place beneath, (that is on the right hand of) the place of unities ſignifies tenth parts of 1 or unity, and is called the firſt place of Decimal parts or place of Primes; likewiſe the ſecond place above the place of Unites, ſignifies hundreds of Unites, but the ſecond place beneath the place of Unities ſignifieth hundredth

dredth parts of 1 or unity, and is called *the second place of Decimals*, or place of seconds, so that as the values of the places in Integers do ascend in a decuple proportion from the place of Units towards the left hand, so the values of the places of Decimals do descend in a subdecuple proportion beneath the place of units towards the right hand: all which will be evident by the following *Table*.

A Table for the *Notation* of *Integers* and *decimals*.

&c.		&c.	
Fifth place	7	*Ten thousands*	of Unites.
Fourth place	3	*Thousands*	
Third place	2	*Hundreds*	
Second place	8	*Tens*	
First place	5	*Unites (under 10)*	
	.		
First place	8	*Tenth parts*	of 1 or Unity.
Second place	2	*Hundredth parts*	
Third place	3	*Thousandth parts*	
Fourth place	7	*Ten thousandth parts*	
&c.		&c.	

In

In the foregoing Table, you may observe that the places of Integers or whole numbers are separated from the places of *Decimal parts* of 1. (or unitie) by a point; so the number on the left hand of the point expresseth 73285 Integers or unities, but the number on the right hand of the point expresseth onely 8237 parts of 1 (or an Integer) supposed to be divided into 10000 equal parts. In like manner this number 5 . 8 signifies 5 Integers and eight tenth parts of an Integer, and this number 285 . 82 signifies 285 Integers (or Unities) and $\frac{82}{100}$ parts of an Integer.

CHAP. XXIII.

Concerning the Reduction of Vulgar Fractions to Decimal Fractions.

I. IF the greatest integer of *money*, as also of *weight*, *measure*, &c. were subdivided decimally, to wit, a *pound* of English money into ten equal pieces of coyn, and every one of these into ten other equal pieces, &c. and *weights*, *measures*, &c. after the same manner; the doctrine of Arithmetick would be taught with much more ease and expedition than now it is; but it being improbable that such a reformation will ever be brought to pass, I shall proceed in directing a course to the studious for obtaining the frugal use of such Decimal fractions as are in his power.

II. Forasmuch as in *Arithmetical* questions, some of

of the given numbers do for the most part happen to be fractions, a way must be shewn how to reduce a *Vulgar Fraction* to a *Decimal Fraction*; yet in some cases there is no need of this *Reduction*; for example, a *foot* in length is vulgarly subdivided into 12 inches, an inch into 4 quarters, and each quarter into 2 half quarters; but a *foot* may as easily, and a great deal more commodiously be divided, first into ten equal parts, and then each of those into ten other equal parts, and each of these into ten other equal parts; (or at least such division must be supposed or imagined when it cannot actually be made) this *foot* in length so divided, being applyed to the sides of *superficial figures*, or of *solid bodies*, will at first sight give the quantities of lines in *feet* and *decimal parts* of a *foot*; (as readily as a *foot* vulgarly divided will shew you how many *feet*, *inches*, *quarters*, and *half quarters* are contained in any line) from whence the *superficial* or *solid content* may be found in *feet* by *multiplication* only; and how much this excels the *vulgar way*, I shall partly manifest in the fifth rule of the 26th Chapter. The like subdivision I would have to be made of a *Yard*, *Perch*, &c.

How to reduce a vulgar fraction to a decimal fraction.

III. A single fraction which is no decimal fraction may be reduced into a decimal of the same value, or infinitely near, (for all vulgar fractions cannot be exactly reduced to decimals) by the rule of Three direct; for as the denominator of any single fraction whatsoever, is to the Numerator thereof, so is any other Denominator to his correspondent Numerator: *Example*, let it be required to reduce $\frac{3}{4}$ into a Decimal, whose

M Denomina-

Denominator is assigned to be 1000, say by the *Rule* of *three*, if the Denominator 8 hath 5 for a Numerator, what will the denominator 1000 require for a Numerator? Multiply and divide as the Rule of *Three direct* doth require, so will the fourth *proportional* be found to be 625, which is the Numerator sought; therefore $\frac{625}{1000}$ or .625, is a *decimal fraction* equal in value to $\frac{5}{8}$. Another *Example*, let it be required to reduce $\frac{7}{240}$ into a *decimal fraction*, whose Denominator shall be 100000, say by the Rule of three, if 240 the Denominator give 7 for a Numerator, what will the *Denominator* 100000 require for a *Numerator*? *Answ.* 2916 and somewhat more, but that which the said 2916 wants of being a true *Numerator* is less than $\frac{1}{100000}$ part of an Integer, therefore the *decimal fraction* $\frac{2916}{100000}$ or .02916 is almost equal to $\frac{7}{240}$, which $\frac{7}{240}$ cannot be exactly reduced into a *Decimal Fraction*, the like will happen in the reduction of most *vulgar Fractions* to *decimals*, in which case, the *Denominator* of the *decimal* must be assigned to be so great, that what is wanting in the *Numerator* may be an inconsiderable value.

IV. Upon the aforesaid ground, the known or accustomary parts of *Money*, *Weight*, *Measure*, *Time*, &c. may be reduced to *decimals*: for if you desire to know what *decimal fraction* of a pound *sterling* is equal in value to one *shilling*, consider first that a pound is the *Integer*, and that 20 *shillings* are equal to that *Integer*, therefore 1 shilling is $\frac{1}{20}$ of a *pound*; now if we conceive one pound to be divided into 100000 parts *viz.* if we assign 100000 for the *Denominator* of a *decimal fraction*, the *Numerator* will be found

found by the last rule to be 5000, so that $\frac{5000}{100000}$ or .05000 or .05 (for cyphers at the end of a *decimal* are of no use, as hath been shewn in the 6th *Rule* of the 22 *Chapter*) is a *decimal fraction* of a *pound*, and is exactly equal to 1 s. or $\frac{1}{20}$ part of a pound *sterling*.

In like manner forasmuch as 240 *pence* are equal to a pound of English money, 7 *pence* are $\frac{7}{240}$ parts of a pound, which *fraction* will be reduced into this *decimal* .02916 *l.* Which is very near equal to $\frac{7}{240}$ *l.* for it wants not $\frac{1}{100000}$ part of a pound. Moreover since 960 *farthings* are equal to a *pound* English, one *farthing* is $\frac{1}{960}$ part of a pound, which will be reduced into the *decimal* .00104 *l.* very near; but if you please to proceed nearer the truth, you will find this *decimal* .00104166 to answer a farthing, and so by augmenting the Denominator with cyphers, you may proceed infinitely near, when you cannot attain unto the truth it self. After the same method may the vulgar *Sexagenary fractions* used in *Astronomy* be reduced to *decimals*, for since a *degree* is usually subdivided into sixty parts called *minutes* or *primes*; a *prime* or *minute* into sixty parts called *seconds*; a *second* into sixty *thirds*; a *third* into sixty *fourths*, &c. and consequently a *degree* is equal unto 60 *minutes* (or Primes) or unto 3600 *seconds*, or 216000 *thirds* or 12960000 *fourths*, &c. It is evident that 7 *minutes* (or Primes) are $\frac{7}{60}$ parts of a *degree*, which by the third *Rule* of this *Chapter* may be reduced into the Decimal .1166, &c. Also 29 *thirds* are $\frac{29}{216000}$ parts of a *degree* which may be reduced into the *decimal* .000134, &c. Moreover,

$58 : \overset{..}{33} : \overset{...}{14} : \overset{....}{12}$, that is, 58 *Primes* 33 *seconds*,

14 *thirds*, and 12 *fourths* may be reduced to a *decimal* in this manner, *viz.* reduce them all into *fourths* (according to the sixth *Rule* of the seventh *Chapter*) so will you find 12647652 *fourths*, which are $\frac{12647652}{12960000}$ parts of a *degree*, which *vulgar fraction* may be reduced into this *decimal* of a *degree*, to wit, .975899, &c. (by the third *Rule* of this *Chapter*.)

This to the ingenious will be a sufficient light for the finding of the *Decimals* congruent to the *shillings*, *pence*, and *farthings* which are under a *pound sterling*; also the *decimals* of the known parts of *Weight*, *Measure*, *Time*, &c. as they are exprest in the following *Table*, wherein you may observe, that most of the *decimals* consist of 7 or 8 figures, yet in ordinary practice, you shall have occasion to use only the first five, and sometimes fewer.

THE

THE TABLE OF REDUCTION.

TABLET I.
Of English money, the Integer being a Pound.

Shillings	*Decimals of a pound.*
19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05

pence with Farthings.	*Decimals of a pound.*
	.0489583
	.0479166
	.046875
11	.0458333
	.0447916
	.04375
	.0427082
10	.0416666
	.040625
	.0395833
	.0385416
9	.0375
	.0364583
	.0354166
	.034375
8	.0333333
	.0322916
	.03125
	.0302083
7	.0291666
	.028125
	.0270833
	.0260416
6	.025

	.0239583
	.0229166
	.021875
5	.0208333
	.0197916
	.01875
	.0177708
4	.0166666
	.015625
	.0145833
	.0135416
3	.0125
	.0114583
	.0104166
	.009375
2	.0083333
	.0072916
	.00625
1 pen. & 1 far.	.0052083
Penny 1	.0041666
3. Farth.	.003125
2. Farth.	.0020833
1. Farth.	.0010416

TABLET II.

Of Troy weight, the Integer being an Ounce.

Penny weights.	*Decimals of an Ounce.*
19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05

Grains	*Decimals of an ounce.*
23	.0479166
22	.0458333
21	.04375
20	.0416666
19	.0395833
18	.0375
17	.0354166
16	.0333333
15	.03125
14	.0291666
13	.0270833
12	.025
11	.0229166
10	.0208333
9	.01875
8	.0166666
7	.0145833
6	.0125

5	.0114166
4	.0083333
3	.00625
2	.0041666
1	.0020833

TABLET III.

Of Averdupois great weight, the Integer being an hundred weight, to wit, 112 pounds.

quarters of 1 hundred.	*decimals of 1 hundred.*
3	.75
2	.5
1	.25

Pounds.	*decimals of 1 hundred.*
27	.2410714
26	.2321428
25	.2232142
24	.2142857
23	.2053571
22	.1964285
21	.1875
20	.1785714
19	.1696428
18	.1607142
17	.1517857
16	.1428571
15	.1339285
14	.125
13	.1160714
12	.1071428
11	.0982142
10	.0892857
9	.0803571
8	.0714285
7	.0625
6	.0535714
5	.0446428
4	.0357142
3	.0267857
2	.0178571
1	.0089285

Ounces.	*decimals of 1 hundred.*
15	.0083705
14	.0078125
13	.0072544
12	.0066964
11	.0061383
10	.0055803
9	.0050223
8	.0044642
7	.0039062
6	.0033482
5	.0027901
4	.0022321
3	.0016741
2	.0011160
1	.0005580

quarters of 1 Ounce.	*decimals of 1 hundred.*
3	.0004185
2	.0002790
1	.0001395

TABLET IV.

Of Averdupois little weight, the Integer being a pound.

Ounces.	*decimals of a pound.*
15	.9375
14	.875
13	.8125
12	.75
11	.6875
10	.625
9	.5625
8	.5
7	.4375
6	.375
5	.3125
4	.25
3	.1875
2	.125
1	.0625

Drams.	*decimals of a pound.*
15	.05859375
14	.0546875
13	.05078115
12	.046875
11	.04296875
10	.0390625
9	.03515625
8	.03125
7	.02734375
6	.0234375
5	.01953125
4	.015625
3	.01171875
2	.0078125
1	.00390625

quarters of a dram.	*decimals of 1 pound.*
3	.0029296
2	.0019531
1	.0009765

TABLET V.

Of liquid measures, the Integer being a gallon.

Pints.	*decimals of 1 gallon.*
7	.875
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125

quarters of a pint.	*decimals of a gallon.*
3	.09375
2	.0625
1	.03125

TABLET VI.

Of dry measures, the Integer being a quarter.

Bushels.	*decimals of a quarter.*
7	.875
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125

Pecks.	*decimals of a quarter.*
3	.09375
2	.0625
1	.03125

quarters of a Peck.	*decimals of a quarter.*
3	.0234375
2	.015625
1	.0078125

Pints.	*decimals of a quarter.*
3	.005859
2	.003906
1	.001953

TABLET VII.

Of long measures, one Yard or one Ell being the Integer.

quarters of 1 yard or 1 ell.	*decimals of 1 yard or 1 ell.*
3	.75
2	.5
1	.25

Nails.	*decimals of 1 ya. or 1 ell*
3	.1875
2	.125
1	.0625

quarters of 1 nail.	*decimals of 1 ya. or 1 ell*
3	.046875
2	.03125
1	.015625

TABLET VIII.

Of the Reduction of inches, &c. to decimals, the integer being a foot in length.

Inches.	*decimals of a foot.*
11	.9166666
10	.8333333
9	.75

8	.6666666
7	.5833333
6	.5
5	.4166666
4	.3333333
3	.25
2	.1666666
1	.0833333

quarters of an inch.	*decimals of a foot.*
3	.0625
2	.0416666
1	.0208333
half a quarter of an inch.	.0104166

TABLET IX.
Of dozens, the integer being a gross.

dozens.	*decimals of a gross.*
11	.9166666
10	.8333333
9	.75
8	.6666666
7	.5833333
6	.5
5	.4166666
4	.3333333
3	.25
2	.1666666
1	.0833333

parts of a dozen.	*decimals of a gross.*
11	.076388
10	.069944
9	.0625
8	.055555
7	.048611
6	.041666
5	.034722
4	.027777
3	.020833
2	.013888
1	.006944

TABLET X.
Of Time, a day being the integer.

Hours.	*decimals of a day.*
23	.9583333
22	.9166666
21	.875
20	.8333333
19	.7916666
18	.75
17	.7083333
16	.6666666
15	.625
14	.5833333
13	.5416666
12	.5
11	.4583333
10	.4166666

9	.375	38	.0263888
8	.3333333	37	.0256944
7	.2916666	36	.0249999
6	.25	35	.0243055
5	.2083333	34	.0236111
4	.1666666	33	.0229166
3	.125	32	.0222222
2	.0833333	31	.0215277
1	.0416666	30	.0208333
	decimals of	29	.0201388
Minutes.	*a day.*	28	.0194444
59	.0409722	27	.01875
58	.0402777	26	.0180555
57	.0395833	25	.0173611
56	.0388888	24	.0166666
55	.0381944	23	.0159722
54	.0375	22	.0152777
53	.0368055	21	.0145833
52	.0361111	20	.0138888
51	.0354166	19	.0131944
50	.0347222	18	.0125
49	.0340277	17	.0118055
48	.0333333	16	.0111111
47	.0326388	15	.0104166
46	.0319444	14	.0097222
45	.0312500	13	.0090277
44	.0305555	12	.0083333
43	.0298611	11	.0076388
42	.0291666	10	.0069444
41	.0284722	9	.00625
40	.0277777	8	.0034722
39	.0270833	7	.0048611
		6	.0041666
		5	.0034722

4	.0027777
3	.0020833
2	.0013888
1	.0006944

V. This Table aforegoing consists of ten several Tablets, of which the first (intituled *English money*) contains in the first column thereof the particular Fractions (*viz.* the *shillings*, *pence*, and *farthings*) of a pound *sterling*; and in the other column the *decimals*, unto which they may be respectively reduced: So in the same Tablet .65 is the *decimal*, answerable to 13 *s.* .0208333 to 5 *d.* and .003125 to 3 *f.* Likewise .0489583 is the decimal of 11 *d.* together with 3 farthings; Also .03125 is the decimal of 7 pence half penny.

The Tablet. 1. Of English money.

VI. The next Tablet (intituled *Troy weight*) contains in the first column thereof the particular Fractions, (*viz.* the *Penny weights*, and *Grains*) of an ounce *Troy*, and in the other their respective decimals: so 6 is the correspondent *decimal* of 12 penny weight, and .0020833 of 1 grain. Likewise .025 is the decimal of 12 grains.

2. Of Troy weight.

VII. The third Tablet (intituled *Averdupois great weight*) contains in the first column thereof the Fractions, (*viz.* the *Quarters*, *Pounds*, *Ounces*, and the *quarters of Ounces*) of an Hundred according to *Averdupois weight*, and in the other their proper decimals: so .5 is the *decimal* of two quarters or half a hundred, .1517857 of 17 pounds: .0033482

3. Of Averdupois great weight.

.0033482 of 6 Ounces, and .0004185 the *decimal* of 3 quarters of an Ounce.

VIII. The fourth (intituled *Averdupois little weight*) sheweth you the Fractions (*viz.* the *Ounces*, *drams*, and *quarters* of *drams*) of a pound *Averdupois*, together with their respective decimals: so the *decimal* of 3 Ounces is .1875, the *decimal* of 9 Drams is .03515625, and the *decimal* of one quarter of a Dram is .0009765.

4. Of *Averdupois little weight.*

IX. The fifth (intituled *Liquid measures*) hath the fractions (viz. the *Pints* and *quarters* of *pints*) of a Gallon, and likewise their several decimals: so the *decimal* of 5 Pints is .625, and the *decimal* of two quarters or half a pint is .0625.

5. Of *Liquid measures.*

X. The sixth (intituled *Dry measures*) gives you the fractions (viz. the *Bushels*, *Pecks*, *quarters* of *Pecks* and *pints*) of a quarter, together with their peculiar decimals: so .375 is the *decimal* of three Bushels, .03125 of one Peck, .0234375 of ¾ of a peck, and .003906 of two pints.

6. Of *Dry measures.*

XI. The seventh (intituled *Yards* and *Ells*) offers you the fractions (viz. the *Quarters*, *Nails*, and *quarters* of *Nails*) of Yards or Ells, and their respective decimals: so .25 is the *decimal* of one quarter of a Yard or Ell, .125 of two Nails, and .046875 of three quarters of a Nail.

7. Of *Long measures.*

XII. The eighth (intituled *Reduction of inches, &c. to decimals of a foot*) presents unto you the fractions (to wit, the Inches, quarters of Inches and half quarters of an Inch) of a *foot*, together with

with their correspondent decimals: so .4166666 is the *decimal* of 5 Inches, .0625 of $\frac{1}{4}$ of an Inch, and .0104166 of $\frac{1}{8}$ or half a quarter of an Inch.

XIII. The ninth Tablet (intituled *Dozens*) yields you the Fractions (viz. the Dozens and particulars) of a Gross; as also their respective *decimals*: so .25 is the *decimal* of 3 Dozen, and .048611 of 7 particulars.

8. Of things accompted by the Dozen.

XIV. The tenth and last Tablet (intituled *Time*) gives you the fractions (viz. the Hours and Minutes) of a Day: so .625 is the *decimal* of 15 hours, .0375 of 54 minutes, and .0006944 of one minute.

9. Of Time

XV. When a single Fraction of any of the premised Tablets is propounded to be reduced to a decimal, find it in the first Column of the Tablet, unto which it belongs; this done, just against that Fraction so found, you shall have the decimal required: so 13 *s.* being propounded, taking the first premised *Tablet*, I find 13 *s.* in the first Column of the Tablet of *money*, and just against the same thirteen shillings, I *observe* 65, before which having prefixed a point, and by that means signed it for a *decimal* (according to the third Rule of the 22 Chapter of this Book) I conclude the same .65 so ordered, to be the correspondent *decimal* of thirteen shillings the fraction propounded: In like manner .0229166 is the *decimal* of 11 Grains in the Tablet of *Troy* weight; and .0357142 the *decimal* of 4 *lb.* in the Tablet of *Averdupois great weight*, &c.

The use of the same Table for the Reduction. 1. Of single Fractions to decimals.

XVI. When

XVI. When two or more fractions are propounded, and it is required to find a decimal equivalent unto the ſum of them, find the decimal of each of the fractions given according to the laſt Rule; then adding together the decimals ſo found, that intire ſum is the decimal ſought: ſo 13 *s.* 5 *d.* being reduced to a decimal, is .670833; for the decimal of 13 *s.* is .65, and the decimal of 5 *d.* .020833, which being added together (by the ſecond rule of the 24th Chapter of this Book) amount to .670833, *viz.* the decimal which repreſents 13 *s.* 5 *d.* the fraction propounded: In like manner the decimal of 9 penny weight, and 13 Grains is .4770833, and the decimal of ½ C. 19 *lb.* 7 Ounces is .67354, &c.

13 *s.*	.65
5 *d.*	.020833
	.670833
9 *p.w.*	.45
13 *gr.*	.027083
	.477083
½ *C.*	.5
19 *lb*	.16964
7 *ounc.*	.00390
	.67354

And here as you ſee meer fractions reduced, ſo likewiſe may the fractions of mixt numbers be reduced to decimals: for example, theſe numbers 97. *lb.*

lb. 7 ounces 13 $\frac{1}{4}$ drams. *Item* of 67 Gallons 5$\frac{1}{4}$ pints. *Item* 28 Quarters, 0, Bushels, 2$\frac{1}{2}$ Pecks, and 3 Pints after reduction are 97, .4891, .67, .71875, and 28 .0781.

97.4375	67.625	28.0625
.0507	.0937	.0156
.0009	67.7187	28.0781
97.4891		

Again 22$\frac{1}{2}$ yards, 3$\frac{1}{4}$ Nails; *Item* 36 Gross, 3 Dozen and 5 particulars, being reduced, are 22 .7031, 36 .2847.

22 .5	
.1875	36.25
.0156	.0347
22.7031	36.2847

3. Of Decimals to single Fractions.

XVII. When a decimal is propounded to know what Fraction it represents, search the same decimal in the second Column of the Tablet, unto which it belongs, where if you find it expressly, the number just against it in the first Column is the fraction you look for: so .65 (representing the fraction of a pound sterling) being given, I find it in the second Column of the Tablet of Money, and over against it in the first Column I find 13 *s.* which is the fraction represented by .65, the decimal propounded. In like manner 3 .025 (representing 3 ounces and .025 of an ounce *Troy*) being

being propounded, the number represented by it, is 3 Ounces, 0 *p.w.* 12 grains.

XVIII. When in the second column of the Tablet, unto which you are directed, you cannot precisely find the decimal propounded, search that, which being less, comes nearest unto it, and take the number that answers unto it in the first column for the greatest fraction of the number required: then deducting the decimal so found out of the decimal given, find likewise the remainder, as another decimal, and take his correspondent number for the next fraction of the number required: and so proceed in that order, till you have discovered the intire number represented by the decimal propounded.

Example: .6739 being propounded, I demand the fraction of a pound sterling represented by it; the decimal in the Tablet of money, which being less comes nearest to .6739 is .65, whose correspondent number in that Tablet is 13, which are the shillings of the number required; then subtracting (by the 1 *Rule* of the 25 Chapter of this Book) .65 out of .6739, the remainder is .0239, and the nearest decimal in the same Tablet to .0239 is .0208, whose correspondent number is 5, which are the pence of the number required: last of all deducting .0208 out of .0239, the remainder is .0031, which gives you in the first column 3, being the farthings of the number required: So that I conclude the intire fraction represented by the decimal .6739, is 13 *s.* 5 *d.* 3 *f.*

	.6739 *l. sterling.*
Subtract 13 *s.* ————	.65
	.0239
Subtract 5 *d.* ————	.0208
3 *f.* ————	.0031

In like manner 7.359 *C.* being reduced by the Tablet of *Averdupois great weight* is $7\frac{1}{4}$ *C.* 12 *lb.* 4 ounc. And 94..58 *lb.* reduced by the Tablet of *Averdupois little weight*; is 94 *lb.* 9 ounces and 6 drams.

	7.359 *C.*
Subtract 1 *quarter* ————	.25
	.109
Subtract 12 *lb.* ————	.107
4 *oz.* ————	.002
	94.58 *lb.*
Subtract 9 *oz.* ————	.56
6 *Drams.* ————	.02

CHAP. XXIV.

Addition of Decimal Fractions.

I. TO such as well understand the *Notation* of *Decimal fractions*, all the varieties of their *Numeration*, to wit, *Addition*, *Subtraction*, &c. will be as easie as the operations by whole numbers; therefore he that would be a good Proficient in *Decimal Arithmetick*,

rithmetick, must throughly understand the 22 and 23 *Chapters* aforegoing.

II. When divers decimal fractions are given to be added together, they must first of all be orderly placed one under another according to the doctrine of their Notation. So if these *decimal fractions*, to wit, .125, .39 and .7 were given to be added, they must be written down thus;

.125
.39
.7

or if you will have the same number of places to be in all the *decimals* given, without altering their values, they may be written thus,

.125
.390
.700

Not thus;

.125
. 39
. 7

For the Figures or Cyphers, which are of like degrees or places must be subscribed directly one under another, *viz. tenth parts* or *primes* must be written down directly underneath *tenths*; also *hundredth parts* or *seconds* must be placed under *hundredth parts*, as you see in the first Example, where

where .3 or three tenth parts in the second decimal ſtands directly under .1 or one tenth part in the firſt decimal; likewiſe .7 or ſeven tenths in the third decimal ſtands directly under the tenths in the former, and ſo of the reſt.

In like manner, when mixt numbers, which conſiſt of Integers and decimal parts are given to be added, due reſpect muſt be had of their ſubſcription one under another: ſo if theſe mixt numbers, to wit, 32.056, 7.07, and 1.9 were given to be added, they muſt be written down thus,

32.056
7.07
1.9

III. Having placed the decimals and drawn a line underneath in manner aforeſaid, add them together, beginning with the outermoſt rank towards the right hand (as hath been taught in Addition of whole numbers of one denomination in the third Chapter:) ſo if the decimals in the firſt Example of the ſecond *Rule* of this *Chapter* were given to be added, I firſt ſubſcribe 5, which is all that ſtands in the firſt rank towards the right hand, then proceeding to the ſecond rank, I ſay 9 and 2 make 11, wherefore I write down 1 which is the exceſs of 11 above 10, and for the 10 I carry 1 in mind to the next rank, ſaying 1 in mind added to 7 makes 8, which added to 3 and 1 make 12, wherefore I write 2 which is the exceſs of 12 above 10 under the line, reſerving

.125
.39
.7
1.215

1

1 in mind for the 10, then I prefix a point before 2, which stands in the first place of decimals; and on the left hand of the point, to wit in the place of Units or first place of Integers, I write down 1 (being the 1 in mind) which done, I find that the sum of the Decimals given is 1.215 that is, one Integer (whether it be a Perch, Yard, Foot, &c.) and $\frac{215}{1000}$ parts of an Integer, as you see in the Example. In like manner these mixt numbers 32.056; 7.07 and 1.9 being given to be added, their sum will be found to be 41.026 that is, 41 Integers and $\frac{26}{1000}$ parts of an Integer, as you see in the Margent; more *Examples* for the learners exercise are these.

$$\begin{array}{r} 32.056 \\ 7.07 \\ 1.9 \\ \hline 41.026 \end{array}$$

$$\begin{array}{r} .65 \\ .025 \\ .03 \\ \hline .705 \end{array} \qquad \begin{array}{r} 24.7 \\ 0.35 \\ 5.26 \\ \hline 30.31 \end{array} \qquad \begin{array}{r} 503.75 \\ 0.32 \\ 0.12 \\ \hline 504.19 \end{array}$$

CHAP. XXV.

Subtraction of Decimal Fractions.

I. HAving first written down the greater of the two numbers given, (whether it be a whole number, mixt number, or decimal) and the lesser underneath the greater, according to the directions in the second Rule of the 24th Chapter. Proceed as you are taught in Subtraction of whole numbers:

$$\begin{array}{r} .837 \\ .784 \\ \hline .053 \end{array}$$

 (by

(by the Rules of the fourth Chapter) ſo if this decimal fraction .784 were given to be ſubtracted from this decimal .837, the remainder will be .053, that is, $\frac{53}{1000}$ parts of an Integer; in like manner if this mixt number 78.919 were given to be ſubtracted from 295.094, the remainder will be 216 $\frac{175}{1000}$. In each of which Examples you may obſerve that 10 is borrowed as often as need requires, according to the Rules of Subtraction of whole numbers of one denomination: Note alſo, when the decimals in both the numbers given conſiſt not of the ſame number of places, that decimal which is defective in places towards the right hand, muſt have the void places filled up with cyphers, or at leaſt cyphers muſt be ſuppoſed to be annexed: ſo if this decimal .04338 be given to be ſubtracted from this .65, the remainder will be found to be .60662, and the work will ſtand as in the Margent, where you ſee the three void places are ſupplied with cyphers, and then the operation is as in whole numbers by borrowing 10 as often as the lower figure cannot be ſubtracted from the upper. More Examples of Subtraction of Decimals are theſe following.

295.094
78.919
216.175

.65000
.04338
.60662

24.04338	37.	.394
.65	0.104	.35
23.39338	36.896	.044

CHAP.

CHAP. XXVI.

Multiplication of Decimal Fractions.

I. WHen two numbers are given to be multiplied, and are both mixt numbers, or both decimal fractions, or one of them a whole number, and the other, a decimal or mixt number, (which are all the cases that can happen) there is no necessity of writing them down precisely one under the other as in Addition and Subtraction, for the product or number sought in Multiplication depends not upon any regular placing of the two numbers given: so if this mixt number 56.3 were given to be added to this mixt number 1.30526, they ought to be written one under the other, as you see (according to the second Rule of the 24th Chapter) but if they are to be multiplied one by the other, they may be written thus,

```
 1.30526
56.3
-------
```

```
1.30526
   56.3
-------
```

II. In any of the Cases which may happen in Multiplication of Decimals, multiply the numbers given as if they were whole numbers, then cut off alwayes from the product by a point, comma, or line, so many places towards the right hand, as there are places of decimal parts in both the num-

bers given to be multiplied, that done, the figure or figures (if any happen to be) on the left hand of the ſaid point or line of ſeparation doth declare the Integer or Integers in the product, and thoſe on the right hand of the point are decimal parts of an Integer: ſo if this mixt number 56.3 (that is, 56 Integers and $\frac{3}{10}$ of an Integer) be given to be multiplied by this mixt number 1.30526, the product will be found to be 73.486138, that is, 73 Integers and $\frac{486138}{1000000}$ parts of an Integer; for having choſen that to be the Multiplicator, which will cauſe leaſt work, and ſubſcribed it under the Multiplicand; (to wit, 56.3 underneath 1.30526) I proceed according to the Rules of Multiplication of whole numbers, *viz.* having drawn a line underneath the numbers given, I multiply all the Multiplicand, to wit, 1.30526, as if it were a whole number, by 3 the firſt multiplying figure, and ſubſcribe the product thereof, which is 391578 underneath the line, and proceeding in like manner with the other multiplying figures 6 and 5; at laſt I find the total of the particular products to be 73486138; and becauſe there are 6 places of decimal parts in both the numbers given, (to wit, 5 places of parts in the multiplicand, and 1 place in the multiplicator) I cut off 6 places to the right hand from the total before produced, ſo will it ſtand thus 73|486138: wherefore I conclude that the true product is 73 $\frac{486138}{1000000}$ or 73.486138, that is, 73 Integers and almoſt ½ of an Integer.

```
 1.30526
    56.3
--------
  391578
 783156
652630
--------
73|486138
```

In

In like manner, if this mixt number 246.25 (that is $246\frac{25}{100}$) were given to be multiplied by 35 Integers, the true product will be found to be 8618.75, that is 8618 Integers and $\frac{75}{100}$ parts of an Integer, as you ſee by the operation in the Margent, where you may obſerve that two places are cut off from the total number produced of the multiplication, towards the right hand, becauſe there are two places of decimals in the multiplicand, (the multiplicator conſiſting of Integers only) but if there had been decimal parts alſo in the multiplicator, ſo many more places ſhould have been cut off, as was ſhewed in the firſt *Example.*

```
  246.25
      35
--------
  123125
  73875
--------
  8618|75
```

Again, if theſe two decimals .87 and .9 (to wit $\frac{87}{100}$ and $\frac{9}{10}$) were given to be multiplied one by the other, the true product will be found to be .783, that is $\frac{783}{1000}$ parts of an Integer, as you ſee in the *Example*, where you may obſerve that the product is a fraction only; for after 3 places (being the number of places of decimals in both the numbers given to be multiplied) are cut off to the right hand, there remains no Integer on the left hand.

```
  .87
   .9
-----
 .783
```

III. When the Multiplication is finiſht, if there ariſe not ſo many places in all as ought to be cut off by the ſecond Rule of this Chapter (which may often happen when the product is a fraction) in ſuch caſe, as many places as are wanting, ſo many cyphers muſt be prefixed to the product on the left hand thereof, and then a point muſt be prefixt to

to ſign the product ſo increaſed for a decimal: ſo theſe decimals .0375 and .05 being given to be multiplied one by the other, I multiply 375 by 5, and there ariſeth 1875, now according to the ſecond Rule of this Chapter, I ſhould cut off 6 places to the right hand, and here are but 4 in all, wherefore I prefix two Cyphers, to wit, as many as there are places wanting, and then prefixing a point, the true product will be .001875 or $\frac{1875}{1000000}$; in like manner if this mixt number 5.525 be multiplied by this decimal .0026, the true product will be found to be .0143650 (or $\frac{143650}{10000000}$ as you may ſee by the operation in the Margent, where one cypher is prefixed to the numbers ariſing from the total Multiplication to diſcover the true product.

$$
\begin{array}{r}
.0375 \\
.05 \\
\hline
.001875 \\
\hline
5.525 \\
.0026 \\
\hline
.33150 \\
.11050 \\
\hline
.0143650
\end{array}
$$

To reduce decimals to the known parts of the Integer.

IV. Decimal parts of an Integer may be reduced to the known or accuſtomed parts of ſuch Integer by Multiplication only, for if the decimal fraction given be multiplied by that number, which declareth how many known parts are equal to the Integer, the Product gives the number of known parts required: So this decimal fraction of a pound ſterling, to wit, 8687 *l.* being propounded, I multiply it firſt by 20 (the number of ſhillings contained in a pound) and the product gives 17 ſhillings and .3740 parts of a ſhilling;

ſhilling; which decimal .3740 being multiplied by 12 (the number of pence in a ſhilling) produceth 4 pence, and .488 parts of a penny; laſtly, multiplying .488 by 4 (the number of farthings, which make a penny, the product gives 1 farthing and .952 parts of a farthing, which are very near in value to another farthing, ſo it appears that .8687 parts of a pound ſterling are 17 *s.* 4 *d.* 2 *f.* very near. After the ſame manner, a *decimal fraction* of any Integer whatſoever may be reduced into the known or accuſtomed parts of ſuch Integer.

	.8687 *l.*
	20
Shill.	17|3740
	12
	7480
	3740
Pence	4|4880
	4
Farth.	1|9520

A brief way to find the value of any decimal fraction of a pound of English moneys

A briefer way to value any decimal part of a pound of *Engliſh money*, without loſs of a farthing may be this, *viz.* the figure (if any happen) in the firſt place of the decimal being doubled gives ſhillings, alſo if there be 5, or a figure greater than 5 in the ſecond place, one ſhilling more is to be added to the former; laſtly, when 5 is taken from the figure in the ſecond place, if every unite in the remainder be accounted as ten, and the figure in the third place as unities, theſe tens and units taken as one number and leſſened by 1 give the number of farthings, which with the ſhillings before found declare the value of the decimal propounded; likewiſe if the figure in the ſecond place

(when

(when any happens) be less than 5, every unite in such figure is to be accounted ten as before: so in the decimal before mentioned, to wit, .8687 *l.* the figure 8 in the first place being doubled gives 16 shillings, also because 5 is contained in 6 which stands in the second place, one shilling more is to be added to the aforesaid 16 shillings, which will now be made 17 *s.* that done, the remainder of the said 6 after 5 is subtracted, to wit, 1 being esteemed as 10, and added to 8 (which stands in the third place, and to be esteemed as unites) gives 18, from which abating 1, the remainder is 17 farthings or 4 pence and a farthing; so that the value of the said decimal .8687 *l.* is found as before to be 17 shillings 4 pence 1 farthing. After the same manner this decimal of a pound of *English money*, to wit, .319 *l.* will be reduced to 6 shillings and 18 farthings or 6 shillings 4 pence 2 farthings, which wants less than a farthing of the exact value of the decimal .319 *l.*

V. Having explained all the cases in *Multiplication* of *Decimals*; I shall here give the learner a taste of their excellent use, by some familiar questions, whereby it will be evident, that what is oftentimes performed by many tedious *Multiplications* and *Divisions* in the vulgar way, is effected for the most part by one or two *Multiplications* in *Decimals*.

See the questions from 49 to 73 in the 10th chapter of the Appendix.

The first *Example* may be this, suppose there is a certain piece of *Wainscot* in form a *rectangled paralelogram* commonly called a *long square*, whose breadth is 3 *yards*, $\frac{3}{4}$ of a *yard*, 1 *nail* and $\frac{1}{4}$ of a *nail*; and the length 6 *yards*, and $\frac{1}{2}$ of a *yard*, the question is to know

know how many *square yards* are contained in that piece of *Wainscot*; here because it is desired that the *superficial content* may be given in *yards*, the parts of a *yard* as well in the breadth as in the length of the *Wainscot* which are before exprest by the accustomed parts of *quarters*, *nails*, &c. must be reduced into decimal parts of a *yard*, which are as easie to be found by a yard subdivided decimally, as the common parts of quarters and nails are found by a yard vulgarly subdivided: but for want of a yard subdivided decimally, this *Reduction* may be performed by the seventh *Tablet* of the precedent *Table* of *Reduction*, *viz.* looking into the said *Tablet*, right against ¾ of a yard, I find this decimal ———— .75

Also the decimal correspondent to 1 nail is ———— .0625

And the decimal of ¼ of a nail is ———— .015625

The sum of those three decimals is ———— .828125

Wherefore the breadth of the Wainscot in yards and decimal parts is ———— 3.828125

Again, the decimal of half a yard is .5, wherefore the length of the Wainscot is ———— 6.5

The length and breadth being multiplyed one by the other produce the *superficial content*, therefore the number of square yards required is ———— 24.8828125

Wherefore I conclude that 24 square yards and somewhat more are contained in that piece of Wainscot,

Wainscot,and it is evident by the first place of the *decimal* that what is above 24 yards is more then $\frac{8}{10}$,but less then $\frac{9}{10}$ of a square yard,or more strictly, it is more then $\frac{88}{100}$ but less than $\frac{89}{100}$ of a square yard, but by taking all the places in the decimal you have the exact answer to this question,because the common parts of *quarters*,*nails*,and *quarters* of *nails* may be always exactly reduced into decimals, but that seldom happens in other things ; nevertheless, albeit by decimal operations you cannot alwayes hit the mark, yet you may come as near it as is possibly to be imagined, and that with much more ease then by vulgar computations in questions of this nature , as will appear by comparing the precedent operation with the common way of working here subjected , *viz.* the 3 *yards* , 3 *quarters* of a *yard* , 1 *nail* , and $\frac{1}{4}$ of a *nail* , (which express the breadth before mentioned) must all be reduced into quarters of *nails* by the sixth rule of the seventh Chapter , so there will be found 245 quarters of Nails , as you see by the operation.

	y.	q.	n.	q. n.
	3	3	1	1
	4			
	12			
add	3			
	15			
	4			
	60			
add	1			
	61			
	4			
	244			
add	1			
	245 *qua. of nails.*			

Again the 6 yards and half which express the length aforesaid , must likewise be reduced into quarters of Nails by the afore-

aforesaid *Rule*, so there will be found 416 quarters of nails of a yard, as you see by the operation.

	y.	*q.*
	6	2
	4	
	24	
add	2	
	26	
	4	
	104	
	4	
	416	*quarters of nails.*

Then multiplying the breadth and length one by the other, to wit, 245 by 416, the product will give 101920 for the superficial content of the piece of Wainscot in square quarters of nails of a yard; now these square quarters of nails of a yard must be reduced to square yards, and the readiest way to perform that is to find first of all how many quarters of nails of a yard are contained in one yard in length, *viz.* since there are 16 nails in a yard, there are consequenly 4 times 16 quarters of nails, to wit, 64 quarters of nails in a yard in length, therefore 64 multiplied by 64 produceth 4096 square quarters of nails in a yard square; lastly, I say by the Rule of three, if 4096 square quarters of nails of a yard give 1 yard square, how many

many yards square will 101920 square quarters of nails give: So will the Answer be found $24\frac{3616}{4096}$ yards, which is the same with 24.8828125 before found by the decimal operation (for $\frac{3616}{4096}$ is equal to the decimal .8828125, as will appear by reducing them to a common denominator by the fourteenth *rule* of the seventeenth *Chapter.*) Now I leave it to the Reader to judge which of these two wayes is the more expeditious, and so let him take which liketh him best.

Example 2. There is a squared piece of Timber terminated at both ends with equal long squares, *viz.* the breadth of the piece of Timber is 1 foot 5 inches, $\frac{1}{4}$ of an inch, and 1 half quarter of an inch; the depth or thickness is 1 foot 3 inches, $\frac{1}{4}$ of an inch, and $\frac{1}{8}$ or half quarter of an inch, and the length of the piece is 11 feet 10 inches, and 3 quarters, the question is how many solid or cubical feet are contained in that piece of Timber? The Answer may be found by *decimal Multiplication* in manner following, *viz.* Forasmuch as it is desired that the solid content may be given in feet, the parts of a foot as well in the breadth, depth, and length, which are before exprest by the accustomed parts of inches, quarters and half quarters must be reduced into the decimal parts of a foot, which are as easie to be found by a foot subdivided decimally, as the other common parts by a foot vulgarly subdivided; but for want of a foot subdivided decimally, this *Reduction* may be performed by the eighth *Tablet* of the precedent *Table* of *Reduction, viz.*

The

The decimal correspondent to 5 inches is ———— .416

The decimal of $\frac{3}{4}$ of an inch is ——— .062

The decimal of half a quarter of an inch is ———— .01

The sum of those 3 decimals is ——— .488

wherefore the breadth of the piece of Timber is ———— 1.488

In like manner the common parts of inches, *&c.* in the depth or thickness of the piece of Timber will be reduced by the said *Tablet*, into these decimals, *viz.*

The decimal correspondent to 3 inches is — .25

The decimal of $\frac{1}{4}$ of an inch is ——— .02

The decimal of half a quarter of an inch is — .01

The sum of these 3 decimals is ——— .28

Wherefore the depth or thickness is —— 1.28

Again, the accustomed parts of inches, &c. in the length of the piece of Timber will be reduced to these decimals, *viz.*

The decimal of 10 inches is ——— .833

The decimal of $\frac{3}{4}$ of an inch is ——— .062

The sum of those 2 decimals is ——— .895

Wherefore the length of the piece is —— 11.895

Now if the breadth depth and length be multiplied continually, the last product is the solid content required, *viz.* 1.488 multiplied by 1.28 produceth 1.90464, which multiplied by 11.895 produceth 22.65, *&c.* wherefore I conclude that 22 solid Feet, half a foot, and somewhat more than half a quarter of a foot are contained in that piece of Timber.

Example 3. How many *Equinoctial degrees* are correspondent unto 136 *days*, 22 *hours*, and 40 *minutes*?

nutes? The Answer is found by multiplying the time given by 360, for as 1 *day* is to 360 *degrees*, so 136 *dayes*, 21 *hours*, and 40 *minutes*, to the *Equinoctial degrees* required; but first the 21 *hours* and 40 *minutes* must be reduced to decimal parts of a day, by the tenth *Tablet*, thus.

The decimal of 21 hours is ———————— .875
The decimal of 40 minutes is ——————— .02777
The sum of these 2 decimals is ——————— .90277
Therefore the time propounded is — 136.90277
Which being multiplied by 360 producethe ———————————— } 49284.99 &c.

Wherefore I conclude that 49284.99 or very near 49285 *Equinoctial degrees* are correspondent unto 136 *dayes*, 21 *hours*, and 40 *minutes*, which was required by the question.

CHAP. XXVII.

Concerning Division by Decimal Fractions.

I. IN any of the Cases which may happen in Division, if the Dividend be greater than the Divisor, the quotient will be either a whole number or else a mixt number, but when the Dividend is less than the Divisor, the quotient must necessarily be a fraction; for a lesser number is contained in a greater once at the least, but a greater is not contained once in a lesser.

II. Sometimes the Dividend, whether it be a whole number, mixt number, or decimal fraction,

is to be prepared by annexing a competent number of cyphers thereunto, to make room for the Divisor: so if 32.5 were given to be divided by 17.325 the Dividend 32.5 must be increased with cyphers at pleasure after this manner 32.50000, &c. Likewise if 1 were given to be divided by 360, the Division cannot be made till the Dividend 1 be increased with cyphers, which being annexed, the Dividend will stand thus 1.000000, &c. Here note, that the cyphers annexed in manner aforesaid do supply places of decimal parts, and will be usefull in discovering the quality of the quotient according to the fourth *Rule* of this *Chapter*.

III. After the Dividend is prepared by annexing cyphers, when occasion requires, (as in the last Rule) all the places thereof must be esteemed as one whole number (to wit consisting of unities or Integers) and so is the Divisor to be esteemed whether it be a decimal fraction or mixt number; for in all cases the Division must be performed in every respect according to the rules of Division of whole numbers in the sixth Chapter. So if this mixt number 326.25 were given to be divided by this mixt number 12.3, you must divide in the same manner, as when you divide 32625 Integers by 123 Integers, also if this decimal .8356 were given to be divided by this decimal .05, you are to divide in the same manner, as when you divide 8356 Integers by 5 Integers; and after the quotient is found, the degree or place of the first figure which ariseth in the quotient must be inquired after; *viz.* you must know how far such first figure is distant from the place of units, to the end that the point or line which is used to separate between the

place of unites (or first place of Integers) and the first place of decimals may be duly placed: This is the only knot in decimal Division, and may be resolved by the following Rule, *viz.*

A general rule to discover the quality of the quotient in all cases of Division by decimal Fractions.

IV. In any of the Cases which may happen in Division of decimals, the first figure which ariseth in the Quotient, will be alwayes of the same place or degree with that figure or cypher of the Dividend, which at the first question standeth over, or at least belongeth unto the place of units in the Divisor. To illustrate this Rule I shall give examples in all cases, and first let a mixt number be given to be divided by a mixt number, *viz.* Let it be required to divide 172.5 by 3.746, here (according to the second Rule of this Chapter) the Dividend must be increased with cyphers at pleasure, so will it stand thus 172.500000, &c. then Division being made according to the rules of Division of whole numbers in Chapter 6, the Quotient arising will be 46049.

3.746) 172.500000 (46049, &c.

Now it remaineth to separate the Integers in this quotient from the decimal parts; to perform which, I subscribe the Divisor 3.746 orderly underneath

3.746) 172.500000 (46,049

3.746

the first Dividual 172.50 (being that part of the Dividend

Dividend whereof the firſt queſtion muſt be asked) or at leaſt I imagine the Diviſor to be ſo ſubſcribed, and ſo I find that the figure 3 which ſtands in the place of Units in the Diviſor will be placed under 7, which is the place of tens, (or ſecond place of Integers) in the Dividend, wherefore by the fourth Rule before given, I conclude that the firſt figure ariſing in the quotient muſt likewiſe ſtand in the place of tens (or ſecond place of Integers) and conſequently the next place on the right hand muſt be the place of *Units*; ſo it is evident that the ſeparating point or line muſt be placed between the figure 6 and 0 in the quotient, that done, the true quotient is found to be 46.049, &c. to wit, 46 Integers and $\frac{49}{1000}$ parts of an Integer, and ſomewhat more, for $46\frac{49}{1000}$ is leſs than the true quotient, but $46\frac{50}{1000}$ is greater than it, and therefore albeit, after the aforeſaid Diviſion of 172,500000 by 3.746 is ended, there will be a remainder, to wit 446 which ſeems to be great, yet here it is leſs in value than $\frac{1}{1000}$ part of an Unit or Integer, and if to that remainder you annex another cypher and continue the Diviſion, you will proceed nearer the truth and not miſs $\frac{1}{10000}$ part of an unit of the true quotient, and in that order you may proceed infinitely near, when you cannot obtain the quotient exactly by Diviſion of Decimals.

Example 2. Suppoſe this mixt number 2.34 be given to be divided by this mixt number 52.125; (where you may obſerve that the Dividend is leſs than the Diviſor) firſt (as before) annex cyphers at pleaſure to the Dividend, to make room for the Diviſor, then the diviſion being proſecuted as in

whole numbers, at length these figures will arise in

52.125) 2.3400000 (.0448, &c.

52.125

the quotient, to wit, 448; and to the end the degree or quality of the first figure 4 may be discovered, I subscribe the Divisor 52.125 under the first dividual 2.34000 (for so far the first question did extend in the Division) and thereby I find that the figure 2 which stands in the place of units in the divisor will be seated under 4, which is in the second place of decimals, wherefore I conclude that the first figure arising in the quotient must also stand in the second place of decimals, and consequently the first place of decimals (which is next on the left hand to the second) must be supplied with a cypher; so that if a cypher be prefixed on the left hand of 4, and then a point placed before that cypher, the quotient will at length be discovered to be .0448, &c. or $\frac{448}{10000}$ and somewhat more; that is to say, $\frac{448}{10000}$ is less than the true quotient, but $\frac{449}{10000}$ is greater than it, and if you will proceed nearer the truth, you may continue the division, as is directed in the first Example of this Rule.

Example 3. Where a whole number is divided by a decimal fraction, *viz.* suppose 82 Integers were given to be divided by this decimal .056; After cyphers are annexed to the dividend at pleasure, and

.056) 82.000000 (1464|28, &c.

the

the division prosecuted as in whole numbers (to wit, 8200000 being divided by 56) these figures 146428 will arise in the quotient, now to the end the degree or seat of 1, the first figure in the quotient may be known. I subscribe the divisor .056 under the first dividual 82 (for so far did the first question in the division extend) and because the divisor is less than unity, I supply the place of units by a cypher or 0 prefixed on the left hand of the point of separation in the divisor; also I pre-

.056) 0082.00000 (1464.28, &c.

0.056

fix cyphers before, (to wit on the left hand of) the Integers in the dividend to represent a succession of places of Integers, (for the order of places in Integers is from the right hand towards the left) then I find that the cypher or 0 which represents the place of units in the divisor, doth stand under that cypher, which represents the fourth place of Integers in the *dividend*, (as you see by the example in the Margent) wherefore I conclude that the first figure arising in the quotient must also be seated in the fourth place of Integers, and consequently the 4 first places in the quotient will be Integers, and the rest a decimal, so that the true quotient is 1464 Integers, and $\frac{28}{100}$ parts of an Integer, and somewhat more, *viz.* 1464.28 is less than the true quotient, but 1464.29 is greater than it.

Example 4. Suppose this decimal .0125 be given to be divided by this decimal .5: after division is finished according to the Rules of division of

.5) .0125 (25

whole numbers, (to wit after 125 is divided by 5) these figures 25 will arise in the quotient; now to discover the degree or seat of 2 the first figure in the quotient, I subscribe the divisor .5 under the first dividual .012, and having (as in the last example) prefixed a cypher on the left hand of the point of separation in the divisor, to denote or represent the place of units, I find that such cypher or place of units doth stand under the figure 1, which is seated in the second place of decimals in the dividend, wherefore I conclude by the Rule, that the first figure which ariseth in the quotient must also be in the second place of decimals, and therefore prefixing a cypher to supply the first place of decimals, and putting a point before that cypher, the quotient is at length discovered to be .025 or $\frac{25}{1000}$.

.5) .0125 (.025
.
0.5

Example 5. Suppose this decimal .8564 be given to be divided by this .008, first I annex cyphers to the dividend at pleasure then prosecuting the division as in whole numbers, to wit dividing 856400 by 8, the quotient arising is 107050, now to discover the degree or place of 1, the first figure in the quotient, I subscribe the divisor .008 under the first dividual .8, then I prefix a cypher to set forth or supply the place of units in the divisor, also I prefix cyphers to represent places of Integers in the dividend, that done, I find that the cypher or 0 which supplieth

.008) .856400 (107.050

.008) 000.85640 (107.05
.
0.008

plieth the place of units in the divisor, doth stand under that cypher which is seated in the third place of Integers in the dividend, wherefore I conclude by the Rule, that the first figure arising in the quotient must be also in the third place of Integers, and consequently the three first places in the quotient will be Integers, and the rest a decimal, so that the true quotient is 107.05 or 107 $\frac{5}{100}$.

Example 6. Let it be required to divide this decimal fraction .73952 by this .32, first dividing 73952 by 32 as if they were whole numbers, the figures arising in the quotient will be 2311. Now to discover the quality or value of the said figures I subscribe the *Divisor* .32 under the first *dividual* .73, then prefixing a cypher as well on the left hand of the *dividend*, as of the *divisor* so subscribed (or imagined to be subscribed) as aforesaid, to represent the place of units in each of them, I find the cypher or 0 which supplyeth the place of units in the *divisor* to stand under the 0 which represents the place of units in the *dividend*, wherefore I conclude by the preceding fourth *Rule*, that the first figure arising in the *quotient* will stand in the place of units, and consequently the following places of the *quotient* will be a *decimal* fraction, so that the true *quotient* is 2.311 or 2$\frac{311}{1000}$.

.32) 0.73952 (2.311
0.32

The reason of the foregoing fourth Rule will appear from the following considerations.

1. If the *divisor* be multiplied by the *quotient*, the *product* is equal to the *dividend*.

2. If

2. If the *diviſor* be multiplied by the firſt figure which ariſeth in the *quotient*, the *product* is the firſt number to be ſubtracted in the *diviſion* and conſequently every particular place of that *product* is of the ſame degree with that figure or cypher of the *dividend*, which ſtands over ſuch particular place when the ſubtraction is to be made, for numbers of unlike denominations cannot be ſubtracted one from the other.

```
                 14.35
                 18.75
              ---------
                  7175
                10|045
               114|80
               143|5
             ---------
14.35)  269|.0625 (18.75
        143|5
        -------
        125|56
        114|80
        -------
         10|762
         10|045
         -------
             7175
             7175
           -------
                0
```

3. So that to find the degree or ſeat of the firſt figure in the *quotient*, is nothing elſe but to find what degree or ſeat that figure muſt have, which

multi-

multiplying the figure or cypher in any particular place of the *divisor*, will produce the same degree as that figure or cypher of the *dividend* hath, which stands over or at least belongs to such particular place of the *divisor* at the first question.

4. And because if a figure standing or supposed to stand in the place of unities in the *divisor*, be multiplied by a figure of the same place or degree, as that figure or cypher of the *dividend* hath, which at the first question standeth over, or at least belongeth unto the said place of unities, the first place arising in that *product* will be of the same degree or place with the said figure or cypher of the *dividend*; (whether it be a place of Integers or decimal parts) therefore the truth of the said fourth *Rule* of this *Chapter* is manifest, namely, look what degree or place that figure or cypher of the *dividend* hath, which stands over or at least belongs to the place of *unities* in the *divisor* at the first question in *Division*, the same degree or place hath the first figure in the *quotient*.

Now that the benefit of *Division* by *decimal* fractions may partly appear, I shall add two questions, and so conclude this Chapter.

Quest. 1. A Merchant bought of gold *Plate* 356 *ounces*, 13 *penny weight*, and and 15 *grains* for 1160 *pounds sterling*, the question is what he paid for an *ounce*? *Answer* 3 *l.*—5 *s.*—$\frac{1}{2}$ *d.* very near. The operation by *decimals* may be after this manner, *viz.*

By the second *Tablet* of *Reduction* the *decimal* of 13 *penny weight* is—— .65.
The *decimal* of 15 *grains* is———— .03125
The *sum* of these 2 *decimals* is——— .68125

Wherefore

Wherefore the quantity of *Plate* in *ounces* and *decimal parts* of an *ounce* is ——————— } 356.68125

Then by the *Rule* of three I say, if 356.68125 *ounces* cost 1160 *pounds*, what 1 *ounce*? Here 'tis evident that if I divide 1160 by 356.68125, the *quotient* will give the value of an *ounce*, to wit, 3.252 *pounds*, or 3 *pounds*, 5 *shillings* and $\frac{1}{2}$ *d.* very near.

356.68125) 1160.0000000 (3,252, &c.

Quest. 2. Suppose the length of the *Tropical year* (or the space of time wherein the Sun running through the whole *Ecliptick* circle, consisting of 360 *degrees* is returned to the same *Equinoctial* or *Solstitial* point from whence he departed) to consist of 365 *dayes*, 5 *hours*, and 49 *minutes*, the question is to know the Suns mean or equal motion for 1 *day*, to wit, what part of 360 *degrees* the Sun moveth in a whole day? The operation by *decimals*, thus,

By the tenth *Tablet* of *Reduction* the *decimal* correspondent to 5 *hours* is ——————— } .2083333

The *decimal* of 49 *minutes* is ——— .0340277

The *sum* of those *decimals* is ——— .2423610

Wherefore the *time* given, in *dayes* and *decimal parts* of a *day* is ——— } 365.2423610

Then by the Rule of three, if 365.242361 *dayes* give 360 *degrees*, (or a total circumference) what will 1 *day* give? Here if I divide 360 by 365.242361, the *quotient* will give the diurnal motion required, which will be found very near .98564, &c. or $\frac{98564}{100000}$ parts of a degree, which *decimal* being reduced into the common *Sexagenary parts* (by the fourth

fourth *Rule* of the 26 *Chapter*) will give 59—8, &c. and such is the Suns diurnal motion very near, according to the aforesaid supposition of the length of the *Tropical year*.

I shall here add the vulgar *Sexagenary* resolution of this question, that by comparing both wayes together, the excellency of *decimal Arithmetick* in calculations of this nature may be the more perspicuous.

The aforesaid question being stated according to the *Rule* of three will stand thus,

dayes	*hours*	*degrees*	*day*
If 365 :	5 : 49	——— 360 ———	1

The first term in the Rule must be reduced into *minutes* (by the sixth *Rule* of the seventh *Chapter*) so there will be found 525949 *minutes*.

```
            D.     h.
            365 —— 5 —— 49
             24
          ————————
           1460
           730
          ————————
           8760
     add      5
          ————————
           8765   hours
             60
          ————————
         525900
     add     49
          ————————
         525949   minutes
```

Like-

Likewise the third term 1 *day* must be reduced into *minutes*, which will be found to be 1440, as you see by the following operation.

1 *Day* or 24 *hours*.
60

1440 *minutes*

Then multiplying the third term by the second, to wit 1440 by 360, the product is 518400, which being divided by the first term 525949 (according to the note in the ninth *Rule* of the 16th *Chapter*) the *quotient* will give $\frac{518400}{525949}$ parts of a degree, which fraction being reduced into the accustomed *Sexagenary* parts (by the ninth *Rule* of the seventeenth *Chapter*) will give as before 59 : 8, &c. for the Suns mean diurnal motion; now which of these two wayes is the more expeditious I leave to him who is verst in both to determine.

CHAP. XXVIII.

The Rule of Three Direct in Fractions.

I. TO repeat such things as have already been declared in reference to the definition of this Rule, and also the due placing of the 3 given numbers would be superfluous; and if respect be had to the Rules of *Multiplication* and *Division* of *fractions* delivered in the 20 and 21 Chapters, the working

working of the Rule of three direct in fractions is the ſame with that in whole numbers, to wit, multiply the ſecond number by the third, (or the third by the ſecond) and divide the product by the firſt number, ſo the quotient is the fourth number ſought; to wit, the anſwer of the queſtion.

Otherwiſe thus,

Multiply the Denominator of the firſt number by the Numerator of the ſecond, alſo multiply that product by the Numerator of the third number and reſerve this laſt product for a new Numerator; again multiply the Numerator of the firſt number by the Denominator of the ſecond, alſo multiply this product by the Denominator of the third number, ſo ſhall this laſt product be a new Denominator; laſtly, the new fraction (whoſe Numerator and Denominator is found as aforeſaid) is the fourth number ſought, which, if it be a proper fraction, may (if occaſion require) be reduced into the known parts of the Integer: (by the ninth Rule of the ſeventeenth Chapter) if an improper fraction, it is to be reduced into its equivalent whole number or mixt number, by the thirteenth Rule of the ſeventeenth Chapter.

Example, If $\frac{3}{4}$ of a yard of Velvet be ſold for $\frac{2}{3}$ of a pound *ſterling*, what ſhall $\frac{5}{6}$ of a yard coſt? *Anſwer* $\frac{40}{54}$ *l.* or 14 *s.* $9\frac{7}{9}$ *d.* For according to the *Rule* I multiply the Denominator 4, by the Numerator 2, and the *product* is 8, this 8 I again multiply by the Numerator 5, and the *product* gives 40 for a new Numerator: moreover I multiply the Numerator 3 by the

y.		*l.*		*y.*		*l.*
$\frac{3}{4}$	—	$\frac{2}{3}$	—	$\frac{5}{6}$	——	($\frac{40}{54}$

the Denominator 3, and the *product* which is 9 I again multiply by the Denominator 6, so the last *product* is 54 for a new Denominator; wherefore I conclude that $\frac{48}{54}$ is the fourth number sought, which if it be reduced (according to the ninth Rule of the seventeenth Chapter) gives 14 *s.* $9\frac{48}{54}$ *d.* (or $9\frac{8}{9}$ *d.*) for the *Answer* of the question.

II. When any of the three given numbers is a whole number or mixt number, such number must first of all be reduced into an improper fraction (by the tenth or eleventh *Rule* of the seventeenth *Chapter*) to the end that all the 3 given numbers may be 3 fractions: moreover if after such Reduction, the first and third numbers be not fractions of Integers of the same particular denomination, such of the said numbers which is of the lesser denomination, must be reduced to a fraction of the greater (by the sixteenth *Rule* of the seventeenth *Chapter*) which preparations being performed, the rest of the work is to be prosecuted according to the first *Rule* of this *Chapter*. An *Example* of this second *Rule* here followeth. If a quantity of *Ambergreece* weighing $1\frac{2}{7}$ *lb. Troy* be sold for 60 *l. sterling*, what are $19\frac{1}{8}$ *grains* worth at that rate? *Answer* $\frac{65940}{552960}$ *l.* or 2 *s.* $4\frac{119}{192}$ *d.*

This question being stated according to the 7 *Rule* of the 8 *Chapter* will stand thus, — which 3 numbers will be reduced (by the tenth and eleventh *rules* of the seventeenth *chapter*) into these *improper fractions*, ——

lb.	*l.*	*gr.*
$1\frac{2}{7}$	60	$19\frac{1}{8}$

lb.	*l.*	*gr.*
$\frac{9}{7}$	$\frac{60}{1}$	$\frac{153}{8}$

But

But since the third number $\frac{157}{8}$ *grains Troy* is not a fraction of an Integer of the same name with the first, (which is a fraction of a pound *Troy*) it must be reduced into a fraction of a pound *Troy*, thus, $\frac{157}{8}$ *gr.* is $\frac{157}{8}$ of $\frac{1}{24}$ of $\frac{1}{20}$ of $\frac{1}{12}$ of a pound *Troy*, which compound fraction will be reduced (by the 16 *Rule* of the 17 *Chapter*) into this single fraction, to wit, $\frac{157}{46080}$ *lb. Troy*, and so the 3 numbers will at length stand thus in the *rule*.

$$\frac{12}{7}\ lb. \text{———} \frac{69}{5}\ l. \text{———} \frac{157}{46080}\ lb.$$

Then working as in the first *example* of this *chapter*, the Answer will be found $\frac{65940}{552960}$ *l.* which being reduced (according the 9 and 4 *rules* of the 17 *Chapter*) is found equal unto 2 *s.* $4\frac{119}{192}$ *d.*

Another *Example*. When the $\frac{2}{5}$ of $\frac{3}{4}$ of a *Ship* is valued at 147 *l.*——11 *s.*——3 *d.* how much is the whole *Ship* worth? *Answ.* 491 *l.*——17 *s.*——6 *d.*

Note, when in any question whatsoever a compound fraction, to wit, a fraction of a fraction, is one of the given numbers, such compound fraction must first of all be reduced to a single fraction; (by the 16 *rule* of the 17 *chapter*) so here, the *compound fraction* $\frac{2}{5}$ of $\frac{3}{4}$ being reduced into a *single fraction* gives $\frac{6}{20}$ or $\frac{3}{10}$; then say, if $\frac{3}{10}$ be worth 147 *l.* 11 *s.* 3 *d.* what is 1 or the whole *Ship* worth?

Ship	*l.*	*s.*	*d.*	*Ship*
$\frac{3}{10}$ ——	147 :	11 :	3 ——	1

After due reduction is made by converting the 147 *l.* 11 *s.* 3 *d.* into pence, and that number of pence, as also the third number 1, into im-

proper fractions, the 3 numbers will stand in the Rule thus,

$$\begin{array}{ccc} \textit{Ship} & \textit{pence} & \textit{Ship} \\ \frac{3}{10} \text{———} & \frac{35415}{1} \text{———} & \frac{1}{2} \end{array}$$

Lastly, proceeding as in the first Rule of this Chapter, the fourth number will be found to be $\frac{354150}{3}$ *d*, which being reduced first by the 13 *rule* of the 17 *chapter*, and then by the 7 *rule* of the 7 *chapter*, the *Answer* at length is 491 *l.*——17 *s.*——6 *d.*

An Example of the Rule of three direct in *Decimals* may be this that follows. If 19 *ounces*, 3 *penny weight*, and 5 *grains* of Gold, be worth 62 *l.*——10 *s.*——6 *d.* what is the value of $1\frac{1}{2}$ *ounce*? *Answer* 4 *l.*——17 *s.*——$10\frac{1}{2}$ *d.* very near.

By the 2. *Tablet* in the *Table* of *Reduction* in the 23 Chapter, the *decimal fraction* correspondent to 3 *penny weight* is —————— } .15

Also, the *decimal* of 5 *grains* is ——— .010416

The sum of those 2 *decimals* is ——— .160416

Wherefore the first number in the rule of three is —————— } *oz.* 19.160416

Again, by the first *Tablet* of the aforementioned *Table*, the *decimal* of 10 *shillings* is —————— } .5

Also the decimal of 6 *pence* is ——— .025

The sum of these two *decimals* is ——— .525

Wherefore the second number in the rule of three is —————— } *l.* 62.525

Moreover by the said *Tablet* 2. the *decimal* of $\frac{1}{2}$ of an *ounce* or 10 *penny weight* is .5, wherefore the third number in the rule of three is ——— } *oz.* 1.5

So

So that after the said reduction is finisht, the 3 given numbers will stand in the rule thus,

oun.	*l.*	*oun.*
19.160416 ——	62.525 ——	1.5

Lastly, multiplying the second number by the third, and dividing the product by the first number (according to the Rules of Multiplication and Division of Decimals delivered in the 26 and 27 Chapters) the fourth number will be this, to wit, 4.894, &c. that is four pounds *sterling* and $\frac{894}{1000}$ parts of a pound, which *decimal* being reduced (according to the fourth *rule* of the 26 *Chapter*) gives 17 *s.* — 10 *d.* — 3 *far.* very near.

The proof of the Rule of three direct in Fractions is the same as in whole numbers, respect being had to the rules of Multiplication in Fractions.

CHAP. XXIX.

The Inverse Rule of Three in Fractions.

I. After a question belonging to this rule is duly stated (according to the seventh rule of the eighth Chapter) and prepared if need require, according to the second rule of the 28 Chapter. The operation will be the same as in the rule of three Inverse in whole numbers, respect being had to the rules of Multiplication and Division in Fractions, *viz.* multiply the first number by the second, and divide the product by the third; the quotient is the fourth number sought, to wit, the answer of the question.

Or thus,

Multiply the Denominator of the third fraction by the Numerator of the second, also multiply that product by the Numerator of the first fraction, and reserve the last product for a new Numerator: again multiply the numerator of the third fraction by the denominator of the second, also multiply this product by the denominator of the first fraction, so is the last product a new denominator; lastly, this new fraction is the fourth number sought, or answer of the question.

Example, If of cloath, which is $1\frac{3}{4}$ yard in breadth, $3\frac{1}{2}$ yards in length will make a Cloak, how much in length of that stuff which is $\frac{5}{8}$ yard in breadth will make a Cloak of the same bigness with the former? *Answer* $9\frac{4}{5}$ yards.

The 3 numbers being duly placed will stand thus——— *brea.* $1\frac{3}{4}$ *y.* — *leng.* $3\frac{1}{2}$ *y.* — *brea.* $\frac{5}{8}$ *y.*

Then, (after the first and second numbers are reduced into improper fractions) the three numbers will stand thus——— $\frac{7}{4}$ —— $\frac{7}{2}$ —— $\frac{5}{8}$.

Lastly, 8, 7 and 7 being multiplied continually give 392 for a numerator; also 5, 2 and 4 being multiplied continually give 40 for a denominator, whereby this improper fraction $\frac{392}{40}$ ariseth, which (by the thirteenth rule of the seventeenth Chapter) will be found to be $9\frac{32}{40}$, or (the fraction being reduced into its least terms) $9\frac{4}{5}$ which is the *Answer* of the question.

*Ex.*2. Suppose when Wheat is at 2 *l.*—00 *s.*—6 *d.* the Quarter, the penny white loaf ought to weigh

8

8 ounces and 1 $\frac{5}{19}$ *penny weight* of *Troy* weight; what ought it to weigh when Wheat is at 36 shillings the Quarter? *Answer* 9 ounces and 1 $\frac{37}{116}$ *penny weight.*

The 3 given numbers being duly placed in the rule and reduced will stand thus, —— } *pence* $\frac{486}{1}$: *p. w.* $\frac{4674}{19}$: *pence* $\frac{438}{1}$

And if the operation be prosecuted according to the rule before given, the *Answer* will be found $181\frac{3996}{12528}$ penny weight, or 9 ounces, 1 $\frac{37}{116}$ penny weight.

CHAP. XXX.

The Double Rule of Three in Fractions.

I. THe *Double Rule of Three* is so called, because it is composed of two single rules, and may either be resolved at one work by the rule compound of 5 numbers, or else by two distinct single rules of three; which latter way, to such as understand the rule of three in fractions is (as I conceive) less troublesome in the stating, and (in the method whereby I intend to prosecute it) the same in operation with the former. This I shall manifest first in whole numbers, then in fractions.

Example 1. If I pay 28 shillings for the carriage of 3 C. weight for 50 miles, how much ought I to pay for the carriage of 17 C. for 84 miles? *Answer* 13 *l.*—6 *s.*—6 *d.* $\frac{18}{25}$.

Of the 5 given numbers I make choice of three such which will make a single rule of three, and say,

	C.	*shil.*	*C.*
If	3 ——	28 ——	17

 Which

Which rule I find (by the third rule of the ninth Chapter) to be direct, and therefore I multiply the third number 17 by the second 28, and the product which is 476 I place as a numerator over the divisor as denominator. Then with this fraction (whether it happen to be a proper or improper fraction) and the remaining two numbers in the question, which have not yet been used, I form a second rule of Three, and say,

$\frac{476}{3}$

	miles		shill.		miles
If	$\frac{50}{1}$	——	$\frac{476}{3}$	——	$\frac{84}{1}$

Which being a rule of three direct, I work as a rule of three in fractions, according to the first rule of the 28 chapter, and so find the fourth number to be $\frac{39984}{150}$ s. or 13 l.—6 s.—6$\frac{18}{25}$ d.

Or the first single rule being varied, the operation will be thus,

	miles	C.	miles	C.
1. *By a rule inverse,*	50 —	3 —	84 —	($\frac{150}{84}$
	C.	sh.	C.	sh.
2. *By a rule direct,*	$\frac{150}{84}$:	$\frac{28}{1}$:	$\frac{17}{1}$:	($\frac{39984}{150}$

Otherwise thus,

	C.	m.	C.	m.
1. *By a rule inverse,*	3 —	50 —	17 —	($\frac{150}{17}$
	m.	sh.	m.	sh.
2. *By a rule direct,*	$\frac{150}{17}$:	$\frac{28}{1}$:	$\frac{84}{1}$:	($\frac{39984}{150}$

Thus you see the 2 single rules to be varied three manner of wayes in resolving the question propounded;

propounded, and each way produceth the same *Answer*; the like diversity may be found in all questions resolvable by the double rule of three, or rule compound of 5 numbers.

Example 2. If $40\frac{3}{5}$ *l.* in $\frac{2}{3}$ of a year gain $2\frac{1}{2}$ *l.* what will 100 *l.* gain after that rate in $\frac{7}{12}$ of a year? *Answ.* 5 l.—7 *s.*—9 $\frac{3}{29}$ *d.*

By 2 Single rules of three, thus,

	l.	*l.*	*l.*	*l.*
1. *By a rule direct,*	$\frac{203}{5}$:	$\frac{5}{2}$:	$\frac{100}{1}$:	($\frac{2500}{406}$
	year	*l.*	*year*	*l.*
2. *By a rule direct,*	$\frac{2}{3}$:	$\frac{2500}{406}$:	$\frac{7}{12}$:	($\frac{52500}{9744}$

Or by these two single rules,

	year	*l.*	*year*	*l.*
1. *By a rule direct,*	$\frac{2}{3}$:	$\frac{5}{2}$:	$\frac{7}{12}$:	($\frac{105}{48}$
	l.	*l.*	*l.*	*l.*
2. *By a rule direct,*	$\frac{203}{5}$:	$\frac{105}{48}$:	$\frac{100}{1}$:	($\frac{52500}{9744}$

Otherwise thus,

	l.	*year*	*l.*	*year*
1. *By a rule inverse,*	$\frac{203}{5}$:	$\frac{2}{3}$:	$\frac{100}{1}$:	($\frac{406}{1500}$
	year	*l.*	*year*	*l.*
2. *By a rule direct,*	$\frac{406}{1500}$:	$\frac{5}{2}$:	$\frac{7}{12}$:	($\frac{52500}{9744}$

Thus by 2 single rules of three varied three several wayes, you see the *Answer* of the question to be $\frac{52500}{9744}$ *l.* to wit, 5 *l.*—7 *s.*—9$\frac{3}{29}$ *d.*

CHAP. XXXI.

The Rule of False in Fractions.

I. WHen a question propounded cannot readily be applyed to the *Rule of three*, or any of the vulgar rules in *Arithmetick*; the best refuge for such as are not acquainted with *Algebra* is the rule of *two false Positions*, which, for that it hath already been handled in *whole numbers*, I shall the more briefly touch upon in *Fractions*.

II. When a number is sought by a question, you are to feign or suppose some number taken by guess to be the number sought, and to make tryal whether that feigned number will answer the conditions in the question or not, by comparing the number resulting at the end of the work, with the given number resulting from the true number sought; and if you find both those results to be the same, then is the number which you first took by guess the true number or answer of the question, but if the number resulting from the supposititious number be either greater or less than the given result, with which it ought to be compared (to see whether you have hit the mark or not) such excess or defect must be noted for the Error of the first Position, to wit, an excess must be signified by this note +, and a defect by this —.

III. In like manner a second number must be feigned, and after tryal is made therewith, to see whether it will perform the conditions prescribed in the question, by comparing the results as afore-

said,

said, the error of this second Position, if too much, is to be noted by +; if too little by —, as before.

W. After the errors of both Positions are discovered, the two numbers before supposed or feigned to be the number sought, must be multiplied by the altern errors, that is, the first Position by the second error, and the second Position by the first error; then if the notes of the errors are unlike, to wit, one of them +, and the other —, the sum of the said products is to be taken for a dividend, and the sum of the errors for a divisor; but if the notes of the errors are both alike, to wit, both of them +, or both —, the difference of the said products is to be taken for a dividend, and the difference of the errors for a divisor; lastly, the quotient arising from the division made by the said dividend and divisor, gives the true number sought, or answer of the question, if it be solvable by the *Rule of False*. These rules are the same in substance with those delivered in the 15 chapter, and may be farther illustrated by the following questions.

Quest. 1. A Gentleman hired a servant for a year for 6 pounds *sterling*, and a livery Cloak valued at a certain rate; but it happened that $\frac{7}{12}$ of the year being expired they fell at variance, and the Gentleman put away his servant, giving him the Cloak together with 50 shillings in money, which was the servants full due for the time of his service, the question is to find what the Cloak was valued at? *Answ.* 2 *l.* —— 8 *s.* —— 0 *d.*

1. I suppose the Cloak to be valued at 3 pounds, and then seek how much thereof was due to the servant,

$\begin{array}{lll} y. & l. & y. \\ 1 - & 3 - & \frac{7}{12}(\frac{21}{12} \text{ or } \frac{7}{4} \text{ li.} \end{array}$ servant, saying, if one year give 3 *l.* how much $\frac{7}{12}$ of the year? *Ans.* $\frac{7}{4}$ *l.*

2. I likewise find what part of the 6 pounds was due to the servant at

$\begin{array}{lll} y. & l. & y. \\ 1 - & 6 - & \frac{7}{12}(\frac{42}{12} \text{ or } \frac{7}{2} \text{ l.} \end{array}$ the end of $\frac{7}{12}$ of the year, saying, if 1 year give 6 pounds, how much $\frac{7}{12}$ of the year? *Answer*, $\frac{7}{2}$ *l.*

3. For as much as the Cloak together with the money which the servant received ought to be equal to the part of the Cloak, together with the part of the 6 pounds wages due to him at the end of $\frac{7}{12}$ of the year; therefore 3 *l.* (the supposed value of the Cloak) together with $2\frac{1}{2}$ *l.* (the money which the servant received) should be equal to $\frac{7}{4}$ of a pound (the value of part of the Cloak due to the servant at the end of $\frac{7}{12}$ of the year) together with $\frac{7}{2}$ *l.* (the wages due for the same time) that is to say, $\frac{11}{2}$ *l.* (the sum of 3 *l.* and $2\frac{1}{2}$ *l.*) should be equal to $\frac{21}{4}$ *l.* (the sum of $\frac{7}{4}$ *l.* and $\frac{7}{2}$ *l.*) but it is greater by $\frac{1}{4}$, wherefore the first Position for the value of the Cloak being 3 pounds, the error is found to be $\frac{1}{4}$ too much.

4. I make a second Supposition guessing the value of the Cloak to be 2 pounds, and proceeding in every respect as with the first supposition I find the *error* to be $\frac{1}{6}$ too little; so that the two *Positions* with their *errors* will be as you see:

Pos.		*Er.*
3	+	$\frac{1}{4}$
2	—	$\frac{1}{6}$

Now

Now in regard the *errors* are *fractions* I may take in their ſtead, whole numbers in the ſame proportion, to wit, multiplying the *Numerator* of the firſt fraction (or firſt *error*) by the *Denominator* of the ſecond, I take the product which is 6 in ſtead of the firſt *error* $\frac{1}{4}$; likewiſe multiplying the *Numerator* of the ſecond fraction, by the *Denominator* of the firſt, I take the product which is 4 inſtead of the ſecond *error* $\frac{1}{6}$; Or inſtead of the ſaid 6 and 4 I may take 3 and 2 which are in the ſame proportion with 6 and 4, (or with $\frac{1}{4}$ and $\frac{1}{6}$:) Then multiplying the *Poſitions* and new *errors* croſswiſe, and adding the products together (becauſe the ſigns are unlike) the ſum is 12 for a *Dividend*, and the ſum of the *errors* 3 and 2 is 5 for a *Diviſor*, ſo the quotient will be found to be $2\frac{2}{5}$ *l.* ſo much therefore was the value of the Cloak, as will eaſily appear if tryal be made with $2\frac{2}{5}$ *l.* in the ſame manner as with the firſt feigned number.

Pos.		*Er.*		
3	+	$\frac{1}{4}$	6	3
2	—	$\frac{1}{6}$	4	2
6				
6				

5) 12 ($2\frac{2}{5}$ *pounds*.

Queſt. 2. *Vitruvius* (*in lib.* 9. *cap.* 3.) reporteth that King *Hiero* having given commandment for the making of a *Crown* of pure *Gold*, was informed that the Workman had detained part of the *Gold*, and mixt the reſt with as much *Silver*, as he had ſtole of *Gold*; The King being much diſpleaſed at the deceit, recommended the examination of the buſineſs to the famous *Archimedes* of *Syracuſe*, who without defacing the *Crown* diſcovered the cheat in this manner: *viz.* Experience telling him that a quantity of *Gold* would poſſeſs leſs room or ſpace than the ſame quantity of *Silver*,

ver, and consequently that a mixt mass of *Gold* and *Silver* of the same quantity, would take up some mean space between the two former, he made a mass of pure *Gold* of the same weight with the *Crown*, likewise another mass of *Silver* of the same weight, then having put the *Crown* as also the other two Masses severally into a vessel filled up to the brim with water, he diligently reserved the water flowing over into another vessel, and from those 3 several quantities of water so expeld, he found out the quantity of *Gold* and of *Silver* in the *Crown*. : But forasmuch as *Vitruvius* delivers not the practical operation, I shall here shew the same after the manner of *Cardanus*, *Gemma*, *Frisius*, and other *Arithmeticians*.

Let us therefore suppose the weight of the *Crown* as also of the two several Masses to have been 5 *l.* Suppose also that by putting of the mass of *Gold* into the vessel, 3 *l.* of water was expeld; by putting in of the *Crown*, $3\frac{3}{4}$ *l.* and by putting in of the mass of *Silver*, $4\frac{1}{2}$ *l.* The question therefore is to know how much *Gold* and how much *Silver* the *Crown* was composed of. This may be resolved after this manner. Suppose 3 *l.* of *Gold* to be in the *Crown*, then there remained 2 *l.* of *Silver*, now say by the rule of 3, if 5 *l.* of *Gold* expel 3 *l.* of water how much 3 *l.* of *Gold*? *Answer*, $1\frac{4}{5}$ *l.* Also if 5 *l.* of *Silver* expel $4\frac{1}{2}$ *l* of water, how much 2 *l.* of *Silver*, *Answer*, $1\frac{4}{5}$ *l.* of water, add therefore the water of the *Silver* and of the *Gold* together, to wit, $1\frac{4}{5}$ and $1\frac{4}{5}$, so there will arise $3\frac{3}{5}$ *l.* of water, this ought to have been $3\frac{3}{4}$ *l.* (for so much over-

5 — 3 — 3 — ($1\frac{4}{5}$
5 — $4\frac{1}{2}$ — 2 — ($1\frac{4}{5}$

flowed

flowed by putting in of the *Crown*) but it is too much by $\frac{7}{20}$, wherefore $\frac{7}{20}$ is to be noted with + for the *error* of the *first Position* 3 *l.* Again, feign another quantity of *Gold* to have been in the *Crown*, to wit, 2 *l.* therefore there remained 3 *l.* of *Silver*, then say if 5 *l.* of *Gold* expel 3 *l.* of water, how much 2 *l.* of *Gold*? *Answ.* $1\frac{1}{5}$ *l.* of water: Also if 5 *l.* of *Silver* expel $4\frac{1}{2}$ *l.* of water, how much 3 *l.* of *Silver*? *Answer*, $2\frac{7}{10}$ then add $1\frac{1}{5}$ unto $2\frac{7}{10}$, the sum will be $3\frac{9}{10}$ *l.* of water, this ought to have been $3\frac{1}{4}$ *l.* but it is too much by $\frac{13}{20}$, wherefore $\frac{13}{20}$ is to be noted with + for the *error* of the *second Position* 2 *l.*

5 — 3 — 2 — ($1\frac{1}{5}$

5 — $4\frac{1}{2}$ — 3 — ($2\frac{7}{10}$

Here because the *errors* are *fractions* having a *common Denominator*, I take their *Numerators* 7 and 13 instead of the *errors*, then multiplying crosswise, to wit, 3 by 13 the product is 39, also 2 by 7 the product is 14 which subtracted from the former Product 39 (because the *errors* are like) leaves 25 for a *Dividend*, also the difference between the *errors* 7 and 13 is 6 for a *Divisor*; Lastly, dividing 25 by 6, the quotient is $4\frac{1}{6}$, so much *Gold* therefore was in the *Crown*, and consequently (because the weight of the *Crown* was 5 *l.*) there was $\frac{5}{6}$ *l.* of *Silver* which may be proved thus; Say if 5 *l.* of *Gold*, expel 3 *l.* of water, how much $4\frac{1}{6}$ *l.* of *Gold*? *Answer*, $2\frac{1}{2}$ *l.* of water, again if 5 *l.* of *Silver* expel

Pos.		*Er.*	
3	+	$\frac{7}{20}$	7
2	+	$\frac{13}{20}$	13

39
14
6) 25 ($4\frac{1}{6}$ *lb.* of *Gold.*

pel $4\frac{1}{2}$ of water, how much $\frac{4}{6}$ of *Silver*? *Answer*, $\frac{3}{4}$ *l.* of water, which being added to $2\frac{1}{2}$ *l.* the sum is $3\frac{1}{4}$ *l.* of water, to wit as much as flowed over, when the *Crown* was put into the vessel.

Here note, that in making a tryal of this nature, there is no necessity that the mass of Gold or of Silver be of the same weight with the Crown or whatsoever thing is to be examined, but of what notable part of weight you please.

Note also, that for the more easie discovering of the Dividend and Divisor by the notes of + and — according to the fourth Rule of this Chapter, the following verse may be a help, to wit.

Addito dissimiles, subtrahitoque pares.

Or thus,

Notes being unlike, Addition make;
If like, lesser from greater take.

The Reader may see more questions to exercise the *Rule* of *False* in the tenth chapter of the *Appendix*, and the demonstration thereof in the ninth chapter of the same.

CHAP.

CHAP. XXXII.

The Extraction of the Square (or Quadrate) Root.

I. THe Extraction of the Square-root is that by which having a number given, we find out another number, which being multiplied by it self, produceth the number given.

II. In the Extraction of the Square-root, the number propounded is alwayes conceived to be a square number, that is, a certain number of little squares comprehended within one intire great square, and the root or number required is the side of that great square, as will readily appear by this Diagram, where you see 25 little squares contained within one great square; now if the said content 25 be given, and the side or root of the square containing the said 25 little squares is required, the invention of such side or root is called the extraction of the square root; which root must be

be such, that if it be squared, that is, multiplied by it self, the product must be equal to the square content first given: So 5 is the square root of 25, for 5 times 5 is 25. Likewise this square number 49 being propounded, his root is 7.

III. Square numbers are either single or compound.

IV. A single square number is that, which being produced by the multiplication of one single figure by it self, is alwayes less then 100: so 25 is a single square number produced by 5, likewise 4 is a square number produced by 2.

A single square number.

V. All the single square numbers together with their respective roots are expressed in the Table following.

Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

Here in the uppermost rank of the Table are placed the single square-numbers of every particular figure, and in the other their respective roots; and therefore if it were demanded what is the square root of 36, the answer will be 6. So the square-root of 16 is 4, the square-root of 9 is 3, &c. And contrarily the square of the root 6 is 36. Also the square of 3 is 9.

VI. When a square number is given, that exceeds not 100, and yet is none of the square numbers mentioned in the Table, for his root you are to take the root of the square number that being less, yet comes nearest unto it: so 45 being given, the the root that belongs unto it is 6, and 10 being given, his correspondent root is 3.

VII. A

VII. A compound ſquare-number is that, which being produced by a number (that conſiſts of more places then one) multiplied by it ſelf, is never leſs then 100: ſo 1024 is a compound ſquare number produced by the multiplication of 32 multiplied by it ſelf.

A compound ſquare number.

VIII. To prepare any ſquare number given for extraction, put a point over the firſt place thereof on the right hand (being the place of Units) then proceeding towards the left hand, paſs over the ſecond place, and put another point over the third place; alſo paſſing over the fourth place put another point over the fifth, and ſo forward in ſuch manner that between every two points which are next one to the other, one place will be intermitted: ſo if the ſquare-root of 1024 be required, the firſt point is to be placed over 4, and the ſecond over 0 as you ſee, and ſo many points as are in that manner placed, of ſo many figures the *root* demanded will conſiſt.

$\dot{1}0\dot{2}4$

IX. Having thus prepared your number, you may ſee it diſtributed by the points into ſeveral ſquares: ſo in the laſt *Example*, 10 is the firſt ſquare and 24 the ſecond, likewiſe if this number 144 were propounded for extraction, after points are duly placed according to the laſt rule, you will ſee 1 to be the firſt ſquare and 44 the ſecond.

$\dot{1}4\dot{4}$

X. Having drawn a crooked line on the right hand of the number propounded for extraction, (after the ſame manner as is uſually done in Diviſion to denote the place of the quotient) find the

root of the first square, and place it in the quotient: so I find, by the sixth rule aforegoing, 3 to be the correspondent root of 10, wherefore I write 3 in the quotient, and then the work will stand as you see.

1024 (3

XI. Subscribe the square of the figure placed in the quotient under the first square of the number given, as you see in the margent.

1024 (3
9

XII. Having drawn a line under the square (of the figure placed in the quotient) subscribed as aforesaid, subtract the same out of the first square of the number propounded, and place the remainder orderly underneath the line, so the square of 3 which is 9 being subtracted from 10, the remainder is 1, and the work will stand as you see in the Margent.

1024 (3
9
———
1

XIII. To the said remainder bring down the next square of the number propounded, that is write down the figures or cyphers standing in the two following places of the number propounded on the right hand of the said remainder: so the square 24 being placed next to the remainder 1, there will be found this number 124, which may be called the *Resolvend*.

1024 (3
9
———
124

XIV. Double the root being the number placed in the quotient, and place the said double on the left hand of the Resolvend, like a Divisor: so the double of 3 is 6, which being placed before a crooked line on the left

1024 (3
9
———
6) 124

left hand of the Reſolvend 124, the work will ſtand as you ſee.

XV. Let the whole Reſolvend except the firſt place thereof on the right hand (being the place of units) be alwayes eſteemed as a Dividend, then demanding how often the Diviſor before found, is contained in the ſaid Dividend, and obſerving in that behalf the rules before taught in Diviſion, write the anſwer in the quotient, and alſo on the right hand of the Diviſor, to wit, between the Diviſor and the crooked line: ſo if you ask how often the Diviſor 6 is found in the Dividend 12, the anſwer is 2, wherefore I write 2 in the quotient, and alſo after the Diviſor 6, as you ſee in the Margent.

```
       . .
       1024 (32
       9
   ------------
   62) 124
         .
```

XVI. Multiply all the number which ſtandeth on the left hand of the Reſolvend, (to wit, before the crooked line) by the figure laſt placed in the quotient, and write the product orderly underneath the Reſolvend (to wit, units under units, tens under tens, &c.) then having drawn a line under the ſaid product, ſubtract it from the Reſolvend, and ſubſcribe the remainder under the line: ſo 62 being multiplied by 2, the product is 124, which if I ſubtract out of the Reſolvend 124, the remainder is 0; and thus the whole work being finiſhed, the ſquare root of 1024 (the number propounded) is found to be 32.

```
       . .
       1024 (32
       9
   ------------
   62) 124
       124
   ------------
         0
```

Note 1. When the product before mentioned exceeds the *Reſolvend* placed above it, the work is erroneous, and then you are to reform it by placing a leſſer figure in the quotient.

Note 2. For every one of the particular ſquares (diſtinguiſhed by the points) except the firſt on the left hand, a *Reſolvend* is to be ſet apart, by bringing down to the remainder the congruent particular ſquare, as is directed in the 13 *rule* ; and as often as a *Reſolvend* is ſet apart, ſo often a new *diviſor* is to be found by doubling or multiplying by 2 all the root in the quotient (conſiſting of what number of places ſoever.)

Note 3. The work of the 10, 11, and 12 *rules* for finding of the firſt figure in the *root*, is but once uſed in the extraction of the root of a number conſiſting of what number of places ſoever ; but the work of the 13, 14, 15, and 16, rules is to be repeated for the finding of every place in the root except the firſt.

The practice of theſe 3 *Notes* will be ſeen in the following *Examples*.

Example 2. Let it be required to extract the ſquare root of 43623.

Having diſtributed the number propounded into ſeveral ſquares by points, as is directed in the eighth rule of this Chapter, I demand the ſquare root of 4 the firſt ſquare, which I find by the 5 rule of this Chapter to be 2, wherefore placing 2 in the quotient and the ſquare thereof, which is 4, under the firſt ſquare 4, I draw a line, and ſubtracting 4 from 4 the remainder is 0, which I ſubſcribe underneath

```
 . . .
43623 (2
4
-----
0
```

derneath the line. This is alwayes the first work, which is no more repeated in the whole Extraction, (as was intimated in the third *Note* aforegoing.)

Then bringing down the next square which is 36, and placing it next after the remainder 0, the *Resolvend* is 36, and doubling the root 2 in the quotient, the product is 4 for a *Divisor* (by the 13 and 14 rules) and the *Dividend* will be 3 (by the 15 rule) wherefore I demand how often the *divisor* 4 is contained in the *dividend* 3, and not finding it once contained in it, I place 0 in the quotient, and also next after the *Divisor* 4, and because the product of 40 multiplied by 0, (the last character in the quotient) is 0, the *resolvend* 36 from which the said product ought to be deducted remains the same without alteration, therefore I bring down 23 the next square, and place it after the remainder 36, so will 3623 be a new *resolvend*, then doubling the whole root in the quotient, which is 20, the *divisor* will be 40 (according to the second *Note* before mentioned) and the *dividend* will be 362 (to wit, all the *resolvend* except the first place on the right hand by rule 15.) wherefore I demand how often the *divisor* 40 is contained in the *dividend* 362 or how often 4 in 36, & though it be 6 times in it, yet (according to the first *Note* aforegoing) I can take but 8, (for if I should take 9, and proceed according to the 15

```
   . . .
   43623 (20
   4
  ---------
40) 036
```

```
    . . .
    43623 (208
    4
  ----------
40) 03623
```

 and

and 16 rules, a number would arise greater than the *resolvend*, from which such number arising ought to be subtracted) wherefore I write 8 in the quotient, and also after the divisor 40; this done, I multiply 408 (the number on the left hand of the *resolvend*) by 8 the figure last placed in the quotient, and the product, to wit, 3264 I subscribe under, and subtract from the *resolvend* 3623, so there will remain 359, thus the work being finished I find 208 to be the number of unities contained in the root sought; and because after the extraction is ended there happens to be a remainder; to wit 359, I conclude that the root sought is greater then the said 208, but less then 209, yet how much it is greater then 208, no rules of Art hitherto known will exactly discover, although we may proceed infinitely near, as in the next rule will be manifest.

```
   . . .
   43623 (208
   4
 ------------
408)03623
     3264
    ------
      359
```

XVII. To find the fractional part of the root very near, a competent number of pairs of cyphers, to wit, 00, 0000, 000000, or 00000000, &c. are to be annexed to the number first propounded, then esteeming the number propounded with the cyphers annexed to be but one intire number, the extraction is to be made according to the precedent rules, and look how many points were placed over the number first given, so many places of Integers will be in the root, the rest of the root towards the right hand will be the Numerator of a decimal fraction, which Numerator consisteth of so many places as there were points over the cyphers

cyphers annexed: so if 43623 were given as before; to find the root thereof (according to this rule) annex cyphers in this manner, and then if you extract it according to the rules aforegoing, you

43623.000000 (208.861, &c.

will find the root arising in the quotient to be 208 .861, that is $208\frac{861}{1000}$; and because after the extraction is finisht there happens to be a remainder, I conclude that $208\frac{861}{1000}$ is less than the true or exact root, but $208\frac{862}{1000}$ is greater than it; so that by annexing three pairs of cyphers to the number propounded, you will not miss $\frac{1}{1000}$ part of an unit of the true *root*; also by annexing 4 pairs of cyphers, you will not miss $\frac{1}{10000}$ part of an unit, and in that order you may proceed infinitely near, when you cannot obtain the true root. The whole operation of the said *Example* here followeth.

```
          43623.000000 (208.861,&c.
          4               The root.
408)      03623
           3264
4168)      35900
           33344
41766)     255600
           250596
417721)     500400
            417721
             82679
```

Again if 10 were propounded to be extracted, you must prepare it thus,

10.00000000000000 (

And then the root thereof being extracted will be——— $3\frac{1622776}{10000000}$,&c.
which (according to the third rule of the 22 chapter) may be written thus——————— 3.1622776,&c.

See here part of the work in the extraction of the *root* of 10, which may give you a light and understanding of the rest.

```
   . . . . . . . .
  10.00000000000000 (3.16227,&c,
   . . . . .
     9
  ----------
  61) 100
       61
  ----------
  626) 3900
       3756
  ----------
  6322) 14400
        12644
  ----------
  63242) 175600
         126484
  ----------
  632447) 4911600
          4427129
  ----------
           484471
```

XVIII. The

XVIII. The extraction of the square root is proved by multiplying the root by it self, for that done, the product (in such case when there is no remainder after the extraction is finished) will be equal to the number whose square root was inquired ; so in the first *Example* of this *Chapter*, the root 32 being multiplyed by it self produceth 1024 the number propounded : but when after the extraction is finished there happeneth to be a remainder, and that the Root is found as near as you please in a mixt number of integers and decimal parts (by annexing cyphers as in the 17. rule) then such mixt number being multiplyed by it self must produce a mixt number less than the number first propounded for extraction, yet so near unto it, that if the figure standing in the last place of the Numerator of the decimal fraction in the root be made greater by 1, and then the mixt number so increased be multiplyed by it self, the product must be greater than the number first propounded : so in the *Example* of the 17. *Rule*, if 208.861 be multiplyed by it self, it produceth 43622.917, &c. which is less than the propounded number 43623, but if 208.862 be multiplyed by it self, the product will be 43623.335, &c. which it greater than 43623.

The Proof.

XIX. The square root of a Fraction is found in this manner, *viz.* extract the root of the Numerator (by the precedent rules of this Chapter) which root shall be a new Numerator. Also the root of the denominator is to be taken for a new denominatur, so the new Fraction shall be the square root of the Fraction first propounded ;

To extract the square root of a Fraction.

ded: thus the *square root* of $\frac{9}{16}$ is $\frac{3}{4}$, *viz.* the *root* of 9 is 3 for a new *numerator*, also the *root* of 16 is 4 for a new *denominator*. In like manner the *square root* of $\frac{1}{4}$ is $\frac{1}{2}$. But here note diligently, that if the *Fraction* whose *square root* is required be not in its least terms, it must first of all be reduced by the 4. *rule* of the 17. *chapter* before any extraction be made; for oftentimes it happens that the *Fraction* first given hath not a perfect *root*, but when such *Fraction* is reduced into its least terms, the *root* thereof may be extracted: so in this *Fraction* $\frac{8}{18}$, each term is *incommensurable* to its *square root*, but the said $\frac{8}{18}$ being reduced to its least terms $\frac{4}{9}$, the *root* of this may be extracted, for the *root* of 4 is 2 for a new *Numerator*; also the *root* of 9 is 3 for a new *Denominator*; so that $\frac{2}{3}$ is found to be the *square root* of $\frac{4}{9}$ (equivalent unto $\frac{8}{18}$.)

XX. When either the Numerator or Denominator of a Fraction hath not a perfect square root, such root is usually exprest by prefixing this Character, $\surd$ or $\surd$q. before the Fraction given: so the *square root* of $\frac{13}{16}$ is signified thus $\surd\ \frac{13}{16}$, or thus $\surd$q. $\frac{13}{16}$, because the *root* of $\frac{13}{16}$ cannot be exprest by any true or rational number whatsoever, yet it may be found very near as in the next *Rule*.

To extract the square root near, of a fraction incommensurable to its square root.

XXI. The *square root* of a *Fraction* which is incommensurable to its *root* may be found near, in this maner, *viz.* reduce the fraction proposed into a decimal by the third rule of the 23 Chapter: the more places are in the decimal, the nearer will the root be found, but the decimal must consist of an even

number

number of places, *viz.* either of two, four, six, eight or ten, &c. places; then extract the *square root* of that decimal as if it were a whole number according to the Rules aforegoing, which *root* found shall be a decimal expressing near the *square root* of the fraction proposed.

So if the *square root* of $\frac{13}{16}$ be required near, reduce the said $\frac{13}{16}$ into a decimal (by the 3d. *rule* of the 23. *chapter*) which will be found .81250000, &c. Then extracting the *square root* thereof as if it were a whole number, it will be found .9013 very near.

To extract the square root of a mixt number.

XXII. The *square root* of a mixt number commensurable to its *root*, is found in the same manner as in the 19 rule of this Chapter, the mixt number being first reduced into an improper fraction by the 10. rule of the 17. Chapter.

So the *square root* of $34\frac{33}{64}$ will be found $5\frac{7}{8}$, *viz.* $34\frac{33}{64}$ being reducad into the improper Fraction $\frac{2209}{64}$, the *square root* of the Numerator 2209 will be 47 for a new Numerator; also the *square root* of the *Denominator* 64 is 8, for a new *Denominator*, so is found $\frac{47}{8}$ which (by the 13. rule of the 17. chapter) is $5\frac{7}{8}$ the *square root* sought. And here the same caution is to be observed as in the 19. Rule of this Chapter; *viz.* the fractional part of the mixt number, or the improper fraction equivalent unto the mixt number, must be in the *least terms* before any extraction be made.

XXIII.

To find the square root near, of a mixt number incommensurable to its root.

XXIII. When the mixt number given is incommensurable to its *square root*, prefix this Character before it, *viz.* √ or √q. So the *square root* of $7\frac{2}{3}$ will be thus expressed: √ $7\frac{2}{3}$ or √q. $7\frac{2}{3}$: but if you desire to find the *square root* near, of a mixt number *incommensurable* to its *root*, reduce the fractional part of the mixt number into a Decimal of an even number of places, as in the 21. Rule of this Chapter, and annex the Decimal so found unto the whole part of the mixt number; then esteeming the said whole number and Decimal as one intire number, extract the *square root* thereof according to the aforegoing Rules of this Chapter, and from the root found, cut off alwayes to the right hand, so many places as there are points over the Decimal annexed, which number so cut off shall be a Decimal, shewing the fractional part of the root, and that on the left hand shall be the whole part of the root; so the *square root* of $7\frac{2}{3}$ will be found 2.7688 very near.

CHAP. XXXIII.

The Extraction of the Cube Root.

I. THe Extraction of the Cube Root is that, by which having a number given, we find another number, which being first multiplyed by it self, and then by the product, produceth the number given.

II.

A Cubical number.

II. In the Extraction of the Cube root, the number propounded is alwaies conceived to be a Cube number, that is a certain number of little Cubes comprehended within one intire great Cube, and the root or number required is the side of that great Cube: what a Cube is may be well exprest by a Die, which indeed is a little *Cube* it self; wherefore if you place four Dice in a square form, that is, laying two and two in a rank, you shall have a square containing four Dice, upon which if you yet erect such another square of Dice you shall have a great intire Cube comprehending two times 4 that is 8 Dice or little Cubes; and here 8 is the Cube number given, and two is the root, or number required: In like manner if you rank 25 Dice in a square form, *viz.* laying 5 in a rank, you have a square containing 25 Dice, now upon this square of Dice if you erect such another square, you shall have a great intire Cube comprehending 5 times 25, that is 125 little Cubes, and in this case 125 is the Cube number propounded, and 5 the root or number required,

III. A Cube number is either single or compound.

A single Cube number.

IV. A single cube number is that, which being produced by the Multiplication of one single figure first by it self, and then by the product, is alwayes less than 1000. So 225 is a single cube number produced by 5 multiplyed first by it self, and then by 25 the product; for 5 times 5 is 25, and 5 times 25 is 125.

V. All the single cube numbers, and square numbers,

bers, together with their respective roots, are expressed in the Tabe following.

Cubes	1	8	27	64	125	216	343	512	729
Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

Here, in the uppermost rank of the Table are placed the single cube numbers of the particular figures 1, 2, 3, 4, 5, 6, 7, 8, 9. in the next the squares of those figures, and in the lowest rank the figures themselves being the respective roots of the cubes and squares in the uppermost ranks; and therefore the *cube root* of 125 being demanded the answer is 5, and the *cube root* of 216 being required, the Table will give you 6, and so of the rest.

VI. When a cube number is given that exceeds not 1000, and yet is none of the Cube numbers mentioned in the Table; for his root you are to take the root of the cube number, that being less comes nearest unto it. So 157 being given, the root that belongs unto it is 5.

A compound Cube number.

VII. A compound cube number is that, which being produced by a number (that consists of more places than one) first multiplyed by it self, and then by the product is never less then 1000. So 157464 is a *compound cube number*, being produced by 54 multiplyed first by it self, and then by 2916 the product, for 54 times 54 is 2916, and then 54 times 2916 is 157464, the *compound cube number propounded.*

VIII. To

VIII. To prepare a cube number for extraction, put a point over the firſt place thereof towards the right hand, (to wit the place of units) then paſſing over the ſecond and third places, put another point over the fourth, and paſſing over the fifth and ſixth put another point over the ſeventh, and in that order (to wit two places being intermitted between every two adjacent points) place as many points as the number will permit: ſo 157464 being given, you are to place the points as in the Margent, and ſo many points as are in that manner placed, of ſo many figures, the *root* demanded will conſiſt.

157464

IX. Having thus prepared your number, you may ſee it diſtributed by the points into ſeveral cubes: ſo in the ſame example 157 is the *firſt cube*, and 464 the *ſecond.* In like manner if this number 7464 were propounded for extraction, after points are duly placed as before, you will ſee 7 to be the *firſt cube*, and 464 the *ſecond.*

157464

7464

X. Having drawn a crooked line on the right hand of the number propounded to ſignifie a quotient, find the cube root of the firſt cube and place it in the quotient: ſo I finding (by the ſixth rule of this Chapter) 5 to be the correſpondent root of 157, I write 5 in the quotient, and then the work will ſtand as you ſee in the Margent.

157464 (5

XI. Subſcribe the cube of the root placed in the quotient, under the firſt cube of the number given: ſo 125 being the cube of 5 the root, (by the

157464 (5
125

fifth

fifth rule of this chapter) I write it under 157 the firſt cube of the number given, as you ſee in the example.

XII. Draw a line under the cube ſubſcribed as aforeſaid (to wit the cube of the root placed in the quotient) and ſubtract this cube from the firſt cube of the number propounded, placing the remainder orderly underneath the line: ſo 125 the cube of 5 being ſubtracted from 157, the remainder is 32, and the work will ſtand as you ſee.

157464 (5
125
———
32

XIII. To the ſaid remainder bring down the next cube of the number propounded, (to wit the figures or cyphers that ſtand in the 3 next places) placing the ſaid cube next after, to wit, on the right hand of the remainder ſo the next *cube* 464 being placed after the remainder 32, there will be found this number 32464, which may be called the *Reſolvend.*

157464 (5
125
———
32464 *reſolv.*
———

XIV. Having drawn a line underneath the *Reſolvend*, ſquare the root in the quotient, that is, multiply it by it ſelf and ſubſcribe the triple of the ſaid ſquare or product, under the reſolvend in ſuch manner, that the firſt place (to wit, the place of units) of the ſaid triple ſquare may ſtand directly under the third place (or place of hundreds) in the Reſolvend: ſo the ſquare of the root 5 is 25, the triple whereof is 75, which I ſubſcribe under the Reſolvend

157464 (5
125
———
32464 *reſolv.*
75

vend in ſuch manner, that the figure 5 which is in the firſt place (to wit the place of units) in the triple product 75, may ſtand under 4 which is ſeated in the third place of the reſolvend, as you ſee in the Margent.

XV. Triple the Root or number in the quotient, and ſubſcribe this triple number in ſuch manner that the firſt place thereof, (to wit the place of units) may ſtand directly under the ſecond place (to wit the place of tens) in the Reſolvend: ſo the triple of the root 5 is 15, which I ſubſcribe in ſuch manner, that the figure 5 which is in the firſt place (to wit the place of units) in the ſaid triple number, doth ſtand directly under 6, which is ſeated in the ſecond place of the reſolvend, and the work will ſtand as in the Margent.

```
 .  .
157464 ( 5
125
----------
 32464 reſolv.
----------
  75
   15
```

XVI. The triple ſquare of the root, and the triple of the root being placed one under the other as is directed in the 14. and 15. rules aforegoing, draw a line underneath and add them together in ſuch order as they are ſeated, and let the ſum be eſteemed as a diviſor: ſo the triple ſquare 75, and the triple number 15, being added together as they are ranked in the work, the ſum will be 765 for a Diviſor.

```
 .  .
157464 ( 5
125
----------
 32464 Reſol.
----------
  75
   15
----------
  765 Diviſor.
----------
```

XVII. Let the whole Resolvend except the first place thereof towards the right hand, (to wit the place of units) be esteemed as a Dividend, then demanding how often the first figure (towards the left hand) of the Divisor is contained in the correspondent part of the dividend, and observing in that behalf the rules before taught in Division, write the answer in the quotient: so if I ask how often 7 (the first figure of the Divisor towards the left hand) is contained in 32, (the correspondent part of the Dividend placed above) the answer will be 4, wherefore I write 4 in the quotient, as you see in the Example.

```
  .  .
157464 ( 54
125
----------
 32464 Resolv.
----------
  75
   15
----------
  765 Divisor.
----------
```

XVIII. Having drawn another line under the work, multiply the triple square before subscribed (as is directed in the 14. rule) by the figure last placed in the quotient, and subscribe this product under the said triple square; (to wit units under units, tens under tens, &c.) so 75 being multiplyed by 4, the product is 300 which I subscribe under 75, (the triple square) and the work will stand as you see in the Margent.

```
  .  .
157464 ( 54
125
----------
 32464 Resolv.
----------
  75
   15
----------
  765 Divisor.
----------
 300
```

Multiply

XIX. Multiply the figure last placed in the quotient first by it self, and then the product by the triple number before subscribed, (as is directed in the 15. rule of this chapter) this done, subscribe the last product under the said triple number; (to wit, units under units, tens under tens, &c.) so 4 being squared or multiplyed by it self, the product is 16, which being multiplyed by the triple number 15, the product is 240, this therefore I subscribe under the aforesaid triple number 15, and the work will stand as you see.

```
157464 (54
125
-----
 32464 Resolvend.
-----
 75
  15
-----
 765 Divisor.
-----
300
 240
```

XX. Subscribe the cube of the figure last placed in the quotient, under the resolvend, in such manner that the first place of this cube, (to wit, the place of units,) may stand under the place of units in the resolvend: So 64 being the cube of 4, I write it under the resolvend 32464, in such manner that the figure 4, which is in the place of units in the *cube* 64, may stand under the figure 4 which is seated in the place of units of the resolvend: observe the work in the Margent.

```
157464 (54
125
-----
 32464 Resolvend.
-----
 75
  15
-----
 765 Divisor.
-----
300
 240
   64
```

XXI. Drawing yet another line under the work, add the three laſt numbers together in the ſame order as they are ſeated, and ſubtract the ſum of them from the reſolvend, placing the remainder orderly underneath: ſo the ſum of the three laſt numbers as they are ranked in the work is 32464, which if you ſubtract out of the reſolvend 32464, the remainder is 0, Thus the whole work being finiſhed the cube root of 157464, (the number propounded) is found to be 54.

```
157464 (54
125
------------
 32464 Reſolvend.
------------
   75
    15
------------
   765 Diviſor.
------------
 300
  240
    64
------------
 32464
------------
     0
```

Note 1. When the ſum of the three laſt numbers before mentioned is greater than the reſolvend, the work is erroneous, and then you are to reform it by placing a leſſer figure in the quotient.

Note 2. For every one of the particular cubes (diſtinguiſhed by the points) except the firſt cube on the left hand, a reſolvend is to be ſet apart, by bringing down to the remainder the next cube (as is directed in the 13. rule.) And as often as a reſolvend is ſet apart, ſo often is a new Diviſor to be found, by adding the *triple* of all the *root* in the *quotient* (conſiſting of what number of places ſoever) to the *triple* of the *ſquare* of ſuch root, after they are orderly placed according to the 14, and 15. *rules.*

Note

Note 3. The work of the 10, 11, and 12. *rules* for finding of the first figure in the *root* is but once used in the extraction of the root of any number whatsoever, but the work of all the following *rules*, is to be used for the finding of every place in the *root* except the first.

The practice of these 3 *Notes* will be seen in the following *Examples*.

Example 2. Let it be required to extract the *cube root* of 8302348.

Having distributed the number given into several *cubes* by points, as is directed in the eighth *rule* of this *Chapter*. I demand the *cube root* of 8 (the first *cube* on the left hand) which I find by the fifth *rule* of this *Chapter* to be 2, wherefore placing 2 in the quotient, and 8 the cube thereof under 8 the first cube, I draw a line, and subtracting 8 out of 8 the remainder is 0, which I subscribe under the line.

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 .  .  .
8302348 (2
8
-------
0
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This is alwayes the first work, and is no more repeated in the whole extraction (as was intimated in the 3 note aforegoing) then bringing down the next cube, (to wit, the figures standing in the three following places of the number propounded) which is 302, I place it after the remainder 0, so is 302 the *resolvend*; this done, having drawn a line underneath the *resolvend*, I seek for the triple of the square of the root, *viz.* the *root* in the *quotient* is 2, which multiplyed by it self produceth the square 4, the triple whereof is 12, this I subscribe under the *resolvend*, in such manner that the figure 2 in

in the units place of this triple square 12, may stand directly under the figure 3, which is seated

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.  .  .
8302348 ( 2
8
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0302 Resolvend.
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12
 06
-----------
126 Divisor.
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in the third place of the *resolvend*, (to wit, the place of hundreds) according to the 14. *rule* aforegoing; Again I triple the root 2, which produceth 6, and subscribe this triple number 6 under the second place (or place of tens) in the *resolvend*, to wit, under 0, (according to the 15. *rule* of this *Chapter*) then drawing a line under the work, and adding together the said two numbers last subscribed, as they are ranked, the sum of them is 126 for a divisor, (according to the 16. *rule* aforegoing)

That done, esteeming 30, to wit, all the places except the first or place of units in the *resolvend*, as a *Dividend*, I demand how often the *divisor* 126 is contained in 30, and not finding it once contained therein, I write 0 in the *quotient*, and now because the sum of the three numbers which ought to have been produced (according to the 18, 19, and 20. *rules* of this *Chapter*) by the multiplication of 0 (which was last placed in the *quotient*) amounts to 0, the *resolvend* 302 out of which the said sum should have been subtracted, remains the same without alteration, wherefore having drawn a line under the work, I write down anew the old *resolvend* 302, and bringing down the next *cube* 348, I annex it to the said

302;

302; ſo there will be a new *reſolvend*, to wit, 302348.

Then ſquaring the root 20, (that is, multiplying of it by it ſelf) the product is 400, which I triple or multiply by 3, and ſubſcribe the product 1200 underneath the new reſolvend, in ſuch manner, that the place of units in this triple quadrate 1200, may ſtand under the place of hundreds or third place of the reſolvend 302348, to wit, under 3 (according to the 14. *rule.*) Again I ſubſcribe the triple of the *root* 20, which is 60, in ſuch manner that the place of units in this triple root 60 may ſtand under the place of tens or ſecond place of the reſolvend; then adding together the two numbers laſt ſubſcribed, to wit, 1200 and 60, in ſuch order as they are ranked in the work, the ſum is 12060 for a Diviſor.

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.  .  .
8302348 ( 202
8
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0302) Reſolvend
----------------
12
 06
----------------
126  Diviſor
----------------
 302348  Reſolvend
----------------
 1200
   60
----------------
 12060  Diviſor
----------------
 2400
  240
   08
----------------
 242408  Ablatitium
----------------
  59940
```

Again, esteeming the whole resolvend except the first place, (or place of units) as a dividend, to wit, 30234, I demand how often 1 (the first figure of the divisor towards the left hand) is contained in 3. the correspondent part of the Dividend, and though it be three times contained in it, yet (according to the first *note* at the end of the 21. *rule* of this *Chapter*) I dare take but 2, for if I should take 3 and proceed according to the 18, 19, 20, and 21 *rules* of this *Chapter*, a number would arise greater than the resolvend, (from which such number arising ought to be subtracted,) wherefore I write 2 in the *quotient*.

Then multiplying the triple square 1200 before subscribed, by 2, (the figure last placed in the quotient) the product is 2400, which I subscribe under the said 1200, (to wit, units under units, and tens under tens, &c.) Also multiplying the triple root 60 before subscribed, by 4 (the quadrate of 2 the figure last placed in the quotient) the product is 240; which I subscribe under the said triple root 60, last of all I subscribe 8 the *cube* of the said new *root* 2, under the place of units or first place of the *resolvend*, to wit, under 8, and having added together those three numbers last subscribed, to wit 2400, 240 and 8 as they stand in ranks in the work, the sum of them is 242408, which being subducted from the *resolvend* 302348, there will remain 59940. Wherefore the work being finished, I find 202 to be the number of unities contained in the *cube root* of 8302348 the number propounded: and because after the extraction is ended there happens

to

to be a remainder, to wit 59940, I conclude that the *cube root* sought is greater then the said 202 but less then 203, yet how much it is greater then 202, no rules of Art hitherto known will exactly discover, although we may proceed infinitely near as by the next *rule* will be manifest.

XXII. To find the fractional part of the root very near, ternaries of cyphers, to wit, 000, 000000, or 000000000, &c, are to be annexed to the number first propounded, then esteeming the number propounded with the cyphers annexed to be but one intire number, the Extraction is to be made according to the preceding *rules* of this *Chapter*, and look how many points were placed over the number first given, so many of the formost places in the Quotient are the Integers or unities contained in the cube root sought, and the rest of the places in the quotient are to be esteem'd as the Numerator of a Decimal fraction, which Numerator consisteth of so many places as there were points over the cyphers first annexed: so if 8302348 were given as before, to find the *cube root* thereof, (according to this rule) annex cyphers in this manner,

$$\dot{8}30\dot{2}34\dot{8},00\dot{0}00\dot{0} \; ($$

And then if you prosecute the extraction according to the rules aforegoing, you shall find the *cube root* sought to be 202.48, &c. that is, $202\frac{48}{100}$ and more, wherefore you may conclude that $202\frac{48}{100}$ is less than the true *root*, but $202\frac{49}{100}$ is greater

greater than it, so that by annexing two ternaries of cyphers, to wit, 6 cyphers, to the number propounded, you will not miss $\frac{1}{100}$ part of an unit of the true *root*, also by annexing 3 ternaries of cyphers, to wit 9 cyphers, you will not miss $\frac{1}{1000}$ part of an unit of the true *root*, and in that order you may proceed infinitely near, when you cannot obtain the true *root*. The whole operation of the said Example here followeth, where you may observe, that for the more certain and easie placing, as well of the numbers which constitute the several Divisors, as of those which constitute the Ablatitious numbers to be subtracted from the several and respective resolvends, down right lines are drawn between the particular *cubes* of the number propounded, first distinguished by points as before.

8	302	348	000	000 (202.48,&c.
8				
0	302			Resolvend
1	2			
	06			
1	26			Divisor
	302	348		Resolvend
	120	0		
		60		
	120	60		Divisor
	240	0		
	2	40		
		08		
	242	408		Ablatitium
	59	940	000	Resolvend
	12	241	2	
		6	06	
	12	247	26	Divisor
	48	964	8	
		96	96	
			64	
	49	061	824	Ablatitium
	10	878	176	000 Resolvend
	1	228	972	8
			60	72
	1	229	033	52 Divisor
	9	831	782	4
		3	886	08
				512
	9	835	668	992 Ablatitium
	1	042	507	008

In

In like manner the *cube root* of 2 will be found to be near equal to 1.25992, &c. that is, $1\frac{25992}{100000}$ and more.

XXIII. The extraction of the Cube root is proved by multiplying the root cubically, to wit, the root being first multiplied by it self, and then the product multiplied by the root, the number arising or last product (in case there be no remainder after the extraction is finished) will be equal to the number propounded: so in the first Example of this *chapter*, the *cube root* 54 being multiplied first by it self produceth 2916, which being multiplied again by 54 produceth 157464, to wit, the number whose *cube root* was inquired. But when after the Extraction is finished, there happeneth to be a remainder, and that the *root* is found as near as you please in *Integers* and *decimal parts*, (by annexing cyphers as in the 22 *rule* of this *chapter*) then such mixt number expressing the *root*, being multiplied cubically, must produce a mixt number less than the number first propounded, yet so near unto it that if the figure standing in the last place of the *decimal fraction* in the *root* be made greater by 1, and the mixt number so increased be multiplied cubically, the product must be greater than the number first propounded: so in the Example of the 22 *rule* of this *chapter*, if 202.48 be multiplied cubically it produceth 8301305.49, &c. which is less than the propounded number 8302348, but if 202.49 be multiplied cubically, there will arise 8302535.49, &c. which is greater than the said given number.

The Proof.

XXIV. The Cube root of a Fraction is found in this manner, *viz.* extract the Cube root of the

Numerator,

Numerator, (according to the aforegoing rules) which root reserve for a new Numerator; also the Cube root of the Denominator shall be a new Denominator: lastly this new Fraction shall be the Cube root of the Fraction first propounded: so the *cube root* of $\frac{8}{27}$ is $\frac{2}{3}$, for the *cube root* of 8 is 2 for a new *Numerator*, also the *cube root* of 27 is 3 for a new *Denominator*. In like manner the *cube root* of $\frac{1}{8}$ is $\frac{1}{2}$. But here note diligently that the *fraction* whose *cube root* is required must be in its least terms before any Extraction be made, for oftentimes it happens that the *fraction* first given hath not a perfect root, albeit, when such *fraction* is reduced into its least terms, the *root* thereof may be extracted; so in this *fraction* $\frac{16}{54}$ neither the *numerator* nor *denominator* hath a perfect *cube root*; yet the said $\frac{16}{54}$ being reduced to its least terms $\frac{8}{27}$, (by the fourth rule of the 17 Chapter) the *cube root* of this may be extracted, for the *cube root* of 8 is 2 for a new *numerator*, also the *cube root* of 27 is 3 for a new *denominator*, so that the *cube root* of $\frac{8}{27}$ (which is equal to $\frac{16}{54}$) is found to be $\frac{2}{3}$.

To extract the cube root of a fraction.

XXV. The Cube root of a fraction which hath not a perfect Cube root may be found near in this manner, *viz.* reduce the Fraction given into a Decimal fraction, by the third rule of the 23 Chapter, the more places are in the Decimal, the nearer will the root be found, but the decimal must consist of ternaries of places, to wit, either of three, six, nine, or twelve, &c. places; then extract the Cube root of the Numerator of that Decimal, as if it were a whole number, (according to the rules before given) which root found shall be a Decimal

expressing

expressing near the Cube root of the Fraction propounded.

So if the *cube root* of $\frac{2}{3}$ were required, I reduce the said $\frac{2}{3}$ into a *decimal* whose *numerator* may consist of *ternaries* of places, to wit, into this, .666666666666 &c. then extracting the *cube root* thereof, I find .8735, which is very near the *cube root* of $\frac{2}{3}$.

XXVI. The Cube root of a mixt number commensurable to its root may be found in the same manner as in the 24 rule of this Chapter, the mixt number being first reduced into an improper fraction (by the 10 rule of the 17 chapter.)

So the *cube root* of $12\frac{19}{27}$ will be found to be $2\frac{1}{3}$, *viz.* reducing $12\frac{19}{27}$ into this improper fraction $\frac{343}{27}$ the cube root of $\frac{343}{27}$ will be found $\frac{7}{3}$ or $2\frac{1}{3}$. And here the same caution is to be observed as in the 24 *rule* of this *chapter*, *viz.* the fractional part of the mixt number, or the improper fraction equivalent unto the mixt number, must be expressed by a *Numerator* and *Denominator* in the least terms before any extraction be made.

XXVII. When the mixt number whose cube root is required hath not a perfect cube root, this character $\surd$c. is usually prefixed before such mixt number; so the *cube root* of $2\frac{5}{8}$ is thus expressed, $\surd$c. $2\frac{5}{8}$. Likewise $\surd$c. $\frac{4}{9}$ denotes the *cube root* of $\frac{4}{9}$ which is a fraction, whose *cube root* is inexpressible by any true or rational number: but if you desire to know the *cube root* near of a mixt number which hath not a perfect *cube root*, reduce the fractional part of the mixt number into a *decimal*, (as in the 25 *rule* of this *chapter*) and annex the *decimal* so found unto the whole part (or Integers) of the mixt number; then esteeming the said Integers with the *decimal* so an-

nexed

nexed as one intire number, extract the *cube root* thereof, and from the *root* found cut off alwayes to the right hand so many places as there were points over the said *decimal* annexed, which places so cut off shall be the fractional part of the *root*, and those remaining on the left hand shall be the Integral part of the *root*: so the *cube root* of $2\frac{3}{8}$ will be found 1.334 and more.

XXVIII. I might here proceed to shew the extraction of other *roots*, as the *Biquadrate*, *Quadrato-cube*, *Cubo-cube*, &c. but in regard their operation is exceeding tedious, and cannot be naturally understood without the knowledge of *Algebra*; I shall only in this place touch upon the Extraction of the *Biquadrate-root*, because it may be extracted by the *rules* delivered in the 32 *Chapter*.

XXIX. A quadrate or square number multiplied by it self produceth a Biquadrate number: So 4 multiplied by it self produceth the *Biquadrate* 16. Therefore if a number be propounded and the *Biquadrate root* thereof be required, first extract the *quadrate* or *square root* of the number propounded, and then extract the *square root* of that *root* for the *Biquadrate root* sought. Thus if 20736 be a number propounded; the *Biquadrate root* thereof will be found 12: for the *square root* of 20736 is 144, and the *square root* of 144 is 12. When the number given hath not a perfect *Biquadrate root*, you are to annex *quaternaries* of *cyphers*, to wit, either 4, 8, 12 or 16, &c. *cyphers*, and then proceed as before; so will you find the *root* near, whose fractional part will be a decimal. Thus the *Biquadrate root* of 7 will be found near 1.62.

To extract the Biquadrate root.

CHAP.

CHAP. XXXIV.

The Relation of Numbers in quantity.

I. THus far single Arithmetick, comparative Arithmetick insues which is wrought by numbers, as they are considered to have Relation one to another.

Boetius Arith. l.1.cap.21.

II. This relation consists in quantity, or quality.

III. Relation in quantity is the reference or respect, that the numbers themselves have one unto another: As when the comparison is made between 6 and 2, or 2 and 6: 5 and 3, or 3 and 5.

IV. Here the Terms or Numbers propounded are alwayes two, whereof the first is called the Antecedent, and the other the consequent: So in the first example, 6 is the Antecedent, and 2 the Consequent: and in the second, 2 is the Antecedent, and 6 the Consequent.

V. Relation in Quantity consists either in the difference, or else in the rate and reason that is found betwixt the Terms propounded.

Difference.

VI. The difference of two numbers is the remainder, which is left after subtraction of the less out of the greater: so 6 and 2 being the Terms propounded, 4 is the difference betwixt them: for if you subtract 2 out of 6, the remainder is 4.

VII. The

VII. The rate or reason betwixt two numbers is the quotient of the Antecedent divided by the Consequent: So if it be demanded what rate or reason 6 hath to 2, I answer, Triple reason: for if you divide 6 the Antecedent, by 2 the Consequent, the quotient is 3; 2 being contained just 3 times in 6. In like manner is there subtriple reason betwixt 2 and 6, for if you divide 2 by 6, the quotient is $\frac{2}{6}$ or, (which is all one) $\frac{1}{3}$, because 6 being not once found in 2, there remains 2 for the Numerator, 6 the Divisor being the Denominator of the Fraction given you in the Quotient; according to the 9 *rule* of the 16 *chapter* foregoing.

Rate or reason.

VIII. This rate or reason of numbers is either equal or unequal.

IX. Equal reason is the Relation that equal numbers have unto one another: as 5 to 5, 6 to 6, 7 to 7, &c.

Equal Reason.

X. Here the one being divided by the other, the quotient is alwayes an Unite: for if it be demanded how often 5 is in 5, the answer is 1.

XI. Unequal reason is the relation that unequal numbers have one unto another: and this is either of the greater to the less, or of the less to the greater.

Unequal Reason.

XII. Unequal reason of the greater to the less, is when the greater Term is Antecedent: as of 6 to 2, 5 to 3, and the like.

XIII. Here the quotient of the Antecedent divided by the Consequent is alwayes greater than an Unit: So 6 divided by 2, the Quotient is

5, and 5 divided by 3 the quotient is $1\frac{2}{3}$.

XIV. Unequal reason of the less to the greater, is when the lesser Term is Antecedent: as of 2 to 6, 3 to 5, &c.

XV. Here the quotient of the Antecedent divided by the consequent is alwayes less than an unit: So 2 divided by 6, the quotient is $\frac{2}{6}$ or $\frac{1}{3}$ and 3 divided by 5, the quotient is $\frac{3}{5}$.

XVI. Each of these kinds of unequal reason is again subdivided into five other kinds or varieties, whereof the three first are simple, and the other two are mixt.

XVII. The simple kinds of unequal reason are 1. Manifold. 2. Superparticular. 3. Superpartient.

XVIII. Manifold reason of the greater to the less is, when the Consequent is contained in the Antecedent divers times without any part remaining: as 4 to 2, 8 to 4, 16 to 8, which is called Double reason, because the less is contained twice in the greater; so 6 to 2, is triple reason, 8 to 2 fourfold reason, &c.

Manifold reason.

XIX. Here the quotient of the Antecedent divided by the consequent is alwayes a whole number: so 8 divided by 2, the quotient is 4.

XX. The opposite of this kind, *viz.* of the less to the greater, is called submanifold: Examples hereof are 2 to 4, 4 to 8, 8 to 16, &c. Likewise 2 to 6, 2 to 8, 2 to 10, &c.

Submanifold.

XXI. Superparticular is, when the Antecedent contains the consequent once, and besides an aliquot part of the consequent;

Superparticular.

quent; that is, an half, a third, a fourth, or a fifth part, &c. of the consequent, as 3 to 2, 4 to 3, 5 to 4, 6 to 5, and the like; here three divided by 2, the quotient is $1\frac{1}{2}$ and 4 being divided by 3, the quotient is $1\frac{1}{3}$. In like manner 5 divided by 4, the quotient is $1\frac{1}{4}$, and 6 divided by 5 the quotient is $1\frac{1}{5}$ wherefore I say 2 and half 2 (that is 1) constitute 3: So likewise 3 and one third part of 3 (*viz.* 1.) constitute 4, and so of the rest.

XXII. Here the quotient of the Antecedent divided by the Consequent is a mixt number, whose whole part, as also the numerator of the fraction annexed, is alwayes an unit: as is observable in the examples last mentioned.

XXIII. The opposite reason of this kind is Subsuperparticular, as 2 to 3, 3 to 4, 4 to 5, 5 to 6, &c.

Subsuperparticular.

XXIV. Superpartient is when the Antecedent contains the Consequent once, and besides divers parts of the consequent: as 5 to 3, 7 to 5, 7 to 4, 8 to 5, 9 to 5, 11 to 7, &c. here 5 divided by 3, the quotient $1\frac{2}{3}$, and therefore 5 contains 3 once, and $\frac{2}{3}$ of 3; for 3 and two thirds of 3 (*viz.* 2) constitute 5.

Superpartient.

XXV. Here the quotient of the Antecedent divided by the consequent is a mixt number, whose whole part being an unite hath alwayes for the Numerator of the fraction annexed unto it a number composed of more units than one: so the conference being made betwixt 5 and 3, and 5 the Antecedent being divided by 3 the consequent, the quotient is $1\frac{2}{3}$.

XXVI. The opposite of this reason is Sub-superpartient: Examples hereof are 3 to 5, 5 to 7, 4 to 7, 5 to 8, 5 to 9, 7 to 11, and the like.

Subsuperparti-ent.

XXVII. The mixt kinds of unequal reason are Manifold-Superparticular, and manifold superpartient.

XXVIII. Manifold Superparticular reason is when the Antecedent contains the consequent divers times, and besides an aliquot part of the consequent: as 5 to 2, 10 to 3, 17 to 4, 21 to 5, and the like.

Manifold Superparticular.

XXIX. Here the quotient of the Antecedent divided by the consequent is a mixt number, whose whole part consisting of more unites than one, hath alwayes an unite for the Numerator of the Fraction annexed unto it; so 5 divided by 2, the quotient is $2\frac{1}{2}$ and 21 divided by 5, the quotient is $4\frac{1}{5}$.

Submanifold Superparticular.

XXX. The opposite of this Reason is Submanifold Superparticular; as 2 to 5, 2 to 7, 3 to 7, 4 to 9, &c.

XXXI. Manifold Superpartient is, when the antecedent contains the consequent divers times, and besides divers parts of the consequent; as 8 to 3, 17 to 5, 19 to 4, 28 to 5, &c.

Manifold Superpartient.

XXXII. Here the quotient of the Antecedent divided by the Consequent is a mixt Number, whose whole part as also the Numerator of the Fraction annexed unto it, is alwayes a Number composed of more unites than one: so 8 divided by 3, the quotient is $2\frac{2}{3}$, and 28 divided by 5, the quotient is $5\frac{3}{5}$.

Submanifold superpartient.

XXXIII. The

XXXIII. The Opposite here, is Submanifold Superpartient: as 3 to 8, 5 to 17, 4 to 19, 5 to 28, and the like.

And these are the several kinds or varieties of the Rates or Reasons that are found amongst Numbers, so that no two Numbers whatsoever can be named, but the Rate or Reason betwixt them is comprehended under one of these five kinds.

CHAP. XXXV.

The Relation of Numbers in Quality, where; of Arithmetical and Geometrical Proportion.

I. RElation in quality (otherwise called Proportion) is either the reference or respect that the Reasons of Numbers have one unto another, or else which the differences of numbers have one to another.

Vide Euclid. l. 3. d. 5. & Alsted. Arith. c. 5.

II. Therefore here the Terms propounded ought alwayes to be more than two, for otherwise there cannot be a comparison of Reasons or differences in the Plural number.

III. This proportion is either Arithmetical, or Geometrical.

IV. Arithmetical proportion is, when divers numbers differ according to an equal difference, as 2, 4, 6, 8, 10, &c. here 2 is the common difference betwixt 2 and 4, 4 and 6, 6 and 8, 8 and 10, &c. So 1, 2, 3, 4, 5, 6, 7, &c. differ by *Arithmetical* Proportion, 1 being the common difference betwixt them.

Arithmetical Proportion.

V. Arithmetical Proportion is either continued or interrupted.

VI. Arithmetical Proportion continued is, when divers Numbers are linked together by a continual progression of equal differences. Such are the examples last propounded, as also these 1, 3, 5, 7, 9, 11, 13, &c. And 100000, 200000, 300000, 400000, &c.

1. *Continued.*

VII. In a rank of numbers that differ by Arithmetical Proportion continued, the sum of the first and last Terms being multiplyed by half the number of the Terms, the Product is the total sum of all the Terms: so it being demanded, how many strokes the Clock strikes betwixt midnight and noon; the Terms of the Progression in this question are Twelve, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. for in that order the Clock strikes, wherefore if I multiply 13 the sum of 12, and 1 (the first and last Terms) by 6 (being half the number of the Terms) the Product is 78, which is the total sum of all the Terms propounded being added together.

VIII. Or thus, Multiply the number of the Terms by the half sum of the first and last Terms, & then likewise the Product will give you the total

of

all the Terms: so 13, 11, 9, 7, 5, 3. being given, their total is 48; for 8 the half sum of 13 and 3, the first and last Terms being multiplyed by 6, the number of the terms, the product is 48.

IX. Three numbers being given, that differ by Arithmetical proportion continued, the mean being doubled, is equal to the sum of the extreams: so 5, 6, 7, being given, 6 being doubled is equal to the sum of 5 and 7 the two extreams.

X. Arithmetical Proportion may be continued either upwards or downwards. *Upwards.*

XI. Upwards, when the Terms of the Progression increase, as these, 2, 4, 6, 8, 10, 12, &c. or these, 1, 2, 3, 4, 5, 6, &c. And this last rank is more particularly termed *Natural Progression.*

XII. Here when the first term is also the common difference of the terms, the last term being divided by the number of the terms, the quotient will give you the first term of the rank: again in this case the first term multiplyed by the number of the terms produceth the last term: so this rank 3, 6, 9, 12, 15, 18, 21. being propounded, wherein 3 is both the first term as also the common difference of the terms; I say 21 the last term being divided by 7 the Number of the terms the quotient is 3 the first term; contrariwise 3 the first term multiplyed by 7, produceth 21, the last term.

XIII. Arithmetical proportion continued downwards is, when the terms of the progression decrease: such as are 35, 32, 29, 26, 23, 20: And 40, 35, 30, 25, 20, 15, 10, 5. *Downwards.*

XIV. Here when the last term is also the common difference of the terms, the first term being divided by the Number of the terms, the quotient will give give you the last term: Again, the last term multiplyed by the Number of the terms, produceth the first term of the rank.

This Rule is in the inverse of the 12. Rule aforegoing.

For example, this rank 40, 35, 30, 25, 20, 15, 10, 5 being propounded, in which 5 is both the last term, and likewise the common difference of the terms, I say, 40 the first term being divided by 8 the number of the terms, the quotient is 5 the last term: on the otherside 5 the last term being multiplyed by 8, the product is 40 the first term.

XV. Arithmetical Proportion interrupted is, when the Progression is discontinued: as in these numbers 2, 4, 8, 10; here 2 and 4 being compared with 8 and 10 differ according to Arithmetical proportion, but so do not 4 and 8 differ, for 2 is the common difference betwixt 2 and 4, 8 and 10, whereas the difference betwixt 4 and 8 is 4. In like manner 8, 14, 17, 23. differ by Arithmetical proportion interrupted.

2. Interrupted.

XVI. Four numbers being given, that differ by Arithmetical proportion either continued or interrupted, the sum of the two means is equal to the sum of the two extreams: so 5, 6, 7, 8, being given, the sum of 6 and 7, the two mean numbers is equal to the sum of 5 and 8, the two extreams: and 8, 14, 17, and 23, being propounded, the sum of 14 and 17 being added together, is equal to the sum of 8 and 23.

XVII. Geo-

XVII. Geometrical proportion is, when divers numbers differ according to like reason : that is, when the reasons of numbers, being compared together, are equal. So 1, 2, 4, 8, 16, 32, &c. which differ one from another by double reason, are said to differ by *Geometrical* proportion, for as one is half 2, so 2 is half 4, 4 half 8, 8 half 16, 16 half 32, &c. *Geometrical proportion.*

XVIII. Geometrical proportion is either continued or interrupted. *1. Continued.*

XIX. Geometrical proportion continued is, when divers numbers are linked together by a continued progression of the like reason : of this sort is the example last given : for as 1 is to 2, so is 2 to 4, 4 to 8, 8 to 16, 16 to 32, &c. So likewise the numbers 3, 9, 27, 81, 243, 729, &c. differ by *Geometrical* proportion continued, *viz.* by triple reason, each of them being contained three times in the next number that follows it.

XX. In Numbers continually proportional from 1, the first number from 1, is the root or first power, the second is the square or second power, the third the cube or third power, the fourth the biquadrate or fourth power, the fifth the fifth power, the sixth the sixth power, &c. So in this rank of numbers, 1 3, 9, 27, 81, 243, 729, &c. 3 is the root, 9 the square, 27 the cube, 81 the biquadrate, 243 the fifth power, 729 the sixth power, &c.

XXI. The root being multiplyed by it self produceth the square, which being again multiplyed by the root produceth the cube, and so each proportional being multiplyed by the root produceth the *Mean proportionals.*

proportional

proportional next above it, and then the numbers comprehended betwixt 1, and the last number produced are called mean proportionals: so in this rank of proportional numbers, 1, 2, 4, 8, 16, 32, &c. 2 the root being multiplyed by it self produceth 4 the square, which being again multiplyed by 2, produceth 8 the cube, then 8 being multiplyed by 2, the product is 16 the biquadrate, and so of the rest in their order, and here 2, 4, 8, and 16 are the mean proportionals in the rank propounded.

XXII. If you multiply the root by it self, and consequently the subsequent numbers by themselves, the numbers intercepted betwixt 1 and the number last produced may not unfitly be called continual means: so 2 being given for the root, multiplyed by it self, the product is 4, which being again multiplyed by it self produceth 16, then 16 in like manner squared produceth 256, which likewise multiplyed by it self produceth 65536, I say then that 2, 4, 16, and 256 are continual means betwixt 1 and 65536.

Continual means. Briggius Arith. Log. c. 6.

XXIII. The continual means comprehended betwixt any number given, and 1, are discovered by a continued extraction of the square roots; for example 65536 being given, the root thereof extracted is 256, whose root is 16; then the root of 16 is 4, and the root of 4 is 2; so that at last I find 256, 16, 4, and 2 to be continual means intercepted betwixt 65536 and 1, as before.

XXIV. In numbers that increase by Geometrical proportion continued, if you multiply the last term by the quotient of any one of the terms divided

divided by another term, which being less is next unto it, and then deducting the first term out of that product, divide the remainder by a number that is an unit less than the quotient, the last quotient will give you the total of all the terms propounded in the progression; so this rank 2, 6, 18, 54, 162, 486, 1458, being propounded, wherein the proportionals differ by subtriple proportion, I first take 2 and 6 the two first terms, and dividing 6 by 2, I find the quotient 3, wherefore multiplying 1458 the last term, by 3 the quotient, the product is 4374, out of which if I deduct 2 the first term, the remainder is 4372, which being divided by 2 (*viz.* a number which is an unite less than 3 the quotient) the last quotient gives me 2186, which is the total sum of the proportionals propounded.

XXV. Three proportionals being given, the square of the mean is equal to the product of the extreams: so 4, 8, and 16 being propounded, 8 times 8 being 64, is equal to 4 times 16, which is likewise 64.

XXVI. Geometrical proportion interrupted is, when the progression of like reason is discontinued; in such sort that four numbers being given, the like reason is not found betwixt the second and third, that is betwixt the first and second, and the third and fourth; of this sort are these numbers 2, 4, 16, 32. here as 2 is to 4, so is 16 to 32, for they differ by double reason; but as 2 is to 4, so is not 4 to 16, for 4 and 16 differ by fourfold reason, 4 being contained 4 times in 16: so likewise 4, 8, 8, 16, differ according to Geometrical proportion interrupted.

2. Interrupted.

XXVII.

XXVII. The numbers of Multiplication and Division are proportional; for in Multiplication, as 1 is to the Multiplicator, so is the Multiplicand to the product, or as 1 is to the Multiplicand, so is the Multiplicator to the product: Again, in Division as the Divisor is to 1, so is the Dividend to the Quotient: or as the Divisor is to the Dividend, so is 1 to the Quotient.

XXVIII. Four proportional Numbers whatsoever being given, the product of the two means is equal to the product of the two extreams: So 2, 4, 16, 32, being propounded, 4 times 16 (which is 64) is equal to 2 times 32, which is likewise 64.

Here endeth the first book, which containeth all that is absolutely necessary, for the full understanding of *common or practical Arithmetick*. Such as desire to see how the same is performed by artificial, or borrowed numbers, called *Logarithmes*, may peruse Mr. *Wingates second book*, being a distinct *Treatise of artificial Arithmetick*.

A N

AN
APPENDIX,

CONTAINING

Choice knowledge in *Arithmetick*, both *Practical and Theoretical*; the Contents whereof are exprest in the following Page.

Composed by JOHN KERSEY.

Teacher of the

MATHEMATICKS.

At the sign of the *Globe* in *Shandois-Street* in *Covent-Garden*.

Vox audita perit, litera Scripta manet.

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The Contents of the APPENDIX.

CHAP.

1 Of Contractions in the *Rule of Three.*

2. Of Rules of Practice by *aliquot parts.*

3. Of Exchanges of *Coins, Weights,* and *Measures.*

4. Practical questions about *Tare, Tret, Loss, Gain, Barter, Factorship,* and measuring of *Tapestry.*

5. Of *Interest of Money,* and the construction of *Tables* to value *Annuities, &c.*

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7. A *demonstration* of the *Double Rule of Fellowship.*

8. A *demonstration* of the rule of *Alligation*: where also of the *composition* of *Medicines.*

9. A *demonstration* of the *Rule* of *False.*

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11. *Sports* and *Pastimes.*

An

An Explication of such Notes or Characters which for brevity sake are used in this

APPENDIX.

THis + is a note of *Addition*, signifying that the number which followeth such sign is to be added to the number preceding it, so 3 + 4 implyeth that 4 is to be added to 3 : sometimes also, when no number is placed next after the said note, it implyeth that the number preceding is not exactly exprest ; so the *square root* of 2 is 1.414 + or 1.414, &c. that is, $1\frac{414}{1000}$ and somewhat more.

This — is a sign of *Subtraction*, signifying that the number which followeth such sign is to be subtracted from the number preceding it ; so 6 — 2 signifieth the difference between 6 and 2 , or 2 to be subtracted from 6.

This × is a sign of *Multiplication*, signifying that the number which precedeth such sign is to be multiplyed into, or by the number following the sign: so 3 × 4 implyeth that 3 is to be multiplyed by 4 ; likewise by 3 × 4 × 8 is understood the *continual multiplication* of the numbers 3 , 4 and 8 ; *viz.* 3 is to be multiplyed by 4, and the product is to be multiplyed by 8. Sometimes also the said sign hath reference to as many of the preceding or following numbers as have a little line placed over them ; so $3 \times \overline{2+6}$ or $\overline{2+6} \times 3$ signifieth that 3 is to be multiplyed by the *sum* of 2 and 6. Likewise

wise $\overline{8-5}\times 3$, or $3\times\overline{8-5}$ implieth that 3 is to be multiplied by the *difference* between 8 and 5: Moreover if A and B represent two numbers, then A × B or A B implieth the product of the multiplication of those numbers: Likewise $\overline{B-C}\times A$ signifieth the product arising from the multiplication of the excess of the number B above the number C, by (or into) the number A. Again, if A B and A C represent two lines, then $\square$ A B × A C implieth a rectangular Figure or long square made of the lines A B and A C.

Numbers placed as you see in the Margent denote a *Divisor*, a *Dividend* and a *Quotient*, to wit, 3 the *Divisor*, 18 the *Dividend*, and 6 the *Quotient*; the like is to be understood of other numbers so placed. 3) 18 (6

Numbers placed after the manner of a *fraction* denote a *quotient*, which ariseth from dividing the *Numerator* by the *Denominator*; so $\frac{2\times 5\times 6}{3\times 4}$ is equal to the *Quotient*, which ariseth from dividing the *product* of the *continual multiplication* of 2, 5 and 6 by the *product* of 3 multiplied by 4.

Four numbers placed as you see in the Margent are *Geometrical proportionals*, *viz.* As 2 is to 4; so is 6 to 12: or if 2 give 4, then 6 will give 12. Sometimes also they are placed thus, 2 . . . . 4 . . . . 6 . . . . 12. 2.4 :: 6.12

This = is a note of *equality* or *equation*; so by $3+4=5+2$ is signified that the sum of 3 and 4 is equal to the sum of 5 and 2: also $7-3=9-5$ signifieth that the *difference* between 7 and 3 is equal to the *difference* between 9 and 5; that is, 7

lessened by 3 leaves the same remainder, as 9 lessened by 3. Also $4 \times 3 = 12$ implieth that the *product* of the multiplication of 4 by 3 is equal to 12.

$>$ This is a sign of *majority*, signifying that the number on the left hand of such sign is *greater* than the number on the right hand thereof; so $5 > 3$ implieth that 5 is *greater* than 3.

$<$ This is a sign of *minority*, signifying that the number on the left hand of such sign is *less* than the number on the right hand thereof; so $3 < 5$ implieth that 3 is *less* than 5.

This Character $\surd$ or $\surd q.$ signifies the square-root of the number which follows it, so $\surd 144$ implies the square-root of 144, to wit 12.

Also this $\surd c$ signifies the cube root of the number which follows it, so $\surd c.\ 1728$ signifies the cube root of 1728, which cube root will be found to be 12.

AN APPENDIX.

CHAP. I.

Of Contractions in the Rule of Three.

I. SUch as are well verst in the parts of *Arithmetick*, which have been fully laid open in the precedent Book, and are mindful of the *Notes* or *Symbols* before explained, will find no difficulty in the 1, 2, 3, 4, 5, and 10 Chapters of this *Appendix*, wherein divers compendious operations no less delightful than useful are methodically handled; and the rest will be as easie to such as are but meanly acquainted with *Geometrical demonstration*.

II. To repeat the brief wayes of *Multiplication* set forth in the 10, 11, and 12 *rules* of the fifth *Chapter*, or those of *Division*, in the 11, 15, and 16 *rules* of

 the

the sixth *Chapter* aforegoing, would be a superfluous work, and therefore I shall presuppose the *Reader*, to be throughly acquainted with them, as also with competent knowledge in the operations of fractions both *vulgar* and *decimal*.

III. It will be no small advantage to the Practical *Arithmetician*, to have by heart not onely the common *Table* of *Multiplication*, but this also in the *Margent*, to the end that when a number is given to be multiplied or divided by 12, (which happens in the *Reduction* of *shillings* to *pence* and the converse) the *product* or *quotient* may be written down in one line onely, as in the Examples following.

2		24
3		36
4		48
5		60
6	× 12 =	72
7		84
8		96
9		108

$$\begin{array}{r} 3472 \\ 12 \\ \hline 12)\ 41664\ (3472 \end{array} \qquad \begin{array}{r} 4736 \\ 12 \\ \hline 12)\ 56832\ (4736 \end{array}$$

IV. When a whole number is given to be divided by a Divisor, which is equal to the product of the Multiplication of two single figures, instead of dividing by that Divisor you may first divide by one of those single figures, and then divide the quotient by the other, so will the last quotient be the same as if the Division had been finisht by the Divisor first given: thus if 3466 *farthings* be given to be reduced to *shillings*, because 8 × 6 = 48 I first divide 3466 by 8,

8, so there will arise 433 for a new *Dividend*, and 2 *farthings* remain, then I divide the said 433 by 6, so there will arise $72\frac{1}{6}$, or

8) 3466
s. d.
6) 433 (72 . . $2\frac{1}{2}$

72 *shillings* 2 *pence*, which with the 2 *farthings* remaining of the first *Division* make in all 72 *s.* : $2\frac{1}{2}$ *d.* which is the very *quotient*, when 3466 farthings are divided by 48. Note that you are to reserve a farthing for every unit remaining of the first Division by 8, and two pence for every unit remaining of the second Division by 6. The reason of the operation is evident, for $\frac{1}{6}$ of $\frac{1}{8} = \frac{1}{48}$.

In like manner, if 7136 pence are given to be reduced into pounds, because 240 $d = 1$ *l.* also $6 \times 40 = 240$, therefore if 7136 pence be first divided by 6, the quotient will give 1189 six pences, and 2 pence remain; then if 1189 be divided by 40, (that is by 4, after 9 the last place of the Dividend towards the right hand is cut off) the quotient will be 29 *l.* and there will remain 29 six pences or 14 *s.* 6 *d.* which together with the

6) 7136
l. s. d.
40) 118|9 (29 : 14 : 8

2 pence remaining of the first Division, and the said 29 *l.* makes in all 29 *l.* : 14 *s.* : 8 *d.* which is the same with the quotient, when 7136 pence are divided by 240, for $\frac{1}{40}$ of $\frac{1}{6} = \frac{1}{240}$.

Again, suppose 3463 pence are given to be reduced into shillings; forasmuch as $4 \times 3 = 12$, I first divide 3463 by 4, so there will arise 865 for a new Dividend and 3 pence remain: then I divide the said 865 by 3, so there will arise $288\frac{1}{3}$ or 288 *s.*

4) 3463

$\quad\quad$ s. d.

3) 865 (288 . 7

4 d. which with the 3 pence before remaining make 288 s. 7 d. which is the same with the quotient, when 3463 pence are divided by 12, for $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$.

V. In the rule of Three as well direct as inverse, when the Divisor with either of the other two given numbers may be severally divided by some common measure, without leaving any remainder, the quotients may be taken for new terms and proceeding in like manner as often as is possible; the operation according to the tenth rule of the eighth Chapter, or the second rule of the ninth Chapter, will be much contracted: so if it be demanded what 52 yards of Cloath will cost at the rate of 21 *l.* for 14 yards; the *Answer* will be found 78 pounds, in manner following.

y.	l.	y.	
14 ...	21 ...	52	
2 ...	3 ...	52	
1 ...	3 ...	26 ..	(78

In the first rank you may observe that the Divisor 14 and the second term 21, being severally divided by their common measure 7, the three new terms in the second rank) will be 2, 3, 52. Again in the second rank the Divisor 2 and the third term 52 being severally divided by their common measure 2, the three new terms (in the third rank) will be 1, 3, 26 Lastly, working with these according to the *rule* of *Three direct*, the *Answer* to the question (or fourth term) will be found to be 78.

Another

Another Example, If 21 men will finish a work in 16 dayes, what time must be allowed to 12 men for the finishing of such a work? *Answer*, 28 dayes.

men		*dayes*		*men*	
21	...	16	...	12	
7	...	16	...	4	
7	...	4	...	1	(28 *dayes*.

In the first rank you may observe, that the Divisor 12 (for the rule is inverse) and the first term 21 being severally divided by their common measure 3, the three new terms (in the second rank) will be 7, 16, 4. Again, in the second rank, the Divisor 4, and the second term 16, being severally divided by their common measure 4, the three new terms in the third rank will be 7, 4, 1. Lastly, working with these as the Rule of three inverse requires, the *Answer* to the question (or fourth term) will be found 28.

VI. In the Rule of three, as well direct as inverse, when the Divisor and either of the other two terms are fractions having a common denominator, the said denominators may be rejected, and their numerators retained as new terms: so if it be demanded what is the value of $\frac{7}{8}$ of an *Ell*, when $\frac{3}{8}$ of an *Ell* are worth 66 pence, the *Answer* will be found 154 pence, and the work will stand as you see.

$\frac{3}{8}$	..	66	..	$\frac{7}{8}$	
3	..	66	..	7	
1	..	22	..	7	(154

Another Example. If $3\frac{3}{4}$ *yards* of Scarlet cloath cost 8 *l.* 15 *s.* what is the price of one *yard* at that rate? *Answer* 2 *l.* 6 *s.* 8. *d.*

$$\frac{15}{4} \ldots \frac{35}{4} \ldots 1$$
$$15 \ldots 35 \ldots 1$$
$$3 \ldots 7 \ldots 1 \ldots (2\tfrac{1}{3}\ l.$$

VII. In the Rule of three as well direct as inverse, when the Divisor only is a fraction, either of the other two terms may be reduced to a fraction of the same Denominator, and then the Denominators may be rejected as before in the sixth rule; also when one of the three given terms is a fraction, and is not the Divisor, the Divisor may be reduced to a fraction of the same Denominator with the fraction first given, and then the common Denominators may be likewise cancelled.

An *Example* of the first Case may be this, if $\frac{7}{8}$ of a *yard* cost 14 *s.* what is the price of 1 *yard*? *Answer* 16 *shillings*.

yard	*shill.*	*yard.*	
$\frac{7}{8}$ ...	14 ...	1	
$\frac{7}{8}$ ...	14 ...	$\frac{8}{8}$	
7 ...	14 ...	8 ...	(16 *shill.*

An *Example* of the second Case; if of stuff which is $\frac{3}{4}$ of a yard in breadth, 7 yards in length will make a Garment; how much of that stuff which is one yard in breadth will be sufficient for the same purpose? *Answer* $5\frac{1}{4}$ *yards*.

Rules

Rules of 3 *Inverse.* $\begin{cases} \frac{3}{4} \;\ldots\; 7 \;\ldots\; 1 \\ \frac{3}{4} \;\ldots\; 7 \;\ldots\; \frac{4}{4} \\ 3 \;\ldots\; 7 \;\ldots\; 4 \end{cases}$ (.. $\frac{21}{4}$ or $5\frac{1}{4}$.

CHAP. II.

Rules of Practice by Aliquot parts.

I. AN *Aliquot part* takes its name from the *Latine* word *aliquoties*, for (according to *Euclid*) an *aliquot part* is of a greater number, such a part which being taken (*aliquoties* or) certain times doth precisely constitute that greater number; so 3 is an *aliquot part* of 12, for 3 taken four times doth exactly make 12, without any excess or defect; in like manner 4 is an *aliquot part* of 20, because 4 taken 5 times doth precisely make 20; but 7 is not an *aliquot part* of 20, for 7 taken twice doth want of 20, and being taken thrice doth exceed 20; this kind of *part* last mentioned is by *Euclid* called *pars aliquanta*, of which there will be no use in this place.

II. When the *Rule of Three direct* hath 1 or an *Integer* for the first term, it is commonly called a *Rule of Practice*, either from the great use and practice thereof in common affairs, or else for that questions of this nature, may be resolved by operations more speedy and practical than those of the *Rule of Three*.

III. The

III. The choiceſt of theſe *Rules* of *Practice* may be reduced to 5 Caſes, *viz.*

When the price of 1 or an Integer conſiſts.	1. *Of ſhillings under* 20.
	2. *Of pounds and ſhillings.*
	3. *Of pence under* 12.
	4. *Of ſhillings and pence.*
	5. *Of pounds, ſhillings, pence, with parts of a penny.*

All which caſes with others of the like nature are handled in their order.

IV. Any even number of ſhillings is either $\frac{1}{10}$ of a pound, (that is 2 *ſhillings*) or elſe is compoſed of $\frac{1}{10}$ *l.* (to wit 2 *s.*) taken certain times: ſo 8 *s.* is compoſed of $\frac{1}{10}$ *l.* (or 2 *ſhillings*) taken four times, in like manner 18 *s.* is compoſed of $\frac{1}{10}$ *l.* taken nine times.

V. When the price of 1 or an integer of what name ſoever is 2 ſhillings, the price of as many Integers as one will of that name is diſcoverable at firſt ſight, to wit by accounting the double of the figure which ſtands in the firſt place (towards the right hand) of the ſaid number of Integers, as ſhillings and the reſt of the ſaid number as pounds: ſo 345 yards at two ſhillings the yard will coſt 34 *l.* 10 *s.* for the double of 5 is 10, which I write down apart as ſhillings, then eſteeming the remaining figures towards the left hand, to wit 34, as an entire number of pounds; the *Anſwer* will be 34 *l.* 10 *s.* This contraction is nothing elſe, but dividing the number

yard	*ſhill.*	*yards*
1	2	345
Anſwer	34 *l.*	10 *s.*

ber

ber of Integers, whose price is required by 10. More examples hereof are these;

oz.	shill.	oz.
1	2	2057

	l.	s.
Answ.	205	14

yard	shill.	yards
1	2	120

	l.	s.
Answ.	12	0

VI. When the given price of 1 or an Integer is any even number of shillings greater than two shillings, multiply the number of Integers, whose price is required, by half the given number of shillings, with this caution, that the double of the figure which ariseth, in the first place of the product be written apart as shillings, and the rest of the product as pounds: so if it be demanded what 218 yards at 8 shillings the yard will amount unto, the *Answer* will be found 87 *l.* 4 *s.* for I multiply 218 by 4, (which is half 8 the given number of shillings) saying, 4 times 8 is 32, here the double of 2 (to wit, of that figure which is to possess the first place in the product) is 4, which I set apart as shillings, keeping 3 in mind for the three tens, again 4 times 1 is 4, which with

y.	s.	y.
1	8	218
		4
		87 : 4

with three in mind makes 7; lastly, 4 times 2 makes 8, so I conclude that the *Answer* to the question is 87 *l.* 4 *s.* The reason of this contraction is evident from the fourth and fifth rules aforegoing. More examples of this rule are these following.

yard	*s.*	*yards*
1 ...	14 ...	436

	l.	*s.*
Answ.	305 . .	4

yard	*s.*	*yards*
1 ...	18 ...	230

	l.	*s.*
Answ.	207 . .	0

VII. Any odd number of shillings is either compos'd of $\frac{1}{10}$ *l.* (or 2 *s.*) and of $\frac{1}{20}$ *l.* (or 1 *s.*) or else it is compos'd of $\frac{1}{10}$ *l.* (or 2 *s.*) taken certain times, and of $\frac{1}{20}$ *l.* (or 1 *s.*) So 3 *s.* is compos'd of 2 *s.* and 1 *s.* Also 7 *s.* is compos'd of 2 *s.* taken three times and of 1 *s.* Likewise 13 *s.* is compos'd of 2 *s.* taken six times and of 1 *s.*

VIII. When the given price of 1 or an Integer is an odd number of shillings, work for the greatest even number of shillings contained in that odd number, according to the fifth or sixth rule aforegoing, then for the odd shilling remaining, take $\frac{1}{20}$ of the number of Integers, whose price is required (by the 16. rule of the sixth Chapter of the preceding book.) These two results added together give the Answer to the question:

question: so if it be demanded what 2344 ounces at 13 s. the ounce will cost, the answer will be found 1523 l. 12 s. For if (according to the sixth rule of this Chapter) I multiply 2344 by 6, (to wit, by half the remainder, when one is abated from 13 the given number of shillings) there will arise 1406 l. 8 s. Then taking $\frac{1}{20}$ of 2344, there will arise 117 l. 4 s. which being added to the former product gives 1523 l. 12 s. for the *answer* to the question.

oz.	*shill.*	*oz.*	
1 ..	13 ..	2344	
		6	
		l.	*s.*
		1406 ..	8
		117 ..	4
		l.	*s.*
	Answ.	1523 ..	12

Note, When 5 shillings is the given price of 1 or an Integer, the briefest way will be to take $\frac{1}{4}$ of the number of Integers, whose value is required, for such quotient will give the pounds and shillings, which answer the question: so 2347 ounces at 5 s. the ounce amount unto 586 l. 15 s. for $\frac{1}{4}$ of 2347 is $586\frac{3}{4}$ or 586 l. 15 s. But when the given price of 1 is any other odd number of shillings, this eighth rule will be as compendious as any other whatsoever.

More examples of this rule are these following:

yard	*shill.*	*yards*	
1 ...	19 ...	739	
		l.	*s.*
		665 ...	2
		36 ...	19
	Answ.	702 ...	1

yard

yards	*shill.*	*yards*
1	17	345

	l.	
	276	0
	17	0
Answ.	293	15

IX. When the given price of an Integer consists of pounds and shillings, first multiply the number of Integers whose price is required, by the number of pounds in the said given price, and subscribe the product as pounds; then proceed with the shillings in the said given price, according to the sixth or eighth rule of this Chapter, and having subscribed that which ariseth under the aforesaid product of pounds, add them all together for the answer of the question: so if it be demanded what 328 hundred weight will amount unto at 2 *l.* 17 *s. per C.* (or one hundred weight) the *answer* will be found to be 934 *l.* 16 *s.* as by the operation is evident.

C.	*l.*	*s.*	*C.*
1	2	17	328

	l.	*s.*
	656	0
	262	8
	16	8
Answ.	934	16

More

More *Examples* to illustrate this *Rule* are these following.

C.	l.	s.		C.	
1 ...	7	: 12	::	504	
				l.	s.
				3528	..
				302	.. 8
		Answ.		3830	.. 8

C.	l.	s.		C.	
1 ...	5	: 7	::	129	
				l.	s.
				645	..
				38	.. 14
				6	.. 9
		Answ.		690	.. 3

X. Any number of pence under 12 is either an Aliquot part of a shilling, or else compos'd of Aliquot parts thereof; so 3 pence is an Aliquot part to wit, $\frac{1}{4}$ of a shilling. Likewise 4 is $\frac{1}{3}$ of 12; moreover 5 pence are compos'd of 2 Aliquot parts, to wit, of 3 pence which is $\frac{1}{4}$ of a shilling, and of 2 pence which is $\frac{1}{6}$ of a shilling; all which will readily appear by the following Table.

Pence	Aliquot parts of a shilling.
1	$\frac{1}{12}$ (or $\frac{1}{3}$ of $\frac{1}{4}$
1 $\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{4}+\frac{1}{6}$
6	$\frac{1}{2}$
7	$\frac{1}{4}+\frac{1}{3}$
8	$\frac{1}{3}+\frac{1}{3}$
9	$\frac{1}{2}+\frac{1}{4}$
10	$\frac{1}{2}+\frac{1}{3}$
11	$\frac{1}{3}+\frac{1}{3}+\frac{1}{4}$

XI. When the given price of 1 or an Integer is an Aliquot part of a shilling; divide the number of Integers whose value is required by the denominator of such aliquot part, so will the quotient be the number of shillings which answers the question, which number of shillings (when there is occasion) may be reduced to pounds by the brief way

way of dividing by 20: so if it be required to know what 2686 ounces at 4 pence the ounce will amount unto; the answer will be found 44 *l.* 15 *s.* 4 *d.* for since 4 *d.* is an aliquot part, to wit, $\frac{1}{3}$ of a shilling, I divide 2686 by 3, so will the quotient be $895\frac{1}{3}$ *s.* or 895 *s.* 4 *d.* which shillings being divided by 20, give 44 *l.* 15 *s.* 4 *d.* for the answer to the question, as you see by the following operation.

oz.	d.	oz.
1 . . .	4 . . .	2686

		s.	d.
20)	89	5 . .	4
Answ.	44 . .	15 . .	4

More Examples of this Rule are these following:

yard	d.	yards
1 . . .	6 . . .	759

		s.	d.
	20)	37 | 9 . .	6
Answ.	18 . .	19 . .	6

yard	d.	yards
1 . . .	1 . . .	204

Answ. 17 *shillings.*

XII. When the given price of an Integer is compos'd of aliquot parts of a shilling, divide the number of Integers whose price is required by the several denominators of the aliquot parts contained in the given number of pence, then add the quoti-

ents together, and the sum shall be the number of shillings which answer the question: so if it be demanded what 2347 yards of linnen cloth will cost at 9 pence the yard, the answer will be found 88 *l.* 0 *s.* 3 *d.* For since 9 *d.* is compos'd of 6 *d.* and 3 *d.* to wit, of the aliquot parts $\frac{1}{2}$ and $\frac{1}{4}$ of a shilling, I first divide 2347 by 2 (the denominator of the aliquot part $\frac{1}{2}$) so there ariseth $1173\frac{1}{2}$ or 1173 *s.* 6 *d.* Again, dividing the said 2347 by 4 (the denominator of the other aliquot part) there will arise $586\frac{3}{4}$ or 586 *s.* 9 *d.* which two quotients being added together give 1760 *s.* 3 *d.* or 88 *l.* 0 *s.* 3 *d.* which is the answer of the question. More Examples to illustrate this rule are these.

yard	*d.*	*yards*		
1	9	2347		
		s.		*d.*
		1173		6
		586		9
	20)	176 | 0		3
		l.	*s.*	*d.*
Answ.		88	0	3

yard	*d.*	*yards*		
1	8	782		
		s.		*d.*
		260		8
		260		8
	20)	52 | 1		4
		l.	*s.*	*d.*
Answ.		26	1	4

ex.

$$\begin{array}{lllll} oz. & & d. & & oz. \\ 1 & \ldots & 11 & \ldots & 540 \\ \hline & & & & 180 \\ & & & & 180 \\ & & & & 135 \\ \hline & & 20) & 49|5 & s. \quad d. \\ \textit{Answ.} & & & 24 & \ldots 15 : 0 \end{array}$$

XIII. When the given price of an Integer consists of shillings and pence, first multiply the number of Integers whose value is required by the said given number of shillings, and subscribe the product as shillings, then divide the said number of Integers, by the several denominators which are correspondent to the aliquot parts contained in the given number of pence, and subscribe the quotient or quotients underneath the aforesaid product of shillings, all which being added together give the number of shillings which answers the question: so if it be demanded what 347 yards of cloth will cost at the rate of 7 *s.* 10 *d.* the yard, the answer will be found 135 *l.* 18 *s.* 2 *d.* for first 347 being multiplied by 7, (the given number of shillings) produceth 2429 shillings, then dividing 347 by 2 and 3 severally, (because 10 *d.* is com-

$$\begin{array}{llllll} yard & s. & d. & yards & & \\ 1 \,.. & 7 : & 10 & ..\, 347 & & \\ \hline & & & & s. & d. \\ 7 \times 347 & = & & & 2429 & : \\ 2)\ 347 & (.. & & & 173 & : 6 \\ 3)\ 347 & (.. & & & 115 & : 8 \\ \hline & & & 20) & 271|8 & : 2 \\ & & & l. & s. & d. \\ & & \textit{Answ.} & 135 & : 18 & : 2 \end{array}$$

pos'd of $\frac{1}{2}$ and $\frac{1}{3}$ of a shilling) the quotients will be $173\frac{1}{2}$ and $115\frac{2}{3}$, that is 173 *s.* 6 *d.* and 115 *s.* 8 *d.* Lastly, the sum of all is 2718 *s.* 2 *d.* or 135 *l.* 18 *s.* 2 *d.*

More Examples of this kind are these.

yard	*s.*	*d.*	*yards*
1 ...	17 :	9 ...	540

$$
\begin{array}{rr|l}
 & & s. \\
17 \times 540 = & & \begin{cases} 3780 \\ 540. \end{cases} \\
2) & 540 (.. & 270 \\
4) & 540 (.. & 135 \\
\hline
 & 20) & 958|5 \\
 & & l. \quad s. \quad d. \\
\textit{Answ.} & & 479 : 5 : 0
\end{array}
$$

y.	*s.*	*d.*	*y.*
1 ...	14 :	6 ...	313

$$
\begin{array}{rrl}
 & & s. \quad d. \\
14 \times 313 = & & \begin{cases} 1252 \\ 313. \end{cases} \\
2) \; 313 \; (. & & 156 : 6 \\
\hline
 & 20) & 453|8 \\
\textit{Answ.} & & 226 .. 18 : 6
\end{array}
$$

XIV. When the price of an Integer consists of shillings and pence, and that such shillings and pence joyntly considered do make an aliquot part of a pound, it will oftentimes be a briefer way than that in the last rule, to divide the number of Integers whose value is required, by the denominator of such aliquot part, so will the quotient give the answer

answer to the queſtion in pounds and known parts of a pound. Thus if it be demanded what 767 yards will coſt at the rate of 6 *s*. 8 *d*. the yard, the anſwer will be found 255 *l*. 13 *s*. 4 *s*. For ſince 6 *s*. 8 *d*. is an aliquot part, to wit, ⅓ of a pound, I divide 767 by 3, ſo there ariſeth in the quotient 255⅔, or 255 *l*. : 13 *s*. : 4 *d*. which is the anſwer of the queſtion. *Note* that the *Aliquot parts* of a pound convenient for this rule are theſe expreſt in the following *Table*.

y.	*s*.	*d*.		
1	6	8	767	
	l.		*s*.	*d*.
3) 767	(255		13	4

ſh.	*d*.	*Aliquot parts of a pound.*
6	8	$\frac{1}{3}$
3	4	$\frac{1}{6}$
2	6	$\frac{1}{8}$
1	8	$\frac{1}{12}$
1	4	$\frac{1}{15}$
1	3	$\frac{1}{16}$

XV. When the given price of 1 or an Integer conſiſts of pounds, ſhillings and pence, reduce the ſaid pounds and ſhillings all into ſhillings, then proceed according to the 13 Rule of this Chapter: So 517 C. at 3 *l*. : 17 *s*. 5 *d*. *per* C. will be found to amount unto 2001 *l*. 4 *s*. 5 *d*. for having reduced 3 *l*. 17 *s*. into 77 *s*. I multiply 517 by 77, and write down the

particular products; then for the 5 pence which is compos'd of the aliquot parts $\frac{1}{4}$ and $\frac{1}{6}$ of a shilling; I take $\frac{1}{4}$ and $\frac{1}{6}$ of 517, and subscribe the quotients orderly underneath the aforesaid products: Lastly, adding all together the sum is 40024 *s.* 5 *d.* or 2001 *l.* 4 *s.* 5 *d.* for the answer of the question.

C.	*l.*	*s.*	*d.*	C.
1 ...	3 :	17 :	5 ..	517

$$77 \times 517 = \begin{cases} 3619 \\ 3619. \end{cases}$$

4) 517 (.. 129 : 3 *d.*
6) 517 (.. 86 : 2

20) 4002 | 4 : 5
l. *s.* *d.*
Answ. 2001 : 4 : 5.

More *Examples* of this *rule* are these following.

C.	*l.*	*s.*	*d.*	C.
1 ...	5 :	13 :	8 ...	108

s.

$$113 \times 108 = \begin{cases} 324 \\ 108. \\ 108.. \end{cases}$$

3) 108 (.. 36
36

20) 1227 | 6
l. *s.* *d.*
Answ. 613 : 16 : 0

C.

C.	l.	s.	d.	C.
1 ...	2 :	10 :	6 ...	84

59 × 84 = 4200
42

		l.	s.	d.
20)	424 | 2	(212 :	2 :	0

C.	l.	s.	d.	C.
1 ...	1 :	12 :	$4\frac{1}{4}$ ...	306

		s.	d.
32 × 306 =		612	
		918.	
3)	306 (..	102	
48)	306 (..	6 :	$4\frac{1}{2}$
	20)	990 | 0 :	$4\frac{1}{2}$
		l. s.	d.
Answ.		495 : 0 :	$4\frac{1}{2}$

Note, when the given price of an Integer consists of certain pence together with $\frac{1}{2}$ *d.* or $\frac{1}{4}$ *d.* it will be convenient to take due *aliquot parts* of the number of Integers propounded for all the given price of an Integer except 1 *d.* and the said $\frac{1}{2}$ *d.* or $\frac{1}{4}$ *d.* then for that penny, and $\frac{1}{2}$ *d.* take $\frac{1}{8}$ of the said Integers propounded, and if there be yet a farthing, take $\frac{1}{8}$ of the said *quotient* which ariseth by taking $\frac{1}{8}$; both which quotients give the value in shillings correspondent to $1\frac{1}{4}$ *d.* this will be evident by the following *Examples*,

yard	*d.*	*yards*
1 ...	$8\frac{3}{4}$	326

	s.	*d.*
3) 326 (..	108 ..	8
4) 326 (..	81 ..	6
8) 326 (..	40 ..	9
6) 40 (..	6 ..	8
6) 9 (..	0 ..	$7\frac{1}{2}$
20)	23|7 ..	$8\frac{1}{2}$

	l.	*s.*	*d.*
Answ.	11 :	17 :	$8\frac{1}{2}$

---

	s.	*d.*	
1 ...	3 :	$6\frac{1}{2}$ ...	720

	s.
3 × 720 =	2160
4) 720 (..	180
6) 720 (..	120
8) 720 (..	90

	l.	*s.*	*d.*
20) 255|0 (	127 :	10 :	0

XVI. When the price of an Integer is given, and the price of many Integers of the same name together with $\frac{1}{4}$ or $\frac{1}{2}$ or $\frac{1}{4}$ of an Integer is required, the value of those Integers may be first found by some of the precedent rules, and then for the price of $\frac{1}{2}$ of an Integer, take $\frac{1}{2}$ of the given price of

of an Integer; likewise for $\frac{1}{4}$ of an Integer, take $\frac{1}{4}$ of the said given price, also for $\frac{3}{4}$ of an Integer take the composed of $\frac{1}{2}$ and $\frac{1}{4}$ of the said given price: So if it be demanded what 34 *C.* 3 *qu.* (to wit, 34 hundred weight, and $\frac{3}{4}$ of an hundred weight) of Sugar will cost at 4 *l.* 16 *s.* 3 *d. per C.* the *Answer* will be found 167 *l.* 4 *s.* $8\frac{1}{4}$ *d.* as by the subsequent operation is manifest.

C.	*l.*	*s.*	*d.*	C.	q.
1 ...	4 :	16 :	3 ...	34 :	3

		s.		*d.*
96 × 34 =		204		
		306.		
4) 34 (..		8	...	6
the quotients for	$\frac{1}{2}$ C.	48	...	$1\frac{1}{2}$
	$\frac{1}{4}$ C.	24	...	$0\frac{3}{4}$

20) 334 | 4 ... $8\frac{1}{4}$

	l.	*s.*	*d.*
Answ.	167 ..	4 ..	$8\frac{1}{4}$

An example of *Averdupois* greater weight, where the quantity whose price is sought consists of entire hundred weights, quarters of an hundred, and of some number of pounds, which is not an aliquot part of 28 or $\frac{1}{4}$ C.

C.

C.	*l.*	*s.*	*d.*		C.	*q.*	*lb.*
1	5	15	$7\frac{1}{4}$	....	218	3	24

	115 × 218 =	1090 218 218		
	2) 218 (..	109	*d.*	*far.*
	8) 218 (..	27	3	0
	$\frac{1}{6}$ of 27 *s.* 3 *d.* ..	4	6	2
the quotients arising for	$\frac{1}{2}$ C.	57	9	$3\frac{1}{2}$
	$\frac{1}{4}$ C.	28	10	$3\frac{1}{4}$
	14 *lb.*	14	5	$1\frac{1}{4}$
	7 *lb.*	7	2	$2\frac{1}{4}$ +
	3 *lb.*	3	1	0 +
	20) 2532	2	3	2 +
		l.	*s.*	*d.*
	Answ. 1266	2		$3\frac{1}{2}$ +

The example last mentioned being (of those questions which ordinarily happen in trade) one of the hardest to be resolved by the *Rule of Practice*, I shall touch upon the aforegoing operation, where you may observe the price of 218 C. 3 qu. to be found after the manner of former Examples; then for 14 *lb.* part of the 24 *lb.* in the question, I take $\frac{1}{2}$ of the price of $\frac{1}{4}$ C. Likewise for 7 *lb.* I take half the price of 14 *lb.* and so there yet remains 3 *lb.* whose price is found by taking $\frac{3}{7}$ of the price of 7 *lb.* *viz.* the price of 7 *lb.* being very near 7 *s.* $2\frac{1}{2}$ *d.* or $86\frac{1}{2}$ *d.* I multiply $86\frac{1}{2}$ by 3, and divide the quotient by 7, so there ariseth 37 *d.* or 3 *s.* 1 *d.* very near; lastly, all being added together, the sum is found to be

be very near 25322 *s.* $3\frac{1}{2}$ *d.* or 1266 *l.* 2 *s.* $3\frac{1}{2}$ *d.*

Note that a quarter of a farthing (or $\frac{1}{16}$ of a peny) is the ſmalleſt money expreſt in the example, and where any thing ariſeth leſs then a quarter of a farthing it is omitted, but it is ſuppoſed to follow this note †, for which ſurpluſages ſome reſpect ought to be had in adding all together: now albeit, in reſolving queſtions after this practical manner there will be ſome error, yet the loſs for the moſt part will be leſs then a farthing, which is inconſiderable.

XVII. When the price of 1 or an Integer conſiſts of divers denominations, as pounds, ſhillings, pence; and the price of a certain number of Integers which exceeds not a ſingle figure is required, work as in the following Example, *viz.* If it be required to find what 8 C. will coſt at 3 *l.* 13 *s.* $7\frac{1}{2}$ *d.* *per* C. it is evident that 8 C. muſt coſt 8 times 3 *l.*

	C.		*l.*		*s.*		*d.*		C.
	1	...	3	:	13	:	$7\frac{1}{2}$	..	8
							8		
Anſw.			29	:	9	:	0		

13 *s.* $7\frac{1}{2}$ *d.* therefore I multiply $\frac{1}{2}$ by 8, ſaying, 8 half pence make 4 pence, which I reſerve in mind again, 8 times 7 pence make 4 *s.* 8 *d.* (to wit, 8 ſix pences make 4 *s.* and there are 8 pence beſides) to which adding 4 pence in mind, there will ariſe 5 *s.* which I reſerve in mind, and ſubſcribe a cypher under the place of pence; again, I ſay 8 times 13 ſhillings make 5 *l.* 4 *s.* (to wit, 8 Angels make 4 *l.* and 8 times 3 *s.* make 1 *l.* 4 *s.*) to which adding 5 *s.*

in

in mind, the ſum will be 5 *l.* 9 *s.* wherefore I ſubſcribe 9 *s.* (the exceſs above the pounds) under the ſhillings, and keep 5 *l.* in mind; laſtly, I ſay 8 times 3 pounds make 24 pounds, which with 5 pounds in mind make 29 pounds; ſo that the total product or anſwer of the queſtion is found to be 29 *l.* 9 *s.*

More Examples of this kind are theſe.

C.	*l.*	*s.*	*d.*	C.
1 ..	17 :	15 :	$5\frac{1}{4}$ ...	7
			7	
Anſw.	124 :	8 :	$0\frac{1}{4}$	

C.	*l.*	*s.*	*d.*	C.
1 ...	18 :	12 :	$6\frac{1}{4}$ ...	8
			8	
Anſw.	149 :	00 :	6	

XVIII. When the price of 1 *lb.* weight is known, and the price or value of 1 C. (to wit 112 *lb.*) is required, the anſwer may ſometimes be given more ſpeedily then by any of the former rules, by this rule which follows, *viz.* Find the number of farthings contained in the given price of 1 *lb.* weight, then take twice that number of ſhillings, and once that number of groats, and having added them together the ſum will give the value of 1 C. to wit 112 *lb.* weight: So if it be demanded what 1 C. or 112 *lb.* weight of Cheeſe will coſt at the rate of $3\frac{3}{4}$ pence the pound weight, the anſwer will be 1 *l.* 10 *s.*

For

For according to the said rule, the number of farthings contained in $3\frac{1}{4}$ *d.* (the the price of 1 pound weight) is 13, therefore the double of 13 shillings is . . 13 Groats make . . Therefore the sum (which is the price of 1 C. or 112 *lb.* weight) is . . .

	l.	*s.*	*d.*
the double of 13 shillings is	1	6	0
13 Groats make	0	4	4
the sum	1	10	4

The reason of this rule is evident, for if 1 *lb.* weight cost 13 farthings, then 112 *lb.* must necessarily cost 112 times 13 farthings, or (which is the same) 13 times 112 farthings; but 13 times 112 farthings are equal to twice thirteen shillings together with once thirteen groats, because 112 farthings are composed of twice 48 farthings (or two shillings) and of 16 farthings (or one groat) wherefore the truth of the said Rule is evident.

Another Example, when Sugar is at $5\frac{1}{2}$ *d.* the pound weight, what is the value of 1 C. (or 112 *lb.* weight?) *Answ.* 2 *l.* 11 *s.* 4 *d.* For in $5\frac{1}{2}$ *d.* are contained 22 *farthings* therefore the double of 22 *shillings* is . . 22 Groats make . . Which added together give the price of 1 C. or 112 *lb.* to wit . .

	l.	*s.*	*d.*
the double of 22 shillings is	2	4	0
22 Groats make	0	7	4
the price of 1 C.	2	11	4

XIX. When the gain of (or allowance for) 100 integers consist of some number of pounds not exceeding 10, the gain of as many like integers and known parts of an integer as one will, may be found very briefly by the following method, *viz.* If 100 *l.* gain 3 *l.* what is the gain

Compendious wayes of computing interest and Factors allowances.

gain of 246 *l.* 18 *s.* 10 *d.*) *Anſwer* 7 *l.* 8 *s.* 1 $\frac{98}{100}$ *d.* Firſt I multiply 246 *l.* 18 *s.* 10 *d.* by 3 (the ſecond term) after the manner delivered in the 17 Rule of this Chapter, and write down the product which is 740 *l.* 16 *s.* 6 *d.* Then I divide the ſaid product by 100 (the firſt term in this *Rule of Three*) in this manner, *viz.* I divide 740 pounds by 100, which is performed by cutting off towards the right hand

l.	*l.*	*l.*	*s.*	*d.*
100 . .	3 . .	246 :	18 :	10
				3

l. 7|40 : 16 : 06
|20

s. 8|16
|12

d. 1|98

the two laſt places of 740, ſo the quotient gives 7 pounds, and there will be a remainder of 40 pounds, which 40 pounds I reduce into ſhillings, ſo there will ariſe 800 *s.* to which adding the 16 *s.* which ſtand in the place of ſhillings, the ſum will be 816 ſhillings, theſe are alſo to be divided by 100, (by cutting off two places as before) ſo the quotient will give 8 ſhillings, and there will remain 16 ſhillings, which being reduced to pence, and unto them 6 pence being added, (to wit the 6 pence which ſtands in the place of pence, there will ariſe 198 pence; theſe alſo are to be divided by 100, (by cutting off two places to the right hand as before) ſo

ſo the quotient gives 1 peny, and there will remain 98 pence; ſo the exact quotient or Anſwer of the queſtion is found to be 7 *l.* 8 *s.* 1$\frac{98}{100}$ *d.*

More Examples of this Rule are theſe following.

l. *l.* *l.* : *s.* : *d.*
100 ... 6 ... 793 : 12 : 7
6
l. 47 | 61 : 15 : 6
20
s. 12 | 35
12
d. 4 | 26

d. *l.* *l.* : *s.* : *d.*
100 ... 8 ... 43 : 14 : 3
8
l. 3 | 49 : 14 : 0
20
s. 9 | 94
12
d. 11 | 28

After the ſame manner may this following queſtion and ſuch like be reſolved, *viz.* When 100 Ells of Linen cloth coſt 30 *l.* 18 *s.* 9 *d.* what is the price of 1 Ell? *Anſwer* 6 *s.* 2 *d.* 1 *farth.*

Ells

Ells	l.	s.	d.	Ell
100 ..	30 :	18 :	9 ...	1
	20			

Shil.	6	18
		12
Pence	2	25
		4
Farth.	1	00

XX. When the given gain of (or allowance for) 100 integers consists of some number of pounds not exceeding 10, together with some Aliquot part or parts of a pound, the operation will be little different from the last mentioned Examples, as may appear by the resolution of the subsequent question, *viz.* What must be allowed for 2156 *l.* 13 *s.* 4 *d.* at the rate of 6 *l.* 15 *s.* for 100 ? *Answ.* 145 *l.* 11 *s.* 6 *d.* thus found ; first I multiply the said 2156 *l.* 13 *s.* 4 *d.* by 6 (the number of pounds in the given allowance 6 *l.* 15 *s.*) after the manner of the last Examples, and subscribe the product which is 12940 *l.* underneath the line as you see, then since 15 *s.* are equal to ½ *l.* together with ¼ *l.* I take ½ of 2156 *l.* 13 *s.* 4 *d.* which is 1078 *l.* 6 *s.* 8 *d.*; likewise ¼ of the said 2156 *l* 13 *s.* 4 *d.* to wit, 539 *l.* 3 *s.* 4 *d.* and having subscribed these quotients underneath the product first found, and added them all together, I find 14557 *l.* 10 *s.* 0 *d.* for the total product, with which I proceed as in the former examples, and so at length the *Answer* is found to be 145 *l.* 11 *s.* 6 *d.* View diligently the operation

100

l.	*ll.*	*l.*	*s.*	*d.*
100 .	6¼ . .	2156	: 13	: 4
				6¼
		12940	: 00	: 0
		1078	: 06	: 8
		539	: 03	: 4
	l.	145|57	: 10	: 0
		|20		
	s.	11|50		
		|12		
	d.	6|00		

CHAP. III.

Concerning Exchanges of Coins, Weights, and Measures.

1. THe rate or proportion between *Coins*, *Weights*, &c. of different kinds being known, either from some good Author, or rather by experience; it will not be difficult, to such as understand the *Rule of Three*, to know how to exchange a given quantity of one kind, for a quantity of the same value in another kind. But since in some cases, the common way of working may be much con-

tracted, I shall endeavour to shew the most compendious wayes to perform this business.

II. In exchanging of things of different kinds, (whether they be *Coins* or *Weights*, &c.) when two things of different kinds are compared together, the question may be resolved by one single *Rule of Three*, as will be evident by the subsequent examples, *viz.*

Quest. 1. How may *Riders* at 21 *s.* $2\frac{1}{2}$ *d.* *sterling* the piece, ought to be received for 251 *l.* 6 *s.* $4\frac{1}{2}$ *d.* of *sterling* money? *Answer*, 237 *Riders*. For the first and third terms in the *Rule of Three*, which ariseth from this question, being converted into half pence, the proportion will be this,

509 . 1 :: 120633 . 237

Quest. 2. If 100 *Ells* of *Antwerp* make 75 *yards* of *London*, how many yards of *London* measure will 27 *Ells* of *Antwerp* make? *Answer* $20\frac{1}{4}$ *yards*.

100 . 75 :: 27 . $20\frac{1}{4}$

III. When more than two different *Coins, Weights, Measures* &c. are compared together *viz.* when one kind of *Coin* is compared with a second of another kind; that second with a third; the third with a fourth; the fourth with a fifth, &c. two different cases are ordinarily raised from such comparison, *viz.*

It may be required to know,

1. *How many pieces of the first Coin are equal in value to a given number of pieces of the last coin:* or

2. *How many pieces of the last Coin are equal in value to a given number of pieces of the first kind of coin.*

An

An Example of the first case.

If 35 ells of *Vienna* make 24 ells at *Lyons*; 3 ells of *Lyons* 5 ells of *Antwerp*; and 100 ells of *Antwerp* 125 ells at *Frankfort*; how many ells of *Vienna* are equal unto 50 ells at *Frankfort*? *Answer*, 35 ells of *Vienna*.

For the more easie understanding of the resolution of this question and others of like nature. Let *a* represent an ell at *Vienna*; *b* an ell at *Lyons*; *c* an ell at *Antwerp*, and *d* an ell at *Frankfort*; then may the given terms in the question be stated in the following order;

Suppositions	35 *a*	=	24 *b*
	3 *b*	=	5 *c*
	100 *c*	=	125 *d*
The question	50 *d*	=	? *a*

Which order of placing the said given numbers (or terms) being observed, it appears that if 35 *a* be accounted to stand in the first place; 24 *b* in the second; 3 *b* in the third; 5 *c* in the fourth; 100 *c* in the fifth, &c. then all the terms which stand in odd places, to wit, in the first, third, fifth, and seventh places, will necessarily fall under the first row or column on the left hand, and all the terms which stand in even places, to wit, in the second, fourth, and sixth places, will fall under the latter column.

These things premised, all questions which fall under Case 1. before mentioned may be resolved by this Rule, *viz.*

Rule I.

Multiply all the given terms which stand in odd places (to wit, in the first column) according to the rule of continual multiplication, and reserve the last product for a dividend: Again, multiply continually all the terms which stand in even places, so shall the product be a divisor, and the quotient arising from the said Dividend and Divisor shall be the answer of the question.

So in the last mentioned question, if all the numbers in the first column, to wit 35, 3, 100, and 50 be multiplied continually, the product will be 525000 for a Dividend; also if all the numbers in the latter column, *viz.* 24, 5 and 125 be multiplied continually, the last product will be 15000 for a Divisor, and the quotient arising from the said Dividend and Divisor will be 35, which is the number of ells of *Vienna* required.

35	24
3	5
100	125
50	

525000 : 15000) 525000 (35

The reason of the said Rule I. will be manifest by solving the question propounded by three single Rules of three, thus,

I. 24 *b*. 35 *a* :: 3 *b*. $\frac{35 \times 3}{24}$ *a* (= 5 *c*.

II. 5 *c*. $\frac{35 \times 3}{24}$ *a* :: 100 *c*. $\frac{35 \times 3 \times 100}{5 \times 24}$ *a* (= 125 *d*.

III. $\frac{125}{1}$ *d*. $\frac{35 \times 3 \times 100}{5 \times 24}$ *a* :: $\frac{50}{1}$ *d*. $\frac{35 \times 3 \times 100 \times 50}{125 \times 5 \times 24}$ *a*.

which Fourth proportional last found, to wit, $\frac{35 \times 3 \times 100 \times 50}{125 \times 5 \times 24}$ being well viewed and compared with the before mentioned order of placing the terms given in the question gives the very Rule I. before exprest in words.

An Example of the latter of the two Cases before mentioned.

If 10 *lb*. of *Averdupois* weight at *London* be equal to 9 *lb*. of *Amsterdam*; 45 *lb*. at *Amsterdam*, 49 *lb*. at *Bruges*; and 98 *lb*. at *Bruges* equal to 116 *lb*. at *Dantzick*; how many *lb*. of *Dantzick* are equal to 112 *lb*. of *Averdupois* weight at *London*? *Answer*, 129.92 *lb*. of *Dantzick*.

That the operation may be the more clear, let *a* represent one pound of *Averdupois* weight; *b* one *lb*. of *Amsterdam*; *c* one *lb*. of *Bruges*, and *d* one *lb*. of *Dantzick*; then let the question be stated after the order in the first Case, *viz*.

$$\textit{Suppositions}\begin{cases} 10\ a = 9\ b \\ 45\ b = 49\ c \\ 98\ c = 116\ d \end{cases}$$
$$\textit{The question}\ 112\ a = ?\ d$$

These things premised, all questions which fall under Case 2. before mentioned may be solved by this Rule, *viz.*

Rule II.

Multiply all the given terms which stand in even places, (to wit in the latter column) and the last odd term in the first column according to the rule of continual multiplication, and reserve the last product for a Dividend; again, multiply continually the rest of the terms which stand in odd places (to wit in the first column) for a Divisor, so shall the quotient arising be the answer of the question.

Or in this latter case if you place the last of the given terms in the same column with the even terms, the rule for solving questions, which fall under the latter case will be this which followeth, *viz.*

Multiply continually all the numbers in the latter column for a Dividend; also multiply continually all the numbers in the first column for a Divisor, so shall the quotient arising be the answer of the question. Thus the answer of the last mentioned question will be found 129, 92, to wit, 129 $\frac{92}{100}$ *lb.* of *Dantzick*, as is evident by the subsequent operation.

10	9
45	49
98	116
	112

44100) 5729472 (129.92

The reason of the said Rule II. will be manifest by solving the question propounded, by three single Rules of three, thus,

$$\text{I. } 9\,b \,.\, 10\,a :: 45\,b \,.\, \frac{45 \times 10}{9}\,a \,.\, (=49\,c.$$

$$\text{II. } \frac{49}{1}c \,.\, \frac{45 \times 10}{9}a :: \frac{98}{1}c \,.\, \frac{45 \times 10 \times 98}{49 \times 9}a (=116\,d.$$

$$\text{III. } \frac{45 \times 10 \times 98}{49 \times 9}a \,.\, \frac{116}{1}d :: \frac{112}{1}a \,.\, \frac{49 \times 9 \times 116 \times 112}{45 \times 10 \times 98}d.$$

Which fourth proportional last found, to wit, $\frac{49 \times 9 \times 116 \times 112}{45 \times 10 \times 98}$ being well viewed and compared with the before mentioned order of placing the terms given in the question discovers the very Rule II. before exprest in words.

Note, when the same numbers happen to be Multiplicators in the Dividend, and also in the Divisor, such Multiplicators may be cancelled in both, and thereby much labour will oftentimes be spared.

Such which have much practice in calculating *Exchanges*, and do exactly know the rate or proportion between two different weights or measures or coins, which they would compare together, may by the *Rule of Three* frame Tables of proportions for the more speedy reducing of a given quantity of one kind of weight, measure, &c. into a quantity of the same value in another kind of weight, &c. In the expressing of which proportions it will be very convenient that the first number or Antecedent of each proportion be made 1, or unity, and the second term or consequent a Decimal, or else a mixt a number whose Fractional part is a Decimal, for then the Coin, Weight, &c. of the one place, (whose term is 1) may be reduced into that of the other place, by help of those Tables and of Multiplication of Decimals without sensible error: For Example, It hath been observed by some ingenious Merchants that 100 *lb.* of *Averdupois* weight at *London* are equal unto 89 *lb.* in *Paris* by the Kings beam, and consequently 1 *lb. Averdupois*, is equal to $\frac{89}{100}$ *lb.* or .89 *lb.* at *Paris*, (for if 100 give 89, then 1 will give .89) therefore any number of pounds *Averdupois* being multiplied by .89 (with respect unto Multiplication of Decimals, explained in the 26 Chapter of the preceding Book) will produce pounds of *Paris*: Again, if 89 *lb.* of *Paris* be equal to 100 *lb. Averdupois*, then 1 *lb.* of *Paris* will be near equal to 1.1235 *lb.* of *Averdupois*: therefore any number of pounds of *Paris* being multiplied by 1.1235 will produce pounds *Averdupois* very near.

Upon this ground I have collected the proportions in the following Tables, wherein I would not have any to confide further than they shall know

them

them to be agreeable to truth, for I have only derived them from those delivered by Mr. *Lewes Roberts Merchant* in his *Map of Commerce* printed at *London*; *Anno* 1638. and do herein only aim at the instruction of ingenious *Merchants* and *Factors* in the briefest wayes of calculating their exchanges, the rate or proportion being truly known; in which practice, *Decimal Arithmetick* (which hath no enemy but the Ignorant) will be very serviceable.

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A Table for the Reduction of Averdupois weight at London, to the weights of divers forreign Cities and remarkable places.

		lb.	
One pound of *Averdupois* weight at *London*, makes at	*Antwerp*,		.9615
	Amsterdam,		.9
	Abbeville,		.91
	Ancona,	1	.282
	Avignon,	1	.12
	Burdeaux		.91
	Burgoyne,		.91
	Bollonia,	1	.25
	Bridges,		.98
	Callabria,	1	.3698
	Callais,	1	.07
	Constantinople,		.8474 *Loder*;
	Deepe,		.91
	Dansick,	1	.16
	Ferrara,	1	.3333
	Florence,	1	.282
	Flanders in general	1	.06
	Geneva,		.9345

Genoa,

One pound of *Averdupois* weight at *London*, makes at

Genoa,	1	.[illegible]	*suttle.*
	1	.[illegible]	*gross.*
Hamburg,		.[illegible]	
Holland,		.9[illegible]	
Lixborn,		.8[illegible]	
Lyons,	1	.07	*common weight.*
		.98	*silk weight.*
		.9[illegible]	*customary weight.*
Leghorn,	1	.3333	
Millan,	1	.4285	
Mirandola,	1	.3333	
Norimberg,		.88	
Naples,	1	.4084	
Paris,		.89	
Prague,		.83	
Placentia,	1	.3888	
Rotchel,	1	.12	
Rome.	1	.27	
Rouan,		.875	*by vicont.*
		.9017	*common weight.*
Sivil,	1	.08	
Tholousa,	1	.12	
Turin,	1	.2195	
Venetia,	1	.5625	*suttle.*
		.9433	*gross.*
Vienna,		.813	

The

The use of the preceding Table will be manifest by the subsequent *example*, viz.

How much weight at *Dansick* do 320 *lb*. *Averdupois* make? *Answer*, 371.2 *lb*. Seek in the preceding Table for *Dansick*, and right against it you shall find 1 . 16 which shews that 1 *lb*. *Averdupois* is equal to 1 . 16 *lb*. at *Dansick*, therefore multiply 320 by 1 . 16, so will the product be 371 . 2 *lb*. of *Dansick*, as by the Operation is manifest.

Aver. *Dans*.   *Aver*. *Dans*.

1 : 1 . 16 :: 320 : 371 . 2

A Table for the Reduction of the weights of divers forreign Cities and remarkable places to Averdupois weight at London.

One pound weight in		make at London of Averdupois weight	lb.
Antwerp			1.04
Amsterdam			1.1111
Abbeville			1.0989
Ancona			.78
Avignon			.8928
Burdeaux			1.0989
Burgoyne			1.0989
Bollonia			.8
Bridges			1.0204
Callabria			.73
Callais			.9345
Deepe			1.0989
Dansick			.862
Ferrara			.75
Florence			.78
Flanders in general			.9433
Geneva			1.97
Genoa	suttle,		.71
	gross,		.7

The

One pound weight in		makes at London of Averdupois weight lb.
Hamburg		1.0865
Holland		1.0526
Lixborne		1.135
Lyons	common weight.	.9345
	silk weight.	1.0204
	custom. weight.	1.1111
Leghorn		.75
Millain		.7
Mirandola		.75
Norimberg		1.1363
Naples		.71
Paris		1.1235
Prague		1.2048
Placentia		.72
Rotchel		.8928
Rome		.7874
Rouan	by Vicont,	1.1428
	common weight.	1.1089
Sivill		.9259
Tholousa		.8928
Turin		.82
Venetia	suttle,	.64
	gross,	1.06
Vienna		1.23

The

The use of the last mentioned Table, will be manifest by this example, *viz.*

In 224 *lb.* weight at *Hamburg*, how many pounds *Averdupois*?

Answ. 243.376 *lb.*

Seek in the Table for *Hamburg*, and right against it you will find 1.0865, which sheweth that 1 *lb.* of *Hamburg* makes 1.0865 *lb.* *Averdupois*; therefore if 1.0865 be multiplied by 224 the product will be pounds *Averdupois*.

1 ... 1.0865 ... 224

```
   1.0865
      224
  -------
    43460
   21730
  21730
  -------
  243|3760
```

A Table for the Reduction of English Ells to the Measures of divers forreign Cities, and remarkable places.

One ell at *London*, makes at

Amsterdam		1.6940	Ells.
Antwerp		1.6666	Ells.
Bridges		1.64	Ells.
Arras		1.65	Ells.
Norimberg		1.74	Ells.
Colen		2.08	Ells.
Lisle		1.66	Ells.
Mastrich		1.57	Ells.
Frankford		2.0866	Ells.
Dansick		1.3833	Ells.
Vienna		1.45	Ells.
Paris		.95	Aulnes.
Rouan		1.03	Aulnes.
Lions		1.0166	Aulnes.
Callais		1.57	Aulnes.
Venice	linnen,	1.8	Braces.
Venice	silk :	1.96	Braces.
Lucques		2.	Braces.
Florence		2.04	Braces.
Millan		2.3	Braces.
Leghorn		2.	Braces.
Madera Isles		1.0328	Braces.

Sivill

One Ell at *London* makes at			
	Sivil	1.35	Vares
	Lisbone	1.	Vares
	Caſtilia	1.3875	Vares
	Andoluxia	1.3625	Vares
	Granado	1.3625	Vares
	Genoa	4.8083	Palms
	Saragoſa	.55	Canes
	Rome	.56	Canes
	Barſelona	.7125	Canes
	Valentia	1.2125	Canes

The uſe of the aforeſaid Table will be manifeſt by the ſubſequent example, *viz.*

In 325 ells of *London*, how many ells at *Antwerp*? *Anſw.* 541.645 ells: Seek in the Table for *Antwerp*, and right againſt it you ſhall find 1.6666 which being multiplied by 325 produceth 541.645 ells of *Antwerp*, as by the operation is manifeſt.

```
1 . . . . 1.6666 . . . 325
            325
        ----------
          83330
         33332
        49998
        ----------
        541|6450
```

A Table for the Reduction of the Measures of divers forreign Cities, and remarkable places to English *Ells.*

One Ell at	*Amsterdam*	makes at *London*	.59	Ells.
	Antwerp		.6	
	Bridges		.6097	
	Arras		.606	
	Norimberg		.5747	
	Colen		.4807	
	Lisle		.6024	
	Mastrich.		.6369	
	Frankford		.4792	
	Dansick		.7228	
	Vienna		.6896	
One Auln at	*Paris*		1.0526	
	Rouan		.9708	
	Lions		.9836	
	Callais		.6369	
One Brace at	*Venice* linnen		.5555	
	Venice silk		.5102	
	Lucques		.5	
	Florence		.4901	
	Millan		.4347	
	Leghorn		.5	
	Madera Isles		.9681	

Sivil

One Vare at	*Sivill*	makes at *London*	.7407	Ells.
	Lisbone		1.	
	Castilia		.7207	
	Andaluzia		.7339	
	Granada		.7339	
One Palm at *Genoa*			.2079	
One Cane at	*Saragossa*		1.8181	
	Roma		1.7857	
	Barsalona		1.4035	
	Valentia		.8247	

The use of the said Table will be manifest by the subsequent example, *viz.*

In 730 Aulnes at *Lions*, how many ells at *London* ?

Answ. 718.028. Seek in the Table for *Lions*, and right against it you shall find .9836 which being multiplied by 730 produceth 718.028 ells of *London*, as by the operation is manifest.

1 : : .9836 . . . 730
730

295080
68852

718 | 0281

Note, that one and the same kind of Weight or Measure, doth seldom or never alter from its peculiar quantity, in the Kingdom or Common wealth, where such weight or measure was first established; but one and the same kind of money doth often rise and fall in its value in forreign parts: for which cause I have spared the pains of calculating *Decimal Tables* for *Coins*, yet to give some light to such as read modern relations, and want experimental knowledge in this matter, I shall here insert a *Table*, in the same estate as I find it in the aforesaid *Map* of *Commerce*, and refer the Reader, for further satisfaction, to the *Tables* in *Riders Dictionary*, concerning *Coins*, *Weights* and *Measures*, both antient and modern,

Of

Of Exchanges of London, with divers forreign Cities.

London doth exchange with

	Pence			
Placentia sterl.	64	for	1	*Crown*
Lyons	64	for	1	*Crown*
Rome	66	for	1	*Ducat*
Genoa	65	for	1	*Crown*
Millan	$64\frac{1}{4}$	for	1	*Crown*
Venice	50	for	1	*Ducat*
Florence	$53\frac{1}{2}$	for	1	*Ducaton*
Naples	50	for	1	*Ducat*
Lecchie in *Callabria*	50	for	1	*Ducat*
Barri	51	for	1	*Ducat*
Palenma	$57\frac{1}{2}$	for	1	*Ducat*
Mesina	$56\frac{1}{2}$	for	1	*Ducat*
Antwerp & *Colen*	1 *l. sterl.*	for	$34\frac{1}{2}$	*shill. flem.*
Valentia	$57\frac{1}{2}$	for	1	*Ducat*
Saragosa	59	for	1	*Ducat*
Barcelona	64	for	1	*Ducat*
Lixborn	$53\frac{1}{2}$	for	1	*Ducat*
Bollania	$53\frac{1}{2}$	for	1	*Ducaton*
Bergamo	52	for	1	*Ducaton*
Frankfort	$59\frac{1}{2}$	for	1	*Florin*
Genoa	83	for	1	*Crown*

London exchangeth in the denomination of *pence sterling* with all other Countries, *Antwerp* and those neighbouring Countries of *Flanders* and *Holland* excepted, with which it exchangeth by the entire *pound* of 20 *shillings English* (or *sterling.*)

CHAP. IV.

Practical Questions about various things; viz. Tare, Tret, Loss, Gain, Barter, Factorship, and Measuring of Tapestry.

Of abatements and allowances in Traffick, viz. 1. of Tare.

IN the trade of Merchandize there are in use various allowances, and abatements, known by the names of *Tare, Tret, &c.* concerning which I shall give a few examples, whereby the practical *Arithmetician* will easily see, that there is more difficulty in the name than in the thing; for the rate, or proportion agreed upon, in any allowance or abatement, (be it called by what name soever) being once known, the Arithmetical work will quickly be dispatcht by the *Rule of Three,* or else by that and some of the former rules mixtly used, as will partly appear by the following questions.

Gross weight is composed of the neat weight of the commodity, and also of the Tare, to wit, the Chest, Bag, But, &c. which containeth the commodity.

Quest. 1. A Factor buyeth 4 Chests of Sugar marked A. B. C. D. The gross weight of each Chest in *Averdupois* greater weight is as followeth,

A.

	C.	q.	lb.
A.	11 ...	1 ...	19
B.	10 ...	3 ...	20
C.	11 ...	2 ...	13
D.	10 ...	1 ...	17
The total groſs weight	44 ...	1 ...	13

Now ſuppoſing the *Tare* or weight of each Cheſt, when it is empty, to be 37 *lb.* the queſtion is what neat weight of Sugar will remain, when the total *Tare* is ſubtracted? *Anſw.* 43 C. 0 q. 5 *lb.*

	C	q.	lb.	
from	44 ..	1 ..	13	*the total groſs weight.*
Subtr.	1 ..	1 ..	08	*the total Tare.*
Rem.	43 ..	0 ..	05	*the neat weight of Sugar.*

Queſt. 2. If from 990 C. 3 qu. 21 *lb. groſs weight, Tare* is to be ſubtracted after the rate of 14 *lb. per* C. (or 112 *lb.*) of *groſs weight*, how many C. neat will remain? *Anſw.* 867 C. 0 qu. 7⅛ *lb.*

I. The groſs weight being converted into pounds by the 6th. rule of the 7th. Chapter of the preceding Book, will give 110985 *lb.*

II. Then by the *Rule of Three.*

112 . 14 : : 110985 . 13873⅛
or 8 . 1 : : 110985 . 13873⅛

		lb.					
III.	*From*	110985	*the gross weight.*				
	Subtr.	$13873\frac{1}{8}$	*the total Tare.*				
				C.	qu.	lb.	
	Rest neat	$97111\frac{7}{8}$	=	867 ..	0 ..	$7\frac{7}{8}$	

Note, when the number of *lb.* to be abated *per* C. for *Tare*, is an *aliquot part* of 112, as in the last mentioned example, where $14 = \frac{1}{8}$ of 112, the operation may be thus ;

C.	C.	C.	q.	lb.	C	q.	lb.
1 .	$\frac{1}{8}$ ::	990 :	3 :	21 .	(123 :	3 :	$13\frac{1}{8}$

$$\frac{1}{8}\text{ of}\begin{cases} 990\text{ c.} = 123 : 3 : 00 \\ 3\text{ q.} = 00 : 0 : 10\frac{4}{8} \\ 21\text{ lb.} = 00 : 0 : 02\frac{5}{8} \end{cases}$$

Total Tare	123 :	3 :	$13\frac{1}{8}$
Rest neat	867 :	0 :	$07\frac{7}{8}$

Of Tret. *Quest.* 3. Suppose at some City, there is a custom in selling of certain merchandize by weight, to allow or cast in as an overplus to the buyer, 4 *lb.* weight for every 100 *lb.* weight that is bought, and in that proportion for a greater or lesser quantity. Now if a Merchant buy 1175 *lb.* weight of some commodity, and is to be allowed thereupon after the aforesaid rate, the question is, how many *lb.* weight ought he to receive in all? *Answ.* 1222 *lb.* weight.

100 . 104 :: 1175 . 1222

This

This kind of allowance is commonly called *Tret*.

Quest. 4. Suppose a Merchant hath 1222 *lb.* weight of a certain commodity, part whereof he bought at a certain rate *per lb.* and the rest was allowed to him or cast in as an overplus, after the rate of 4 *lb.* weight for every 100 *lb.* weight which he bought; the question is, to know how many pounds neat weight he bought? *Answ.* 1175 *lb.* weight.

104 . 100 :: 1222 . 1175

This question is the converse of the former, and sheweth how to make abatement for *Tret*.

Quest. 5. If from 55 C. 1 qu. of gross weight, *Tare* is to be subtracted after the rate of 16 *lb. per* C. and from the remainder *Tret* is to be abated after the rate of 4 *lb. per* 104 *lb.* the question is, what the neat weight is worth in money after the rate of 8 *l.* 8 *s.* for every C. (or 112 *lb* ?) *Answ.* 382½ *l.*

I. The gross weight in *lb.* is 6188 *l.*
II. 112 . 16 :: 6188 . 884
or 7 . 1 :: 6188 . 884.
III. 6188—884=5304
IV. 104 . 100 :: 5304 . 5100
V. 112 . $8\frac{2}{5}$ :: 5100 . 382½

Quest. 6. A Merchant hath bought Linnen cloth at 11 *s. per* ell, which proving worse then he expected, he is willing to sell it at such a price that he may lose precisely after the rate of $1\frac{2}{3}$ *l.* for every 20 *l.* that he laid out; the question is to know at what price he ought to sell the ell, that the proportion in the *Of loss and gain.*

 said

ſaid loſs may be obſerved? *Anſw.* 10 *s.* 1 *d.* *per* ell.

I. $20 - 1\frac{2}{3} = 18\frac{1}{3}$
II. $20 \,.\, 18\frac{1}{3} :: 11 \,.\, 10\frac{1}{12}$ *pence.*

Otherwiſe,

I. $20 \,.\, 1\frac{2}{3} :: 11 \,.\, \frac{11}{12}$
II. $11 - \frac{11}{12} = 10\frac{1}{12}$

Queſt. 7. If 100 *lb.* weight of any commodity coſt 30 *s.* at what price muſt 1 *lb.* weight of that commodity be ſold to gain after the rate of 10 *l.* for every 100 laid out? *Anſw.* $3\frac{24}{25}$ *d. per lb.* weight.

I. $100 \,.\, 110 :: 30 \,.\, 33$
II. $100 \,.\, 33 :: 1 \,.\, \frac{33}{100}$ *s.* (or $3\frac{24}{25}$ *d.*)

Queſt. 8. A Merchant ſelleth a parcel of Jewels which coſt him 250 *l.* ready money, for 559 *l.* payable at the end of 6 months; the queſtion is (his ſecurity being ſuppoſed to be good) what his gain was worth in ready money upon rebate of intereſt at the rate of 6 *l.* for 100 *l.* for an year? *Anſ.* 300 *l.*

$559 - 250 = 309$
$103 \,.\, 100 :: 309 \,.\, 300.$

Of Barter. *Queſt.* 9. How much Sugar at 8 *d. per lb.* weight may be bought for 20 C. of Tobacco at 3 *l. per* C.? *Anſw.* 1800 *lb.* weight of Sugar.

1 . 3 :: 20 . 60

$\frac{1}{30}$. 1 :: 60 . 1800

Quest. 10. A. hath 100 pieces of Silks, which are worth but 3 *l. per* piece in ready money, yet he barters them with B. at 4 *lb. per* piece, and at that rate takes their value of B. in Wools at 7 *l.* 10 *s. per* C. which are worth but 6 *l. per* C. in ready money, the question is to know what quantity of Wools payes for the Silks, and which of the two A. or B. is the gainer, and how much? *Answ.* $53\frac{1}{3}$ C. of Wools payes for the Silks, and A. gaineth 20 *l.* by the barter.

I. $7\frac{1}{2}$. 1 :: 400 . $53\frac{1}{3}$

II. { 1 . 6 :: $53\frac{1}{3}$. 320
{ or $7\frac{1}{2}$. 6 :: 400 . 320

So it is evident that the true worth of the Wool which B. delivered was 320 *l.* for which he received only of A. the worth of 300 *l.* in Silks, and therefore B. loseth 20 *l.* by the barter.

Quest. 11. A Merchant delivereth to his Factor 600 *l.* upon condition that if the Factor add to it 250 *l.* of his own money, and bestow his pains in managing the whole stock, he shall then have $\frac{2}{5}$ parts of the total gain. The question is to know what stock the Factors service was estimated at? *Answ.* 150 *l.*

Of Factorship. See brief rules for computing of Factors allowances in the 19, and 20. rules of the second chapter of this Appendix.

I. The Factors part of the gain being $\frac{2}{5}$, the Merchant must necessarily have the remainder, which is $\frac{3}{5}$.

II. $\frac{3}{5}$. $\frac{2}{5}$ :: 600 . 400

III. 400 — 250 = 150

Quest.

Quest. 12. A Merchant delivereth to his Factor 320 *l.* and permitteth him to add to it 64 *l.* of his own money, to be employed in traffick; and by agreement between them the Factors service is estimated equivalent to a certain stock, which is such, that if the total gain be divided proportionably according to those three stocks, the Factor is to receive $\frac{1}{5}$ of the total gain, in consideration of the said imaginary stock (being the value of his service;) the question is to know the full part of the gain belonging to each, and what stock the Factors service was valued at? *Answ.* the Merchant $\frac{2}{3}$ of the gain, and the Factor $\frac{1}{3}$, whose service was valued at 96 *l. stock*.

I. $320 + 64 = 384$

II. $\frac{4}{5} \cdot \frac{1}{5} :: 384 \cdot 96.$

III.
$$\begin{array}{r} 320 \\ 64 \\ 96 \\ \hline 480 \cdot 1 :: \end{array} \left\{ \begin{array}{l} 320 \cdot \frac{2}{3} \\ 160 \cdot \frac{1}{3} \end{array} \right.$$

Quest. 13. If a piece of Arras hangings, in the form of a long square, hath for its length $6\frac{1}{4}$ yards *English*, and breadth 4 yards; how many square ells, or sticks *Flemish* are contained in that piece, when the length of a *Flemish* ell is equal to $\frac{3}{4}$ yard *English*? *Answer*, $44\frac{4}{9}$ square ells or sticks *Flemish*.

Of Measuring of Tapestry.

Forasmuch as by supposition, a *Flemish* ell in length, hath such proportion to an *English* yard in length, as 3 to 4, and consequently the square of the one to the square of the other, as 9 to 16.

Therefore

Therefore in a direct proportion, as 9 is to 16; so is any given number of square yards *English*, to a number of square ells *Flemish*, which will take up equal space with the said square ells *English*. Also in a direct proportion, as 16 is to 9, so is any given number of square ells *Flemish*, to a number of square yards *English*, which will take up an equal space with the said *Flemish* ells: therefore to resolve the aforesaid question, first find the number of square yards *English* contained in the said piece of Arras, by multiplying the length and breadth in yards mutually one by the other, then proceed according to the aforesaid proportion; so the work will stand thus,

I. $6\frac{1}{4} \times 4 = 25$ square yards *English*.
II. $9 \;.\; 16 :: 25 \;.\; 44\frac{4}{9}$ square ells *Flemish*.

Otherwise,

$6\frac{1}{4}$ yards *English* in length, give by the *Rule of Three* in *Flemish* ells — $8\frac{1}{3}$ *length.*

Also 4 yards *English* give in *Flemish* ells — $5\frac{1}{3}$ *breadth.*

Therefore the product of the said $8\frac{1}{3}$ multiplyed by $5\frac{1}{3}$, gives for the superficial content as before — $44\frac{4}{9}$

Quest. 14. If a piece of Tapestry in the form of a long square be in length $15\frac{1}{4}$ ells *Flemish*, and in breadth $4\frac{1}{3}$ ells *Flemish*, how many square yards *English* are contained in that piece, when 4 ells *Flemish* in length are equal to 3 yards *English*? *Answ.* $37\frac{11}{64}$ square yards *English*.

I. $15\frac{1}{4} \times 4\frac{1}{3} = 66\frac{1}{12}$.
II. $16 \;.\; 9 :: 66\frac{1}{12} \;.\; 37\frac{11}{64}$.

CHAP.

CHAP. V.

Concerning the Intereſt of Money, and the Conſtruction of Tables to that purpoſe.

I. IN reſolving queſtions concerning intereſt of money, four things are to be well obſerved, to wit, firſt, the Principal, or money lent for gain or intereſt; ſecondly, the time for which the ſaid Principal is lent; thirdly, the rate or proportion which the Principal bears to the ſum of the principal and intereſt; and fourthly the intereſt it ſelf: So if 100 *l.* be lent upon condition that 106 *l.* ſhall be repaid at the end of a year, the ſaid 100 *l.* is called Principal; the time for which the ſaid principal is lent is one year; the proportion which the principal bears to the ſum of the principal and intereſt is ſuch as 100 hath to 106; laſtly, the intereſt it ſelf is 6 *l.*

II. Intereſt is either Simple or Compound.

III. Simple Intereſt, is that which ariſeth or is computed from the principal only: So if 100 *l.* be lent for two years, the ſimple intereſt thereof after the rate of 6 pounds for 100 pounds for 1 year will be 12 pounds, *viz.* 6 pound due at the firſt years end, and 6 pounds due at the ſecond years end.

IV. Compound Intereſt is that which ariſeth from the principal, and alſo from the intereſt thereof, and therefore it is called intereſt upon intereſt: So if 100 pounds be lent and forborn 3 years, and compound intereſt thereof is to be computed

puted aft[er the] rate of 6 pounds for 100 *l.* for one year; ther[e] will ariſe beſides the ſimple intereſt of the principal for 3 years, the intereſt of 6 pounds (due at the firſt years end) for 2 years, and the intereſt of 6 pound (due at the ſecond years end) for one year following.

V. Rebate or diſcompt of money is, when a ſum of money due at any time to come, is ſatisfied by the payment of ſo much preſent money, which if it were put forth at a certain rate of intereſt for the ſaid time, would become equal to the ſum firſt due: So if 100 pound be due at the end of two years, and is to be ſatisfied by the payment of preſent money upon rebate, after the rate of 6 pounds *per centum, per annum, ſimple intereſt*, there ought to be ſo much ready money paid, which in two years after the ſaid rate of intereſt would be augmented unto 100 *l.* In like manner if the rebate or diſcompt were to be made after any rate of compound intereſt, ſo much ready money ought to be paid, which at ſuch rate of compound intereſt, for the time agreed on, would become equal to the ſum firſt due. *Examples* of the manner of computation by rebate may be ſeen in the tenth and fourteenth Rules of this Chapter.

The foundation upon which the Rules for computing ſimple intereſt are grounded.

VI. In the taking of intereſt, or uſe-money, for the loan or forbearance of money lent, reſpect muſt be had to the rate limited by Act of Parliament, which now reſtraineth all perſons from taking more than 6 *l.* for the intereſt or uſe of 100 *l.* lent for a year, but what part of 6 *l.* may be taken for the intereſt of 100 *l.* lent for half a year, a quarter of

of a year, a moneth, or any other part of a year, is not exprest in the Act; In this case therefore we must observe custom and daily practice, so we shall find that 3 *l.* is usually taken for half a years interest of 100 *l.* and 30 *s.* for a quarter of a year; &c. by which practice, this following Analogy, (which is the ground or reason of the common rules for computing simple interest) seems to be assumed for a safe exposition of the Statute, *viz.* That such proportion as the whole year, (supposed to consist of 365 dayes) hath to any propounded space of time more or less than a year, such proportion any interest, (not exceeding the rate limited by the Act) for any Principal lent for a year, ought to have to the interest of the same Principal for the time propounded: This Analogy being granted, the manner of computing simple interest, for any Principal lent and forborn any time propounded, will be such as is exprest in the two next Sections.

VII. The interest or gain of 100 *l* principal money forborn for a year being known, the interest of any other principal money for the same time, may be found out by one single *Rule* of *Three*; for as 100 *l* principal, is in proportion to the interest thereof, so is any other principal to its interest: So if it be demanded what 270 *l.* will gain in a year at the rate of 6 *l.* for 100 *l.* for one year, the *Answer* will be found to be 16 *l.* 4 *s.* For,

l.		*l.*		*l.*		*l.*		*l.*	*s.*	*d.*
100	.	6	::	270	.	16,2	(or	16 :	4 :	0

A second Example, What is the interest of 175 *l.* 18 *s.* 11 *d.* for a year, at the rate of 6 *l.* for 100 *l.*

for

for a year? *Anſw.* 10 *l.* 11 *s.* 1$\frac{62}{100}$ *d.* as by the following operation, (which is performed after the practical manner delivered in the nineteenth Rule of the ſecond Chapter of this *Appendix*) is evident:

l.	*l.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*
100 .	6 : :	175 :	18 :	11	(10 :	11 :	1$\frac{62}{100}$

multiply by . . . 6

l. . . . . . . 10|55 : 13 : 6
|20

s. . . . . . . 11|13
|12

d. . . . . 1|62

VIII. If the intereſt of 100 *l.* principal for one whole year, or 365 dayes be known, the ſimple intereſt of any other principal, for any number of dayes more or leſs than 365, may be found out by the following Rule, *viz.*

A Rule for computing ſimple intereſt for any number of dayes.

Multiply theſe three numbers according to the Rule of continual Multiplication, to wit, the given intereſt of 100 *l.* for a year, the principal, whoſe intereſt is required, and the number of dayes preſcribed, reſerving the laſt product for a Dividend: Alſo multiply 365 by 100 and reſerve this product for a Diviſor; Laſtly finiſh Diviſion, ſo ſhall the quotient be the intereſt or gain ſought.

Note here, that the two principals, to wit 100 *l.* and the other propounded, are ſuppoſed to be of one and the ſame denomination; Alſo the intereſt

required will be of the same denomination with the given interest of 100 *l.*

For an example of this Rule, let it be required to find out the interest of 400 *l.* for a week, or 7 dayes, at the rate of 6 *l.* for 100 *l.* for a year, or 365 dayes; First multiplying these three numbers 6, 400, and 7 continually, (*viz.* multiplying 6 by 400, and the product thence arising by 7) the last product will be 16800 for a Dividend, also multiplying 365 by 100, the product is 36500 for a Divisor; lastly, dividing 16800 by 36500 (after cyphers at pleasure are added to 16800) the quotient, (according to the fourth Rule of the 27th. Chapter of the preceding Book) will be discovered to be this decimal .4602, which is equal to 9 *s.* 2 *d.* 1 *farth.* (as will appear by the brief way of valuing a decimal fraction in the fourth rule of the 26th. Chapter.)

The reason of the above mentioned rule for the computing of interest for dayes, will be manifest by this following way of solving the same question by two single *Rules of Three*, viz.

$$\text{I. } 100 \,.\, 6 :: 400 \,.\, \frac{6 \times 400}{100}$$

$$\text{II. } \frac{365}{1} \,.\, \frac{6 \times 400}{100} :: \frac{7}{1} \,.\, \frac{6 \times 400 \times 7}{365 \times 100}$$

Which fourth proportional in the latter *Rule of Three*, to wit, $\frac{6 \times 400 \times 7}{365 \times 100}$, being well viewed, the truth of the rule before delivered will be manifest.

Hence one vulgar error in computing interest

is discovered, for some argue thus, 6 *l.* is the interest of 100 *l.* for a year, therefore 10 *s.* (or $\frac{1}{12}$ of 6 *l.*) is the interest for a moneth, and consequently 2 *s.* 6 *d.* for a week or seven dayes, and so the interest of 400 *l.* for 7 dayes, computed after that manner would be 10 *s.* which exceeds the Answer found by the preceding Rule by $9\frac{1}{2}$ *d.* very near, which fallacy hath its rise from the taking, (or rather mistaking) of 28 dayes for $\frac{1}{12}$ part of the number of dayes in a year, when indeed the just $\frac{1}{12}$ part of 365 dayes consists of $30\frac{5}{12}$ dayes.

Moreover, by the help of this decimal fraction of a pound, to wit, .000164383, which is very near the interest of one pound for a day at the rate of 6 *per cent. per annum* (as will appear by the preceding rule) the interest of any principal, (supposed to be pounds or decimal parts of a pound) for any number of dayes propounded, at the said rate of interest, may be found out by multiplication onely, *viz.* First multiply the said decimal .000164383 by the principal whose interest is required, then multiply that product by the number of dayes propounded, so shall this last product be the interest required; (but in these multiplications respect must be had to the cutting off of places in the products, according to the second and third rules of the 26th. Chapter of the preceding Book,) for example, if it be required to find the interest of 1000 *l.* for 131 dayes, at the rate of 6 *per cent. per ann.* the *Answ.* will be found 21.534+, or 21 *l.* 10 *s.* 8 *d.*+ for according to the rule last given.

Another Rule for computing simple Interest for dayes.

$.000164383 \times 1000 \times 131 = 21.534 +$

But at another rate of interest, a peculiar decimal instead of the said .000164383, (which serves onely for 6 *per cent. per annum*) must be found out by the first rule aforegoing before the latter rule can take place, the reason of which latter rule doth also evidently arise from two single rules of three.

The manner of summing up Annuities in arrear with allowances of simple interest.

IX. When an Annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount unto, simple interest being computed for each particular yearly payment, from the time it became due, until the end of the term of years, the work will be as in this following example, *viz.* If an Annuity, or yearly rent of 134 *l.* 10 *s.* 6 *d.* be all forborn till the end of 4 years, what will it then amount unto, simple interest being allowed at the rate of 6 *per cent. per annum* for each years rent, from the time on which it was due, until the end of the said term of four years? *Answ.* 586 *l.* 10 *s.* $6\frac{96}{100}$ *d.*

It is evident by the question, that at the rate of interest propounded, there must be computed the interest of 134 *l.* 10 *s.* 6 *d.* (due at the third years end) for one year, (to wit, the fourth year) also the interest of the like sum due at the second years end, for two years, (to wit, the third and fourth years) likewise the interest of the same sum due at the first years end, for three years, (to wit, the second, third and fourth years) all which interest being added to the sum of the four years rent, the total sum will shew what the said Annuity will a-

mount unto at the end of the said term of 4 years.

Explication.

	years	l. s. d.
The interest of 134 l. 10 s. 6 d. at 6 per cent. per annum, for	1 is	8 : 1 : 5.16
	2 is	16 : 3 : 10.32
	3 is	24 : 4 : 3.48
The sum of the 4 years rent, (to wit, 4 times 134 l. 10 s. 6 d.)		is 538 : 2 : 0
All which added together give the *Answer* of the question, to wit,		586 : 10 : 6.96

X. When it is required to find out how much ready money will satisfie a Debt due at the end of any space of time to come, by rebating or discompting at a given rate of simple interest, it may be effected by this rule, *viz.* First find out the interest of 100 l. at the given rate of interest, for the time which the ready money is to be paid beforehand, then adding the interest so found to 100 l. make alwayes the sum of that addition the first term in a rule of three; 100 l. the second term; and the debt propounded to be satisfied the third term; lastly, the fourth proportional found out by the said *Rule of Three* shall be the ready money which ought to be paid in satisfaction of the debt propounded.

Of rebate or discompt of money at simple interest.

Example 1. If a debt of 100 l. be payable at the end of a year to come, how much ready money will discharge that debt by rebating or discompting at the rate of 6 *per cent. per annum*? *Answ.* 94 l.

6 s. 9 d. 2/5 very near; for by this Rule of Three

$$106 \;.\; 100 \;::\; 100 \;.\; 94.3396 +$$

That is to say, if 106 *l.* (which is compos'd of 100 *l.* principal and 6 *l.* interest) proceeds from 100 *l.* principal forborn for a year, from what principal forborn for a year doth 100 *l.* (compos'd of principal and interest) proceed from? *Answ.* 94.3396 *l.* + (or 94 *l.* 6 *s.* 9½ *d.* very near) principal money: therefore 94 *l.* 6 *s.* 9½ *d* in ready money, is of equal value with 100 *l.* due at the end of a year to come; for if the said 94 *l.* 6 *s.* 9½ *d.* be put forth at interest for a year, at the rate of 6 *per cent. per annum*, it will gain 5 *l.* 13 *s.* 2½ *d.* very near, which together with the said 94 *l.* 6 *s.* 9½ *d.* makes the 100 *l.* the debt first propounded to be discharged by rebate.

Example 2. If 150 *l.* 10 *s.* be payable at the end of 73 dayes to come; how much present money will discharge the said debt, by rebating after the rate of 6 *per cent. per annum*? *Answ.* 148 *l.* 14 *s.* 3½ *d.* + as by the following operation is manifest.

	dayes	l.	dayes	l.
I.	365 .	6 ::	73 .	1.2
	l.	l.	l.	l.
II.	101.2 .	100 ::	150.5 .	148.7154 +

That is to say, First I seek by a single *Rule of Three*, the interest of 100 *l.* for 73 dayes, at the rate of interest propounded, saying, if 365 dayes (or a year) gain 6 *l.* what will 73 dayes gain? *Answ.* 1 2/10 *l.* or 1.2 *l.* Then adding the said 1.2 to 100; I say, by

by a second *Rule of Three*, if 101.2 *l.* principal and interest, (payable at the end of 73 dayes to come, be equivalent to 100 *l.* ready money, what ready money is 150 *l.* 10 *s.* (or 150.5) payable at the end of 73 dayes to come equivalent unto? so by multiplying and dividing according to the rules of Decimal multiplication and Division, (explained in Chapter 26, and 27. of the preceding Book) the quotient or answer of the question will be found 148.7154 +, that is, 148 *l.* 14 *s.* 3½ *d.* + (for the decimal .7154 being valued according to the brief way at the end of the fourth rule of the 26th. Chapter, will by inspection onely be discovered to be 14 *s.* 3½ *d.* which rule I shall here once for all, advise the Learner to be well acquainted with.)

The proof.

Seek (by the *Rule of Three*) what the ready money found as aforesaid will gain, in so much time as it is paid before hand, at the rate of interest propounded; then having added this gain to the said ready money, if the sum be equal to the debt first propounded to be satisfied by rebate, the ready money was rightly found out. So the last example will be thus proved.

l. *l.*
100 . 1.2 :: 148.7154 . (1.7845

Which fourth proportional 1.7845 being added to 148.7154, the sum will be 150.4999 + which doth not want a farthing of 150 *l.* 10 *s.* the debt first propounded.

Of the present worth of Annuities by discompting at simple interest.

XI. When it is required to find the present worth of an Annuity, by rebating or discompting at a given rate of simple interest, the operation will be as in the following example, *viz.* How much present money is equivalent to an Annuity or rent of 100 *l.* *per annum* to continue five years, rebate being made at the rate of 6 *l.* for 100 *l.* for one year, at simple interest? *Answ.* 425 *l.* 18 *s.* 9½ *d.* very near.

It is manifest that there must be computed the present worth of 100 *l.* due at the first years end, also the present worth of 100 *l.* due at the second years end, and in like manner for the third, fourth and fifth years; all which particular present worths being added together, the aggregate or sum will be the total present worth of the Annuity, to wit, in the example above propounded, $425 \frac{8286180}{8821267}$ *l.* that is, 425 *l.* 18 *s.* 9½ *d.* very near.

The operation by decimals (which will come near enough to the truth) will be as followeth, *viz.*

	l.	*l.*	*l.*	*l.*
1.	106 .	100 ::	100 .	94,33962 +
2.	112 .	100 ::	100 .	89,28571 +
3.	118 .	100 ::	100 .	84,74576 +
4.	124 .	100 ::	100 .	80,64516 +
5.	130 .	100 ::	100 .	76,92307 +

Answ. 425,93933 +

Here

Here by the way, from the manner of resolving the last mentioned question, that *Rule* commonly called *Equation of payments*, which is insisted on by divers *Arithmetical Writers*, will be found erroneous, which I thus prove.

1. Since that rule aims at the reducing of several dayes of payment, upon which particular sums of money are due, unto a mean time upon which the aggregate or total of those particular sums ought to be paid, without dammage to the *Debitor* or *Creditor*, there must be necessarily some rate of interest implied; for otherwise why may not any day at pleasure be assigned for one intire payment.

2. If some rate of interest be implied, then equity requires that the present worth of the total sum payable at one intire payment, rebate or discompt being made according to that rate of interest, may be equal to the sum of the present worths of the particular sums of money, rebate being made at the same rate of interest.

3. In regard the said Rule doth mention no particular rate of Interest, it ought to be true at any rate of interest whatsoever.

4. Let us therefore examine the said Rule according to the rate of 6 *per centum, per annum*, simple interest, by taking the last mentioned question for an example, which (according to the accustomed manner) will be thus stated, *viz.* If 500 *l.* ought to be paid by five equal yearly payments, to wit, 100 *l.* at each years end, what time ought to be given for the payment of the said 500 *l.* at one entire payment, without loss either to the *Debitor* or *Creditor*.

5. By proceeding according to the said rule of *Equation* of *payments* (which saith, If the sum of the products,

products, arising from the multiplication of each particular sum of money by its respective time, be divided by the sum or aggregate of the said particular sums of money, the quotient will be the mean time to be assigned for one intire payment) there will be found three years, which time (according to the said rule) ought to be given for the payment of the whole 500 *l.*

6. Now if 500 *l.* due at the end of three years to come be worth as much in present money, as is the present worth of an *Annuity* of 100 *l.* to continue five years, then the said *Rule* of *Equation* is true; otherwise false; but the present worth of 500 *l.* due at the end of three years to come, rebate being made at the rate of 6 *per centum, per annum, simple interest* will be found (by the tenth rule of this Chapter) to be 423 *l.* 14 *s.* 6 *d.* 3 *f.* very near; also the present worth of the said *Annuity*, rebate being made as before, is found (as appeareth by the resolution of the last mentioned question) to be 425 *l.* 18 *s.* 9¼ *d.* very near; wherefore it is evident that the *Creditor* loseth 2 *l.* 4 *s.* 2¾ *d.* very near, by receiving the whole 500 *l.* at three years end: moreover at 6 *per centum, per annum, compound interest*, he would lose 1 *l.* 8 *s.* 6 *d.* very near, as will be manifest by the *Tables of compound interest* hereafter expressed: so that the loss will be either more or less according as the rate of interest doth differ: and therefore I conclude the said *Rule*, (as also all other rules or resolutions of questions which have dependance thereon) to be erroneous.

Although questions of this nature seldom come into practice, yet he that will take the pains, may find out such a mean time as is required by the said

Rule

Rule of Equation of *payments*, at any rate of simple interest by this following rule, *viz.*

First, by the preceding tenth Rule of this Chapter find out the present worth of every particular sum in the question payable at a time to come, by rebating at the rate of interest agreed on; then find in what time, the sum of those present worths will be augmented unto the total of all the particular sums payable at times to come, according to the first agreement, so shall the time found out be the mean time for the payment of the whole debt: thus the mean or equated time in the last example will be found to be 2.8979, &c. years, (not three years, as the said *Rule* of *Equation* of *payments* would have it) for by rebating at 6 *per cent. per annum*, simple interest, 500 *l.* payable at the end of 2.8979, &c. years to come, (that is 2 years and 328 dayes very near) is worth in ready money 425 *l.* 18 *s.* 9½ *d.* very near, and the same ready money is also the present value of 100 *l.* Annuity for 5 years, at the same rate of interest, as before hath been manifested. But to return to the path from which I have made a digression.

From the preceding tenth rule of this Chapter the following *Tables* I. and II. are deduced, whose construction and use are afterwards declared.

Years

Years	*Table* I. Which sheweth in decimal parts of a pound, the present worth of one pound due at the end of any number of years to come, not exceeding 7 years, at the rate of 6 *per centum, per annum,* simple interest.	Years	*Table* II. Which sheweth in pounds and decimal parts of a pound, the present worth of one pound Annuity, to continue any number of years not exceeding 7, at the rate of 6 *per centum, per annum,* simple interest.
1	.943396	1	943396
2	.892857	2	1.836253
3	.847457	3	2.683710
4	.806451	4	3.490167
5	.769230	5	4.259393
6	.735294	6	4.994687
7	.704225	7	5.698912

The Construction of Table I.

The numbers in the first Table which are placed right against the numbers of years 1,2,3,4,5,6,and 7,are decimal fractions,one pound of English money being the Integer, and are thus found.(according to the preceding tenth Rule of this Chapter:) *viz.*

106 . 100 :: 1 . ,943396 +
112 . 100 :: 1 . ,892857 +
118 . 100 :: 1 . ,847457 +

whereby

whereby it appears, that 1 *l.* due at the end of a year to come, is worth in ready money .943396 + that is, 18 *s.* 10 *d.* 1 *f.* and somewhat more. Also 1 *l.* due at the end of two years to come, is worth in ready money .892857 +, or 17 *s.* 10$\frac{1}{4}$ *d.* rebate being made at the rate of 6 *per centum, per annum,* simple interest; the like is to be understood of the rest of the numbers in Table I. which may be continued to more years, and other Tables also of rebate may be framed upon the same ground, for moneths, or dayes, by the ingenious Artist.

The use of Table I.

The practical use of the said first Table will be manifest by solving this following question; *viz.* How much ready money will discharge 345 *l.* 15 *s.* 6 *d.* due at the end of five years to come, by rebating simple interest at the rate of 6 *per centum, per annum,* *Answer,* 265 *l.* 19 *s.* 7$\frac{1}{4}$ *d.* which is thus found out; *viz.* In the preceding Table I. right against 5 years, I find the decimal .76923 which shews that 1 *l.* due at the end of five years to come is worth in ready money .76923, (that is, 15 *s.* 4$\frac{1}{2}$ *d.*) then instead of 15 *s.* 6 *d.* mentioned in the question propounded, taking the decimal .775 which is equal to 15 *s.* 6 *d.* (the same being reduced according to the fifth rule of the 23 chapter of the preceding book) I say, by the *Rule of Three.*

1 : .76923 :: 345.775 . (265.9805 +

That is to say if 1 *l.* give .76923 *l.* what will 345.775 *l.* give? *Answ.* 265.9805 *l.* for multiplying 345.775 by .76923, according to the second Rule of the 26 Chapter of the preceding Book. the product will be 265.9805, that is, 265 *l.* 19 *s.* 7$\frac{1}{4}$ *d.*

The

The Construction of Table II.

The numbers in the second *Table* are found out by the addition of those in the first, *viz.* the first number in the latter *Table* is the same with the first number in the former, the second in the latter is the sum of the first and second in the former; the third in the latter is the sum of the first, second and third in the former, and in that manner the rest are found; (the reason of which composition is manifest from the example of the eleventh rule aforegoing;) otherwise, the numbers in *Table* II. may be found more easily thus, *viz.* the first number in the said *Table* II. is the same with the first number in *Table* I. the second number in the latter Table is compos'd of the second number in the former and the first in the latter, the third number in the latter Table is compos'd of the third number in the former and the second in the latter, the fourth in the latter is compos'd of the fourth in the former and the third in the latter; the like is to be understood of the rest of the numbers in *Table* II. which might be continued to more years, and fitted to other rates of interest, but I shall spare that labour, in regard a more equal way of finding out the present worth of an Annuity, agreeable to the accustomed and practical rates of buying and selling Annuities or Rents, for terms of years, is grounded upon a computation of interest upon interest, as will hereafter be made manifest, for at simple interest an Annuity will be overvalued.

The use of Table II.

The use of *Table* II. will appear by this following

ing example ; *viz.* What is the present worth of an Annuity of 100 *l. per annum* payable yearly during the term of five years, discompt or rebate being made at the rate of 6 *per centum, per annum*, simple interest ? *Answer*, 425 *l.* 18 *s.* 9½ *d.* very near, which is thus found out, *viz.* In the preceding *Table* II. right against five years, I find this number 4.259393, which shews that an Annuity of 1 *l.* payable yearly during five years, is worth in ready money 4.259393 *l.* (that is 4 *l.* 5 *s.* 2 *d.* and somewhat more) therefore, I say, by the *Rule of Three* ;

l.	*l.*		*l.*	*l.*
1 .	4.259393	::	100 .	(425.9393

That is to say, if 1 *l.* give 4.259393 *l.* what will 100 *l.* give ? *Answer*, 425 *l.* 18 *s.* 9½ *d.* very near, for by multiplying 4.259393 by 100, the product (according to the second rule of the 26 Chapter of the preceding Book) is 425.9393, that is, 425 *l.* 18 *s.* 9½ *d.* very near. Which operation being compared with the manner of solving the same question before mentioned in the eleventh Rule of this Chapter, the great benefit of Tables of this kind in point of expedition will be apparent.

Of the forbearance of money at compound interest.

XII. When it is required to know, unto what sum of money any propounded principal forborn any number of years will at the end of such term be augmented unto, interest upon interest being computed at a given rate, there must be found a rank of continual proportionals, more in number by one then is the number of years in the question ; of which proportionals the first is the principal assigned, the second must increase

or proceed from the first, the third from the second, &c. in such manner or rate, as 106 proceeds from 100 (or as 108 from 100, if the rate of interest be 8 *per centum*) then will the last proportional be the Answer of the question: So if 300 pounds principal money be put forth at interest upon interest, at the rate of 6 *l.* for 100 *l.* for one year, and all forborn untill the end of 4 years, there will then be due 378.743088, or 378 *l.* 14 *s.* $10\frac{1}{2}$ *d.* very near, as by the four following *Rules of Three* is manifest.

$$100 \;.\; 106 :: \begin{cases} 300 & . \; 318 \\ 318 & . \; 337.08 \\ 337.08 & . \; 357.3048 \\ 357.3048 & . \; 378.743088 \end{cases}$$

For the said 300 *l.* will at the first years end be augmented unto 318 *l.* which 318 *l.* being put forth as a *principal* for 1 year, (will at the second years end) be augmented unto 337.08, again this 337.08 being put forth as a *principal* for 1 year, will (at the third years end) be augmented unto 357.3048, in like manner 357.3048 being put forth as a *principal* for 1 year, will (at the fourth years end) be augmented unto 378.743088 which is the number required by the question. And if the work be well examined, it will appear, (as was before declared) that the *principal* first assigned, to wit 300 *l.* and the numbers resulting successively at the ends of the several years are *continual proportionals*; *viz.* these five numbers are so qualified, that if the second be multiplied

300 | 318 | 337.08 | 357.3048 | 378.743088

tiplied

tiplied by it self, the product will be equal to the product of the first and third; also if the third be multiplied by it self, the product will be equal to the product of the second and fourth; In like manner, if there were more *continual proportionals* in a rank, if any one proportional which is placed between two next on each side of such one, be multiplied by it self; the product will be equal to the product of those two extreams (which is a property peculiar to continual proportionals.)

Two numbers being given to find a third, a fourth, a fifth, &c. in continual proportion.

Note here by the way, that if any two numbers be propounded, suppose 300 and 318, and it be required to find to them a third, a fourth, a fifth, &c. in continual proportion, multiply the second proportional 318 by it self, and divide the product 101124 by the first proportional 300, so shall the quotient 337.08 be a third in continual proportion; In like manner if you multiply the third proportional 337.08 by it self, and divide the product 113622.9264 by the second proportional 318 the quotient 357.3048 shall be a fourth in continual proportion, and after the same manner a fifth, a sixth, or as many as you please may be found out.

From what hath been said by way of explication of the preceding twelfth Rule, the following *Table* III. is deduced, the construction and use whereof is afterwards declared.

TABLE III.

Which sheweth what one pound will amount unto, being forborn unto the end of any term of years under 31, compound interest being computed yearly, at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, *and* 12 per centum, per annum.

Years.	4	5	6	7	8	9	10	11	12
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	1.08160	1.10250	1.12360	1.14490	1.16640	1.18810	1.21000	1.23216	1.25440
3	1.12486	1.15762	1.19101	1.22504	1.25971	1.29502	1.33100	1.36763	1.40[illegible]92
4	1.16985	1.21550	1.26247	1.31079	1.36048	1.41158	1.46410	1.51807	1.57351
5	1.21665	1.27628	1.33822	1.40255	1.46932	1.53862	1.61051	1.68505	1.76234
6	1.26531	1.34009	1.41851	1.50073	1.58687	1.67710	1.77156	1.87041	1.97382
7	1.31593	1.40710	1.50363	1.60578	1.71382	1.82803	1.94871	2.07616	2.21068
8	1.36856	1.47745	1.59384	1.71818	1.85093	1.99256	2.14358	2.30453	2.47596
9	1.42331	1.55132	1.68947	1.83845	1.99900	2.17189	2.35794	2.55803	2.77307
10	1.48024	1.62889	1.79084	1.96715	2.15892	2.36736	2.59374	2.83942	3.10584
11	1.53945	1.71033	1.89829	2.10485	2.33163	2.58042	2.85311	3.15175	3.47854
12	1.60103	1.79585	2.01219	2.25219	2.51817	2.81266	3.13842	3.49845	3.89597
13	1.66507	1.88564	2.13292	2.40984	2.71962	3.06580	3.45227	3.88328	4.36349
14	1.73167	1.97993	2.26090	2.57853	2.93719	3.34172	3.79749	4.31044	4.88711
15	1.80094	2.07892	2.39655	2.75903	3.17216	3.64248	4.17724	4.78458	5.47356

A continuation of the preceding Table III.

Years.	4	5	6	7	8	9	10	11	12
16	1.87298	2.18287	2.54035	2.95216	3.42594	3.97030	4.59497	5.31089	6.13039
17	1.94790	2.29201	2.69277	3.15881	3.70001	4.32763	5.05447	5.89509	6.86604
18	2.02581	2.40661	2.85433	3.37993	3.99601	4.71712	5.55991	6.54355	7.68996
19	2.10684	2.52695	3.02559	3.61652	4.31570	5.14166	6.11590	7.26334	8.61276
20	2.19112	2.65329	3.20713	3.86968	4.66095	5.60441	6.72749	8.06231	9.64629
21	2.27876	2.78596	3.39956	4.14056	5.03383	6.10880	7.40024	8.94916	10.80384
22	2.36991	2.92526	3.60353	4.43040	5.43654	6.65860	8.14027	9.93357	12.10031
23	2.46471	3.07152	3.81975	4.74053	5.87146	7.25787	8.95430	11.02626	13.55234
24	2.56330	3.22509	4.04893	5.07236	6.34118	7.91108	9.84973	12.23915	15.17862
25	2.66583	3.38635	4.29187	5.42743	6.84847	8.62108	10.82470	13.58546	17.00006
26	2.77246	3.55567	4.54938	5.80735	7.39635	9.39915	11.91817	15.07986	19.04007
27	2.88336	3.73345	4.82234	6.21386	7.98806	10.24508	13.10999	16.73864	21.32488
28	2.99870	3.92012	5.11168	6.64883	8.62710	11.16713	14.42099	18.57990	23.88386
29	3.11865	4.11613	5.41838	7.11425	9.31727	12.17218	15.86309	20.62369	26.74993
30	3.24339	4.32194	5.74349	7.61225	10.06265	13.26767	17.44940	22.89229	29.55992

The Construction of the preceding Table III.

The numbers 1, 2, 3, 4, &c. to 30, in the first column on the left hand signifie years; the numbers 4, 5, 6, 7, 8, 9, 10, 11, and 12, placed at the head of the rest of the colums signifie rates of interest, for 100 *l.* lent for a year, and the numbers placed in the several columns underneath those rates of interest, are found out by the *Rule of Three* in decimals, in manner following; *viz.*

I.	100	.	104	: :	1	..	(1.04
II.	100	.	104	: :	1.04	..	(1.0816
III.	100	.	104	: :	1.0816	..	(1.12486

That is to say, First, If 100 *l.* put forth at interest for a year be augmented to 104 *l.* at the years end, what will 1 *l.* be then augmented unto at the same rate? *Answ.* 1.040 *l.* (that is 1 *l.* 0 *s.* 9 *d.* 2 *f.* and somewhat more) which 1.04 (or 1.04000, the cyphers after the 4 being of no value in decimals) is the first number in the second column belonging to 4 *per centum*, and is placed right against 1 year in the first column.

Secondly, say if 100 *l.* lent for a year be augmented to 104 *l.* at the years end, what will 1.04 *l.* be then augmented unto at the same rate? *Answ.* 1.0816 *l.* (that is 1 *l.* 1 *s.* 7 *d.* 2 *f.* +) which 1.0816 is the second number in the said column of 4 *per cent.* and is placed right against 2 years in the first column.

Thirdly,

Thirdly, as 100 is to 104, so is 1.0816 to 1.124864 (or 1 l. 2 s. 5 d. 2 f. +) which 1.12486 is the third number in the column of 4 *per centum*, and is placed right against 3 years in the first column. Hence it appears, that 1 l. at 4 *per centum, per annum*, compound interest, will at the end of 3 years be augmented unto 1.124864 l. (that is, 1 l. 2 s. 5 d. 2 f. and somewhat more.)

After the same manner the rest of the numbers in the second column, as also in the other columns are found out (*mutatis mutandis.*)

The use of the preceding third Table.

Quest. 1. What will 136 l. 15 s. 6 d. be augmented unto, being forborn 20 years, interest upon interest being computed at the rate of 6 *per centum, per annum*? *Answ.* 438 l. 13 s. 1 d. very near, which is thus found out.

First, looking into the fourth column of the said third *Table*, to wit, that column which hath the figure 6 placed at the head of it, I find right against 20 years the number 3.20713, which shews that 1 l. being continued 20 years at 6 *per centum, per annum*, compound interest, and all forborn until the end of the said term will be augmented unto 3.20713 l. (that is 3 l. 4 s. 1 d. 2 f. and somewhat more) therefore after the 15 s. 6 d. in the question is reduced to the decimal .775 (by the sixteenth rule of the 23 Chapter of the preceding book) I multiply the said tabular number 3.20713 by 136.775 (the sum propounded in the question) according to the second rule of the 26th Chapter, so the Product is

found to be 438.665,&c.that is,438 *l*.13 *s*.1 *d*. for the Answer of the question. View the operation here following.

1	:	3.20713	: :	136.775	(438.655 +

136.775

1603565
2244991
2244991
1924278
962139
320713

438|65520575

Quest.2. If 320 *l*. be forborn 11 years, at interest upon interest at 5 *per centum*, *per annum*, what will be due at the end of those eleven years for principal and interest? *Answer*, 547 *l*. 6 *s*.1 *d*. +. For in the third column of the third Table, under the figure five at the head of the column and right against 11 years you will find this number 1.71033 which shews that 1 *l*. at the end of 11 years will at 5 *per centum*, *per annum*, compound interest, be augmented to 1.71033 (that is 1 *l*. 14 *s*. 2 *d*. 1 *f*. and somewhat more) wherefore by multiplying the said 1.71033 by 320 (the number of pounds propounded in the question) the product will be 547.305,&c.that is 547 *l*.6 *s*.1 *d*. + for the Answer of the question. See the following operation.

1 . 1.71033 :: 320 . (547.305+
320

3420660
513099

547|30560

After the same manner the numbers belonging to any of the other rates of interest mentioned in the third Table are to be used.

The manner of summing up Annuities in arrear with allowances of interest upon interest.

XIII. When an Annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount unto, compound interest being computed for each particular Annuity from the time it became due, until the end of the term of years, the work will be as in the following example, *viz.* Suppose an Annuity of 300 *l.* payable at yearly payments be forborn, and all unpaid untill the end of four years, the question is, what will then be due, compound interest being computed at the rate of 6 *per centum, per annum,* for each yearly payment from the time it becomes due to the end of the said term of four years? *Answ.* 1312 *l.* 7 *s.* 8 *d.* very near.

It is evident by the question, that there must be computed what 300 *l.* due at the third years end will be augmented unto in one year (to wit the fourth year) at 6 *per centum*. Also what 300 *l.* due at the second years end will be augmented unto in two years; (to wit the third and fourth years) like-

wise what 300 *l.* due at the first years end, will be augmented unto, in the three following years, (to wit the second, third, and fourth years) all which sums being added to 300 *l.* (the payment due at the end of the fourth year, which is incapable of any improvement) the aggregate or sum will be the total money in Arrear at the end of the fourth year, to wit, $1312\frac{3848}{10000}$ *l.* as may appear by the following operation, *viz.*

The last payment of the Annuity due at the end of the fourth year is	300.
Again, the 300 *l.* due at the third years end, will in one year after the rate of 6 *per centum*, be augmented unto	318.
Also 300 *l.* due at the second years end, will in two years at the rate of 6 *per centum, per annum*, compound interest, be augmented unto, (as appears by the first example of the twelfth Rule aforegoing.)	337.08
In like manner, 300 *l.* due at the first years end, will in three years be augmented unto	357.3048
The sum due at four years end	1312.3848

The invention of the numbers before mentioned being well examined, it will appear that if an Annuity or Rent payable at yearly payments be improved

proved to the utmost at interest upon interest, and all forborn or respited unto the end of certain years, the total then due will be the sum of a rank of continual proportionals as many in number as there are yearly payments, the first of which proportionals is the first (or any one) years rent, and the second proportional proceeds from the first in the same rate as 106 proceeds from 100, if the rate of interest be 6 *per centum*, (or as 108 proceeds from 100, if the rate of interest be 8 *per centum*, &c.) and so likewise the third from the second, the fourth from the third, &c. (after the manner of the operation in the first example of the twelfth Rule of this Chapter.)

Otherwise,

Find a principal which may have such proportion to 300 as 100 hath to 6, and say by the *Rule of Three*;

6 . 100 : : 300 . 5000

That is to say, as 6 *l.* interest hath 100 *l.* for a principal, so 300 *l.* interest hath 5000 *l* for a principal; then seek what 5000 *l.* will be augmented unto, being forborn four years at 6 *per centum*, *per annum*, compound interest, (after the manner of the first example of the twelfth rule aforegoing) so will you find 6312.3848, from which subtracting the said principal 5000 *l.* the remainder (as before) is 1312.3848 *l.* being the sum which 300 *l.* Annuity will be augmented unto at the end of four years, according to the said rate of interest, the Annuity being payable at yearly payments.

The

The reason of the latter Rule.

If a principal be put forth at interest upon interest payable by yearly payments, and all be forborn until the end of certain years, the total then due is equal to the aggregate or sum of these three numbers, to wit, the said principal first put forth; the sum of the annual simple interests of that principal; and the utmost improvement of those simple interests by computing interest upon interest; wherefore if from the said aggregate the first principal be subtracted, the remainder must necessarily consist of the sum of the annual simple interests, (which are in the nature of an Annuity) and the utmost improvement of those simple interests (or Annuity) by computing interest upon interest.

The Construction of the following Table IV.

Upon the aforesaid grounds, the following *Table* IV. is calculated, to shew what one pound Annuity, payable at yearly payments, and forborn any number of years under 31, will amount unto by computing interest upon interest at any of the rates exprest at the head of the said Table.

But the same *Table* may be more easily composed by the addition of the numbers in the preceding *Table* III. in this manner, *viz.* the first number in each of those columns in the following *Table* IV. at the head whereof are placed the numbers 4, 5, 6, 7, 8, 9, 10, 11, and 12, signifying rates of interest

tereſt *per centum*, is 1 or unity, the ſecond number in each of theſe columns in the latter *Table* is compos'd of 1 or unity, and the firſt number in the reſpective columns of the ſaid preceding *Table* III.

Alſo the third number in each of the ſaid columns of this *latter Table* is compos'd of 1, and the ſum of the firſt and ſecond numbers of the reſpective columns of the *former Table*, and in that order the reſt are found out; or more eaſily thus, the third number in the *latter Table* is compos'd of the ſecond number in the *latter*, and of the ſecond in the *former*; the fourth number in the *latter* is compos'd of the third in the *latter*, and of the third in the *former*, &c. But you are to obſerve that according to either of theſe wayes of compoſing the fourth *Table* by Addition, the numbers in the preceding *Table* III. ought to be continued to more places then are there expreſt to prevent error which may happen by adding of defective decimal fractions.

Table

TABLE IV.

Which sheweth what one pound Annuity, payable by yearly payments, and forborn any number of years under 31, will amount unto, at the end of the term, compound interest being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, *and* 12 per centum, per annum.

Years.	4	5	6	7	8	9	10	11	12
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.04000	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440
4	4.24646	4.31012	4.37461	4.43994	4.50611	4.57312	4.64100	4.70973	4.77932
5	5.41632	5.52563	5.63709	5.75073	5.86660	5.98471	6.10510	6.22780	6.35284
6	6.63297	6.80191	6.97531	7.15329	7.33592	7.52333	7.71561	7.91285	8.11518
7	7.89829	8.14200	8.39383	8.65402	8.92280	9.20043	9.48717	9.78327	10.08901
8	9.21422	9.64910	9.89746	10.25980	10.63662	11.02847	11.43588	11.85943	12.29969
9	10.58279	11.02656	11.49131	11.97798	12.48755	13.02103	13.57947	14.16397	14.77565
10	12.00610	12.57789	13.18079	13.81644	14.48656	15.19292	15.93742	16.72200	17.54873
11	13.48635	14.20678	14.97164	15.78359	16.64548	17.56029	18.53116	19.56142	20.65458
12	15.02580	15.91712	16.86994	17.88845	18.97712	20.14071	21.38428	22.71318	24.13313
13	16.62683	17.71298	18.88213	20.14064	21.49529	22.95338	24.52271	26.21163	28.02910
14	18.29191	19.59863	21.01506	22.55048	24.21492	26.01918	27.97498	30.09491	32.39260
15	20.02358	21.57856	23.27596	25.12502	27.15211	29.36091	31.77248	34.40515	37.27971

A continuation of the preceding Table IV.

Years.	4	5	6	7	8	9	10	11	12
16	21.82453	23.65749	25.67252	27.88805	30.32428	33.00339	35.94972	39.18994	42.75328
17	23.69750	25.84036	28.21287	30.84021	33.75022	36.97370	40.54470	44.50084	48.88367
18	25.64541	28.13238	30.90565	33.99903	37.45024	41.30133	45.59917	50.39593	55.74971
19	27.67122	30.53900	33.75999	37.37896	41.44626	46.01845	51.15909	56.93948	63.43968
20	29.77807	33.06595	36.78559	40.99549	45.76196	51.16011	57.27499	64.20283	72.05244
21	31.96920	35.71925	39.99272	44.86517	50.42292	56.76453	64.00249	72.26514	81.69874
22	34.24796	38.50521	43.39228	49.00573	55.45675	62.87333	71.40274	81.21430	92.50258
23	36.61788	41.43047	46.99582	53.43614	60.89329	69.53193	79.54302	91.14788	104.60289
24	39.08260	44.50199	50.81557	58.17667	66.76475	76.78981	88.49732	102.17415	118.15524
25	41.64590	47.72709	54.86451	63.24903	73.10593	84.70089	98.34705	114.41330	133.33387
26	44.31174	51.11345	59.15638	68.67646	79.95441	93.32397	109.18176	127.99877	150.33393
27	47.08421	54.66912	63.70576	74.48382	87.35076	102.72313	121.09994	143.07863	169.37400
28	49.96758	58.40258	68.52810	80.69769	95.33882	112.96821	134.20993	159.81728	190.69888
29	52.96628	62.32271	73.63979	87.34652	103.96593	124.13535	148.63092	178.39718	214.58275
30	56.08493	66.43884	79.05818	94.46078	113.28321	136.30753	164.49402	199.02087	241.33268

The use of the preceding Table IV.

The use of the said fourth *Table* will be manifest by the manner of solving this Question, *viz.* If an Annuity of 20 *l.* payable by yearly payments for 15 years, be all forborn or unpaid until the end of the said term, what will it then amount unto, upon a computation of interest upon interest, at the rate of 6 *per cent. per annum*? *Answ.* 465 *l.* 10 *s.* 4 *d.* 2 *f.* very near, as by the following operation is evident; For in the column belonging to 6 *per centum*, (to wit, that column which hath the figure 6 placed at the head of it) right against 15 years, you will find 23.27596, which shews that an Annuity of 1 *l.* payable at yearly payments for 15 years, will at the end of the said term (compound interest being computed at 6 *per cent. per annum*) amount unto 23.27596 *l.* (or 23 *l.* 5 *s* 6 *d.*+) Therefore multiplying the said tabular number 23.27596 by 20, (20 because the Annuity propounded is 20 *l.*) the product will be 465.519+, that is 465 *l.* 10 *s.* 4 *d.* 2 *f.* which is the Answer of the question; view the following operation.

$$1 \;.\; 23.27596 :: 20 \;.\; (465.519+$$

$$\begin{array}{r} 20 \\ \hline 465|51920 \end{array}$$

In the same manner the numbers in the other columns are to be used.

Of rebate at compound interest. *XIV.* When a sum of money is due at a time to come, and it is required to know what it is worth in ready money, rebate being made at a given rate of

of compound interest, the work will not be much different from the twelfth Rule of this Chapter, *viz.* there must be found a series or rank of continual proportionals more in number by one, than is the number of years in the question; of which proportionals, the first is the money propounded to be rebated, the second must decrease or lessen from the first, the third from the second, &c. in such manner or rate as 100 decreaseth from 106 (or as 100 from 108 if the rate of interest be 8 *per centum*) then will the last proportional be the answer of the question: So if 378 $\frac{743088}{1000000}$ *l.* be due at the end of four years wholly to come, it will be found to be worth in ready money 300 *l.* rebate being made at compound interest at 6 *per centum*, as by the four following *Rules of Three* is manifest, which may be proved by the preceding twelfth rule, where it will appear that 300 *l.* being forborn four years, will at the said rate of compound interest be augmented unto 378.743088 *l.*

			l.		*l.*
106 .	100	::	378.743088	.	357.3048
			357.3048	.	337.08
			337.08	.	318
			318.	.	300

Upon this ground, the following *Table* V. is calculated to shew what one pound due at the end of any number of years to come, is worth in present money, rebate being made at the rates of compound interest, mentioned in the said *Table*; by the help whereof and of *Multiplication*, questions of rebate for any sum propounded may be performed without considerable error.

Table

TABLE V.

Which sheweth what one pound, payable at the end of any term of years to come under 31, is worth in ready money, discompt or rebate being yearly computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per centum per annum, compound interest.

Years.	4	5	6	7	8	9	10	11	12
1	.961538	.952381	.943396	.934579	.925925	.917431	.909090	.900900	.892857
2	.924556	.907029	.889996	.873438	.857338	.841680	.826446	.811622	.797193
3	.888996	.863837	.839619	.816297	.793832	.772183	.751314	.731191	.711780
4	.854802	.822702	.792093	.762895	.735029	.708425	.683013	.658731	.635518
5	.821927	.783526	.747258	.712986	.680583	.649931	.620921	.593451	.567426
6	.790314	.746215	.704960	.666342	.630169	.596267	.564474	.534640	.506631
7	.759917	.710681	.665057	.622749	.583490	.547034	.513158	.481658	.452349
8	.730690	.676839	.627412	.582009	.540268	.501866	.466507	.433926	.403883
9	.702586	.644608	.591898	.543933	.500248	.460427	.424097	.390924	.360610
10	.675564	.613913	.558391	.508349	.463193	.422410	.385543	.352184	.321973
11	.649580	.584679	.526787	.475092	.428882	.387532	.350494	.317283	.287476
12	.624596	.556837	.496989	.444012	.397113	.355534	.318630	.285840	.256675
13	.600573	.530321	.468839	.414964	.367697	.326178	.289664	.257514	.229174
14	.577474	.505067	.442300	.387817	.340461	.299246	.263331	.231994	.204619
15	.555264	.481017	.417265	.362446	.315241	.274538	.239392	.209004	.182696

A continuation of the preceding Table V.

Years.	4	5	6	7	8	9	10	11	12
16	.533908	.458111	.393646	.338734	.291890	.251869	.217629	.188292	.163121
17	.513373	.436296	.371364	.316574	.270269	.231073	.197844	.169632	.145644
18	.493628	.415520	.350343	.295864	.250249	.211993	.179858	.152822	.130039
19	.474642	.395733	.330512	.276508	.231712	.194489	.163508	.137677	.116106
20	.456386	.376889	.311804	.258419	.214548	.178430	.148643	.124034	.103666
21	.438833	.358942	.294155	.241513	.198655	.163698	.135130	.111742	.092559
22	.421955	.341849	.277505	.225713	.183940	.150181	.122846	.100668	.082642
23	.405726	.325571	.261797	.210947	.170315	.137781	.111678	.090692	.073787
24	.390121	.310067	.246978	.197146	.157699	.126405	.101525	.081705	.065882
25	.375116	.295302	.232998	.184249	.146018	.115967	.092296	.073608	.058823
26	.360689	.281240	.219810	.172195	.135201	.106392	.083905	.066313	.052520
27	.346816	.267848	.207367	.160930	.125186	.097607	.076277	.059742	.046893
28	.333477	.255093	.195630	.150402	.115913	.089548	.069343	.053821	.041869
29	.320651	.242946	.184556	.140562	.107327	.082154	.063039	.048487	.037383
30	.308318	.231377	.174110	.131367	.099377	.075371	.057308	.043682	.033377

The Construction of the preceding Table V.

The numbers 1,2,3,4,&c. to 30, in the first column on the left hand, signifie years; the numbers 4,5,6,7,8,9,10,11, and 12, placed at the head of the rest of the columns signifie rates of interest for 100 *l*. lent for a year, and the numbers placed in the several columns underneath those rates of interest are found out by the *Rule of Three* in decimals, in manner following, *viz.*

I.	104 . 100 : : 1 . (,9615384615 &c.
II.	104 . 100 : : ,9615384615 + . (,9245562, &c.
III.	104 . 100 : : ,9245562 &c. . (,888996 +

That is to say, First, if 104 decrease to 100, or if 104 *l*. payable at the end of a year to come be worth 100 *l*. ready money, what ready money is 1 *l*. due at the end of a year to come worth? *Answer*, ,9615384615 + (or 19 *s*. 2 *d*. 3 *f*. very near) So that ,961538 is the first decimal in the second column belonging to 4 *per centum*, in *Table* V. and is placed right against 1 year in the first column.

Secondly, say in like manner if 104 decrease to 100, what will ,9615384615, &c. (the decimal found by the first rule of three) decrease unto? *Answer*, ,9245562, &c. the first 6 places whereof, to wit, ,924556 are the second decimal in the said column of 4 *per cent*. which is placed right against two years.

Thirdly,

Thirdly, as 104 is to 100, ſo is ,9245562, &c. (the decimal found by the ſecond *Rule of Three*) to ,888996+ (or 17 *s.* 9 *d.* 1 *f.* +) which is the third decimal in the column of 4 *per centum.* Hence it appears that 1 *l.* due at the end of three years to come is worth .888996,+ (or 17 *s.* 9 *d.* 1 *f.* and ſomewhat more) in ready money rebate being made at the rate of 4 *per centum, per annum,* compound intereſt.

After the ſame manner the reſt of the decimal fractions in the ſaid ſecond column, as alſo in the other columns are found out (*mutatis mutandis.*)

The uſe of the preceding Table V.

To exemplifie the ſaid fifth Table, let it be required to find out how much ready money will diſcharge a debt of 356 *l.* payable at the end of ſeven years to come, by rebating at the rate of 7 *per centum, per annum,* compound intereſt? *Anſw.* 221 *l.* 13 *s.* 11 *d.* 3 *f.* very near, For in the fifth column, at the head whereof is placed 7, ſignifying 7 *per centum,* right againſt 7 years, I find .622749, which ſhews that 1 *l.* due at the end of 7 years to come is worth in preſent money .622749 decimal parts of a pound, rebate being made at the ſaid rate of compound intereſt. Therefore multiplying the ſaid tabular number .622749 by the ſaid 356 *l.* (the debt propounded) the product (according to the ſecond rule of the 26th Chapter) will be 221,698, &c. that is, 221 *l.* 13 *s.* 11 *d.* 3 *f.* which is the Anſwer of the queſtion. See the ſubſequent operation.

The Construction of the preceding Table V.

The numbers 1,2,3,4,&c. to 30, in the first column on the left hand, signifie years; the numbers 4,5,6,7,8,9,10,11, and 12, placed at the head of the rest of the columns signifie rates of interest for 100 *l.* lent for a year, and the numbers placed in the several columns underneath those rates of interest are found out by the *Rule of Three* in decimals, in manner following, *viz.*

I. 104 . 100 :: 1 . (,9615384615 &c.
II. 104 . 100 :: ,9615384 615 + . (,9245562, &c.
III. 104 . 100 :: ,9245562 &c. . (,888996 +

That is to say, First, if 104 decrease to 100, or if 104 *l.* payable at the end of a year to come be worth 100 *l.* ready money, what ready money is 1 *l.* due at the end of a year to come worth? *Answer*, ,9615384615 + (or 19 *s.* 2 *d.* 3 *f.* very near) So that ,961538 is the first decimal in the second column belonging to 4 *per centum*, in *Table* V. and is placed right against 1 year in the first column.

Secondly, say in like manner if 104 decrease to 100, what will ,9615384615, &c. (the decimal found by the first rule of three) decrease unto? *Answer*, ,9245562, &c. the first 6 places whereof, to wit, ,924556 are the second decimal in the said column of 4 *per cent.* which is placed right against two years.

Thirdly,

Thirdly, as 104 is to 100, so is .9245562, &c. (the decimal found by the second *Rule of Three*) to .888996+ (or 17 *s*. 9 *d*. 1 *f*. +) which is the third decimal in the column of 4 *per centum*. Hence it appears that 1 *l*. due at the end of three years to come is worth .888996,+ (or 17 *s*. 9 *d*. 1 *f*. and somewhat more) in ready money rebate being made at the rate of 4 *per centum*, *per annum*, compound interest.

After the same manner the rest of the decimal fractions in the said second column, as also in the other columns are found out (*mutatis mutandis*.)

The use of the preceding Table V.

To exemplifie the said fifth Table, let it be required to find out how much ready money will discharge a debt of 356 *l*. payable at the end of seven years to come, by rebating at the rate of 7 *per centum, per annum*, compound interest? *Answ*. 221 *l*. 13 *s*. 11 *d*. 3 *f*. very near, For in the fifth column, at the head whereof is placed 7, signifying 7 *per centum*, right against 7 years, I find .622749, which shews that 1 *l*. due at the end of 7 years to come is worth in present money .622749 decimal parts of a pound, rebate being made at the said rate of compound interest. Therefore multiplying the said tabular number .622749 by the said 356 *l*. (the debt propounded) the product (according to the second rule of the 26th Chapter) will be 221.698, &c. that is, 221 *l*. 13 *s*. 11 *d*. 3 *f*. which is the Answer of the question. See the subsequent operation.

1 . ,622749 : : 356 . (221.698 +
356

3736494
3113745
1868247

221|698644

In the ſame manner the numbers in the other columns are to be uſed.

To find the preſent worth of Annuities by a computation of compound intereſt.

XV. The finding out the preſent worth of an Annuity, is grounded upon this foundation, to wit, if the preſent money which is paid for the purchaſe of an Annuity, to continue any term of years, be put forth at any rate of compound intereſt, and all forborn until the end of the ſaid term, and that the total money then due be put into one Scale: alſo if the total ſum of the utmoſt improvements of the annual payments of the Annuity, put forth at the ſame rate of compound intereſt, from the time thoſe annual payments become due until the end of the term, be put into the other Scale, the ſcales muſt be even; *viz.* the ſaid two total ſums of money muſt be equal one to the other.

Now to find out ſuch a preſent worth of an Annuity, there are divers wayes, ſome of which I ſhall here explain by examples:

Fiſt therefore let it be required to find the preſent worth of an Annuity of 378.743088 *l.* to continue three years compound intereſt being computed at 6 *per cent. per ann.* *Anſwer*, 1012.3848 *l.*

It

It is evident by the queſtion that there muſt be computed (after the manner of the Example upon the fourteenth Rule aforegoing) the preſent worth of 378 $\frac{743088}{1000000}$ *l.* due at the firſt years end; alſo the preſent worth of the like ſum due at the ſecond years end, and in like manner for the third year; all which particular preſent values being added together, the aggregate or ſum will be the total preſent worth of the Annuity propounded, *viz.*

	l.
378.743088 *l.* payable at the end of one year is worth in ready money (as is evident by the fourteenth rule aforegoing.)	357.3048
Alſo the like ſum payable at the end of 2 years to come is worth in ready money	337.08
Again, the like ſum payable at the end of three years to come, is worth in ready money	318.
Therefore the total preſent worth of an Annuity of 378.743088 *l.* to continue 3 years is	1012.3848

Otherwiſe.

Find a Principal which may be in ſuch proportion to the propounded Annuity 378.743088 *l.* as 100 is to 6. Which will be exactly 6312.3848 *l.* for

6 . 100 : : 378.743088 . 6312,3848

Then ſuppoſing this Principal ſo found to be a ſum due at the end of three years to come, find what it will be worth in ready money, by diminiſhing it according to the fourteenth Rule of this Chapter, ſo you will find 5300 *l.* for the ready money equivalent to the ſaid 6312.3848 *l.* due at the end

end of three years, which ready money 5300 *l.* being subtracted from the said 6312.3848 *l.* leaves (as before) 1012.3848 *l.* for the present worth of the said Annuity of 378.743088 *l.* to continue three years, compound interest being allowed at 6 *per centum, per annum.*

The reason of the latter Rule.

It will not be difficult to apprehend that if 6312.3848 *l.* ready money be put forth as a Principal at interest upon interest, it will at three years end be augmented unto an Aggregate or sum compos'd of these three numbers, to wit, the said Principal 6312.3848 ; the sum of the annual simple interests of that Principal, and the utmost improvement of those simple interests by interest upon interest: And because (by the operation aforegoing) 5300 *l.* ready money (part of the said ready money 6312.3848 *l.*) will at three years end be augmented unto 6312.3848 *l.* part of the said Aggregate, therefore 1012.3848 *l.* the complement or remaining part of the said ready money 6312.3848 *l.* must necessarily be augmented unto the complement or remaining part of the said Aggregate, which remaining part last mentioned is composed of the sum of the aforesaid simple interests, and of their utmost improvement at interest upon interest, that is, the said remainder is the utmost improvement of an Annuity of 378.743088 *l.* to continue three years, compound interest being allowed at 6 *per centum, per annum.*

The

The Construction of the following Table VI.

Upon the aforesaid grounds the following Table VI. is calculated, to shew how much ready money an Annuity of one pound to continue any number of years under 31. and payable at yearly payments, is worth, upon a computation of compound interest at any of the rates *per centum*, mentioned at the head of the said Table. But the said Table VI. may more easily be composed by the help of the preceding Table V. in this manner, *viz.* the first number in every of the Columns (except the Column of years) in the following Table VI. is the same with the first number in the like Columns respectively in the preceding Table V. the second number in each of the said Columns of the sixth Table is the sum of the first and second numbers in the respective Columns of the sixth Table; the third number in the said Columns of the fifth Table is the sum of the first, second and third numbers in the respective Columns of the fifth Table: Or yet more easily thus, the third number in the sixth Table, is composed of the third in the fifth Table, and of the second in the sixth; the fourth number in the sixth Table is composed of the fourth in the fifth and of the third in the sixth; the like is to be understood of the rest. But you are to observe that according to this way of composing the sixth Table by Addition, the numbers in the fifth Table must be continued to more places then are there exprest, to prevent error arising by the addition of defective Decimal fractions.

TABLE VI.

Which sheweth the present worth of one pound Annuity, to continue any term of years under 31, and payable by yearly payments, compound interest being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per centum, per annum.

Years.	4	5	6	7	8	9	10	11	12
1	.90153	.95238	.94339	.93457	.92592	.91743	.90909	.90090	.89285
2	1.88609	1.85941	1.83339	1.80801	1.78326	1.75911	1.73553	1.71252	1.69005
3	2.77509	2.72324	2.67301	2.62431	2.57709	2.53129	2.48685	2.44371	2.40183
4	3.62989	3.54595	3.46510	3.38721	3.31212	3.23971	3.16986	3.10244	3.03734
5	4.45182	4.32947	4.21236	4.10019	3.99270	3.88965	3.79078	3.69589	3.60477
6	5 24213	5.07569	4.91732	4.76653	4.62287	4.48591	4.35526	4.23053	4.11140
7	6.00205	5.78637	5 58233	5.38928	5.20636	5.03295	4.86841	4.71219	4.56375
8	6.7327[illegible]	6.46321	6 20979	5.97129	5.74663	5.53481	5 33492	5.14612	4 96763
9	7.43533	7.10782	6.80169	6.51523	6.24688	5.99524	5.75901	5.53704	5.32824
10	8.11089	7.72173	7.36008	7.02358	6.71008	6.41765	6.14456	5.88923	5.65022
11	8.76047	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	6.20651	5.93769
12	9 38507	8.86325	8.38384	7.94268	7.53607	7.16072	6.81369	6.49235	6.19437
13	9.98964	9.39357	8.85268	8.35765	7.90377	7.48690	7.10335	6.74987	6.42354
14	10.56312	9.89864	9.29498	8.74546	3.24423	7.78614	7.36668	6.98186	6.62816
15	11.11838	10.37965	9.71224	9.1079	3.55947	8.06068	7.60608	7.19087	6.81086

A continuation of the preceding Table VI.

Years.	4	5	6	7	8	9	10	11	12
16	11.65229	10.83776	10.10589	9.44664	8.85136	8.31255	7.82371	7.37916	6.97398
17	12.16566	11.27406	10.47725	9.76322	9.12163	8.54363	8.02155	7.54879	7.11962
18	12.65929	11.68958	10.82760	10.05908	9.37188	8.75562	8.20141	7.70161	7.24966
19	13.13393	12.0853[illegible]	11.15811	10.33559	9.60359	8.95011	8.36492	7.83929	7.36577
20	13.59032	12.46220	11.46992	10.59401	9.81814	9.12854	8.51356	7.96332	7.46944
21	14.02915	12.82115	11.76407	10.8355[illegible]	10.01680	[illegible].29224	8.64869	8.07507	7.56200
22	14.45111	13.16300	12.04158	11.06124	10.20074	9.44242	8.77154	8.17574	7.64464
23	14.85683	13.48857	12.30337	11.27218	10.37105	9.58020	8.88322	8.26643	7.71843
24	15.24696	13.79864	12.55035	11.46933	10.52875	9.70661	8.98474	8.34813	7.78431
25	15.62207	14.09394	12.78335	11.65358	10.67477	9.82257	9.07704	8.42174	7.84313
26	15.98276	14.37518	13.00316	11.82577	10.80997	9.92897	9.16094	8.48805	7.89505
27	16.32958	14.64303	13.21053	11.98671	10.93516	10.02657	9.23722	8.54780	7.94255
28	16.66305	14.89812	13.40616	12.13711	11.05107	10.11612	9.30656	8.60162	7.98442
29	16.98371	15.14107	13.59071	12.27767	11.15840	10.19828	9.36960	8.65011	8.02180
30	17.29202	15.39244	13.76482	12.40904	11.25778	10.27365	9.42691	8.69379	8.05518

The use of the preceding Table VI.

The use of the said sixth Table will appear by the manner of solving these two subsequent questions, *viz.*

Quest. 1. What is the present worth of an Annuity or rent of 56 *l. per annum*, payable by yearly payments for 21 years, accompting interest upon interest at the rate of 6 *per centum, per annum?* *Answer*, 658 *l.* 15 *s.* 9 *d.* very near, thus found out; In the fourth Column of the preceding *Table* VI. under the figure 6 at the head, and right against 21 years, I find 11.76407, which shews that an Annuity of 1 *l.* payable by yearly payments for 21 years, is worth in present money 11.76407 *l.* (or 11 *l.* 15 *s.* 3 *d.* 1 *f.* and somewhat more) interest upon interest being computed on both sides at the rate of 6 *per centum, per annum*; therefore multiplying the said tabular number 11.76407 by 56, (56 because the Annuity propounded is 56 pound) the product (according to the second rule of the 26th. Chapter of the preceding Book) will be found to be 658.787, &c. that is 658 *l.* 15 *s.* 9 *d.* very near; Wherefore I conclude that the Answer of the question is 658 *l.* 15 *s.* 9 *d.* view the following operation.

1 . 11.76407 : : 56 . (658,787 +
56

7058442
5882035

658|78792

Queſt. 2. What is the preſent worth of an annual rent of 45 *l.* payable by yearly payments for 21 years, intereſt upon intereſt being computed at 10 *per centum, per annum?* *Anſw.* 389 *l.* 3 *s.* 10 *d.* very near; for in the Column of 10 *per centum*, in the ſaid ſixth *Table*, right againſt 21 years, and under 10 at the head I find this number 8.64869; which ſhews that at 10 *per centum*, compound intereſt, an Annuity or rent of 1 *l.* payable by yearly payments for 21 years, is worth in ready money 8.64869 *l.* that is 8 *l.* 12 *s.* 11 *d.* 3 *f.* therefore multiplying the ſaid tabular number 8.64869, by 45 (the rent propounded) the product will be 389.191 +, that is 389 *l.* 3 *s.* 10 *d.* very near, which is the Anſwer of the Queſtion.

1 . 8.64869 : : 45 . (389.191 +
45

4324345
3459476

389|19105

In the ſame manner the numbers in the other Columns of *Table* VI. are to be uſed.

Moreover

Moreover the numbers in the said sixth *Table* will at first sight shew how many years purchase an Annuity to continue any number of years under 31 is worth, to be sold for present money, compound interest being computed on both sides, at any of the said rates 4, 5, 6, 7, 8, 9, 10, 11 and 12 *per centum*: so if you desire to know how many years purchase an Annuity issuing out of Lands for 21 years, to begin presently, is worth, if it were to be sold for ready money, when the current rate of interest is 6 *per centum*: Seek in the first Column of *Table* VI. for 21 years, and carry your eye from thence equidistant to the head-line of the *Table* till you come under 6, which (as before hath been said) signifies 6 *per centum*. So in the fourth Column you will find 11.76407, whereof you need only consider 11.76, which shews that the said Annuity is worth 11 years purchase, (or 11 times one years rent whatever it be) and 76 parts of one years purchase divided into 100 parts, or $11\frac{3}{4}$ years purchase and a little more. The same Annuity when money was at 8 *per centum* was worth 10 years purchase and about $\frac{1}{100}$ part of a years purchase more, as the number in the Column of 10 *per centum* right against 21 years will discover.

To find how many years purchase an Annuity or a Lease for years is worth.

In like manner supposing 10 *per centum* to be a fit rate to be allowed in the valuation of Leases of houses, the Lease of a house for 21 years will be found by the said *Table* to be worth 8 years purchase and $\frac{64}{100}$ parts of a years purchase, or 8 years

purchase

purchase and an half, and half a quarter of a years purchase, and somewhat more; But note here, that in valuing of Leases, the rate *per centum* is to be set higher or lower according to the goodness of the thing leased, and the certainty or uncertainty of the rent.

XVI. When a sum of money is propounded, and it is required to know what Annuity (to continue any number of years, and according to any given rate) that sum will buy, you may presuppose at pleasure an Annuity for the term of years propounded, and find the value of that Annuity in ready money (according to the fifteenth Rule aforegoing) at the rate assigned; then will the proportion be as followeth;

Of the purchase of Annuities at compound interest.

As the value found is in proportion to the supposed Annuity, so is the sum of money propounded, to the Annuity required.

So if it be required to find what Annuity to begin presently, and to continue 3 years, 500 *l.* in present money will purchase, compound interest being computed at 6 *per centum, per annum*: The *Answer* will be 187 *l.* 1 *s.* 1 *d.* very near.

For presupposing an Annuity at pleasure, to wit, 378.743088 *l.* payable yearly for 3 years, the value thereof in present money will (by the fifteenth Rule of this Chapter) be found to be 1012.3848 *l.* Therefore by the *Rule of Proportion*, say,

1012.3848 . 378,743088 :: 500 . 187,054

That

That is to say, if 1012.3848 *l.* in ready money will buy an Annuity of 378.743088 *l.* (to continue three years) then 500 *l.* in present money will purchase an Annuity (to continue the same term of years, and at the same rate of interest) of 187.054, &c. that is 187 *l.* 1 *s.* 1 *d.* very near.

The Construction of the following Table VII.

Upon this ground the following *Table* VII. is calculated to shew what Annuity (to continue any term of years under 31, and at any rate of interest mentioned at the head of that *Table*) one pound will purchase, by which *Table*, and by the help of *Multiplication*, questions concerning the *purchase of Annuities*, *Rents* or *Pensions*, by any sum of ready money propounded, may be resolved without considerable error. But a more ready way to make the said *Table* VII. may be this following, *viz.*

Forasmuch as it is evident by the construction of the third *Table* aforegoing, that one pound ready money is equivalent unto 1.06 *l.* payable at the end of a year to come, at the rate of 6 *per centum, per annum*; therefore this 1.06 is to be the first number in the Column intituled 6 *per centum* in the subsequent *Table* VII. Again, the present value of one pound Annuity to continue two years at the said rate will be found by the preceeding *Table* VI. to be near 1.83339 *l.* Therefore by the Rule of Proportion, say,

1,83339 . 1 :: 1 . 54543, &c.

That is, if 1,83339 *l.* ready money will purchase an Annuity of 1 *l.* (to continue two years;) what Annuity to continue the same time will 1 *l.* in present money purchase? *Answer*, an Annuity of .54543 *l.* that is 10 *s.* 11 *d.* very near, to continue two years; therefore the said Decimal .54543 *l.* is to be placed as the second number in the fourth Column of the subsequent *Table* VII. Hence it follows, that if 1 or unity be divided by every one of the numbers in all the Columns of *Table* VI. except the first Column of years, the quotients will give the respective numbers to be placed in the like Columns of the following *Table* VII. In which operation it will be requisite, that the numbers in the preceding *Table* VI. be continued to more places then are there exprest, to prevent error that will arise by adding of defective decimals.

Table

TABLE VII.

Which sheweth what Annuity payable by yearly payments, to continue any term of years under 31, one pound will purchase; compound interest being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per centum, per annum.

Years.	4	5	6	7	8	9	10	11	12
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	.53019	.53780	.54543	.55309	.56076	.56846	.57619	.58393	.59169
3	.36034	.36720	.37411	.38105	.38803	.39505	.40211	.40921	.41634
4	.27549	.28209	.28859	.29519	.30192	.30866	.31547	.32232	.32923
5	.22462	.23097	.23739	.24389	.25045	.25709	.26379	.27057	.27740
6	.19076	.19701	.20336	.20979	.21631	.22291	.22960	.23637	.24322
7	.16660	.17281	.17913	.18555	.19207	.19869	.20545	.21221	.21911
8	.14852	.15472	.16103	.16746	.17401	.18067	.18744	.19432	.20130
9	.13449	.14069	.14702	.15148	.16007	.16679	.17364	.18060	.18767
10	.12329	.12950	.13586	.14237	.14902	.15582	.16274	.16980	.17698
11	.11414	.12038	.12679	.13335	.14007	.14694	.15396	.16112	.16841
12	.10655	.11282	.11927	.12590	.13269	.13965	.14676	.15402	.16143
13	.10010	.10645	.10296	.11965	.12652	.13356	.14077	.14815	.15567
14	.09466	.10102	.10758	.11434	.12129	.12843	.13574	.14322	.15087
15	.08994	.09634	.10296	.10979	.11682	.12405	.13147	.13906	.14682

A continuation of the preceding Table VII.

Years.	4	5	6	7	8	9	10	11	12
16	.08581	.09226	.09895	.10585	.11298	.12029	.12781	.13551	.14339
17	.08219	.08869	.09544	.10242	.10962	.11704	.12466	.13247	.14056
18	.07899	.08554	.09235	.09941	.10670	.11421	.12192	.12984	.13793
19	.07613	.08274	.08962	.09675	.10412	.11173	.11954	.12756	.13579
20	.07358	.08024	.08718	.09439	.10184	.10954	.11745	.12557	.13387
21	.07128	.07799	.08500	.09228	.09983	.10761	.11562	.12383	.13224
22	.06919	.07597	.08304	.09040	09803	.10590	.11400	.12231	.13081
23	.06734	.07413	.08127	.08871	.09642	.10438	.11257	.12097	.12955
24	.06655	.07247	.97967	.08718	.09497	.10302	.11129	.11978	.12846
25	.06401	.07095	.07822	.08581	.09367	.10180	.11016	.11874	.12749
26	.06256	.06956	.07690	.08456	.09250	.10071	.10715	.11781	.12665
27	.06123	.06829	.07569	.08342	.09144	.09973	.10825	.11698	.12590
28	.06001	.06712	.07459	.08239	.09048	.09885	.10745	.11625	.12524
29	.05887	.06604	.07357	.08144	.08961	.09805	.10672	.11565	.12465
30	.05783	.06496	.07264	.08058	.08882	.09733	.10607	.11502	.12414

The uſe of the preceding Table VII.

Queſt. 1. What Annuity or yearly rent iſſuing out of Lands, to begin preſently, and to continue 14 years, will 320 *l.* purchaſe, compound intereſt being reckoned on both ſides, at the rate of 6 *per centum, per annum?* *Anſw.* 34 *l.* 8 *s.* 6 *d.* very near, which is thus found out, *viz.* In the fourth Column of the preceding *Table* VII. under 6 at the head of that Column, and right againſt 14 years you will find this decimal .10758, which ſhewes that 1 *l.* ready money will purchaſe an Annuity of .10758 *l.* (that is 2 *s.* 1 *d.* 2 *f.* +) therefore multiplying the ſaid decimal .10758 by the ſaid 320; the product (according to the ſecond Rule of the 26th. Chapter of the preceding Book) will be found to be 34.425, &c. that is 34 *l.* 8 *s.* 6 *d.* very near, which is the *Anſwer* of the queſtion.

1 . ,10758 : : 320 . (34.425 +

```
      320
  ---------
   215160
   32274
  ---------
  34|42560
```

In like manner, if 10 *per centum* be thought a fit rate of intereſt to be allowed in purchaſing Leaſes of houſes, 500 *l.* will buy a preſent yearly rent of 63 *l.* 18 *s.* 1 *d.* payable for 16 years out of a houſe. For underneath 10 at the head of the 8th. Column, and right againſt 16 years, (in the preceding *Table* VII.) you will find this decimal .12781, which being

ing multiplyed by 500, (the number of pounds propounded to purchase the Lease) the product will be found to be 63.90500, that is, 63 *l.* 18 *s.* 1 *d.*+ as by the subsequent operation is manifest.

1 . ,12781 : : 500 : (63.905
500

———————

63 | 90500

XVII. Upon the same foundations which have been laid in the 12, 13, 14, 15 and 16 Rules of this Chapter, for the making of Tables which respect yearly payments; Tables may be made for half yearly and quarterly payments, the interest of 100 *l.* for $\frac{1}{2}$ year, and likewise for $\frac{1}{4}$ year being first agreed upon: For if we suppose that at the rate of 6 *l.* for 100 *l.* for a year, the interest of 100 *l.* for $\frac{1}{2}$ year is 3 *l.* the numbers 100 and 103 are to be used in the same manner to calculate *Tables* for half yearly payments, as the numbers 100 and 106 have been before used to form *Tables* for yearly payments. But if at the rate of 6 *per centum, per annum*, the interest of 100 *l.* for $\frac{1}{2}$ year ought to be such, that being added to the said principal 100 *l.* and the whole put forth at interest for the next half year, at the said rate, the sum then due (to wit, at the years end) must exactly amount unto 106 *l.* In this case a Geometrical mean proportional number between the extreams 100 and 106 must be sought, which mean will (by the following 18. Rule) be found to be near 102.956301 +. And then the numbers 100 and 102.956301, &c. are to be used instead of the

The making of Tables for half yearly and quarterly payments.

numbers 100 and 106 in manner aforeſaid. In like manner, if it be ſuppoſed that at the rate of 6 *per centum, per annum*, the intereſt of 100 *l.* for $\frac{1}{4}$ year is 1 *l.* 10 *s.* or 1.5 *l.* the numbers 100 and 101.5 are to be uſed for the calculating of *Tables* for quarterly payments, in the ſame manner as the numbers 100 and 106 for yearly payments. But if at the rate of 6 *per centum, per annum*, the intereſt of 100 *l.* for $\frac{1}{4}$ year ought to be ſuch, that being added to the ſaid 100 *l.* and the whole put forth at the ſame rate of intereſt for the next $\frac{1}{4}$ year, and in that manner for the third and fourth quarters, and that the ſum due at the years end muſt exactly amount unto 106 *l.* In this caſe a ſeries or rank of five numbers in Geometrical proportion continued muſt be conſidered, *viz.* the principal 100 *l.* (which is the leſſer of the two extream proportionals) the three ſums (compoſed of principal and intereſt) due at the end of the firſt, ſecond, and third quarters of the year, (which are the three mean proportionals) and 106 *l.* due at the years end, (which is the greater of the two extream proportionals;) now between the ſaid extreams 100 and 106, the firſt (to wit the leaſt) of the ſaid three mean proportionals is to be ſought, which (by the following Rule of this Chapter) will be found to be near 101.4673 +. And then the numbers 100 and 101.4673, &c. are to be uſed inſtead of the numbers 100 and 106 in manner aforeſaid.

To find a Geometrical mean proportional number between two numbers given.

XVIII. Two numbers being given to find a Geometrical mean proportional between them; multiply the two given numbers one by the other, and extract the ſquare root

root of the product, so is such square root the mean proportional sought: for example, if 8 and 18 are two numbers given, and it is required to find a mean number Geometrically proportion between them, multiply 18 by 8, so is the product 144, whose square root is 12 for the mean proportional sought; so that 8, 12 and 18, are three numbers in Geometrical proportion continued, *viz.* As 8 is in proportion to 12, so is 12 to 18. In like manner a Geometrical mean proportional between the extreams 100 and 106 will be found near 102.956301 +.

To find the first of two Geometrical mean proportional numbers between two extream numbers given.

XIX. Two numbers being given, to find the first of two Geometrical mean proportional numbers between the extreams given: multiply the square of the lesser extream by the greater, and extract the cube-root of the product, so is such cube root the lesser of the two mean proportionals required: for example, if 8 and 27 are assigned for two extreams, the lesser mean will be found 12, for according to the rule, the square of 8 the lesser extream is 64, which being multiplyed by 27 (the greater extream) produceth 1728, whose cube root is 12 the lesser mean sought, then may the greater mean be found more easily by the *Rule of Three*, for 8 . 12 :: 12 . 18, so that 12 and 18 are two means Geometrically proportional between the extreams 8 and 27, *viz.* these four numbers are in Geometrical proportion continued, to wit, 8, 12 . 18 and 27.

To find the first of three Geometrical mean proportionals between two extream numbers given.

XX. Two numbers being given, to find the first of three Geometrical mean proportionals between the extreams given; multiply the cube of the lesser extream by the greater, and extract the Biquadrate root of the product, so is such Biquadrate root the first (to wit, the least) of the three mean proportionals required: for example, if 2 and 32 are two extreams given, the first and least of three Geometrical mean proportionals will be found to be 4; for (according to the *Rule*) the cube of 2 (the lesser extream given) is 8, which being multiplied by 32 (the *greater extream*) produceth 256, the *Biquadrate root* whereof being extracted (according to the 29 *Rule* of the 33 *Chapter* of the preceding *Treatise*) gives 4 for the first and least of the three means sought, the other means may be easily found by the *Rule of Three* for,

2 . 4 :: 4 . 8 :: 8 . 16 :: 16 . 32

So that these five numbers will appear to be in Geometrical proportion continued, to wit,

2 . 4 . 8 . 16 . 32.

In like manner the first and least of three *Geometrical mean proportionals* between the extreams 100 and 106, will be found to be near 101.4673, &c. Thus have I shewed the most easie wayes (raised from clear grounds) to make *Tables* for the resolution of the usual questions which depend upon the computation of interest, by the help of *Multiplication* onely.

Questions

Questions to exercise the precedent Tables, with their use in solving Questions of the same nature when the number of years exceeds 30.

Quest. 1. If the Lease of a house be worth 153 *l.* *Fine*, and 16 *l.* *yearly rent*, payable yearly for 21 years, and the *Lessee* be desirous to bring down the *Fine* to 50 *l.* and so to pay the more *Rent*, the question is what rent the Tenant shall pay, accompting compound interest at the rate of 10 *per centum, per annum*? *Answer*, 27 *l.* 18 *s.* $1\frac{1}{4}$ *d.* near.

First find the difference between the *Fines* which is 103 *l.* Then after the manner of the examples of the use of the preceding *Table* VII. seek what Annuity or rent to continue 21 years, 103 *l.* ready money will purchase at 10 *per centum*, so will you find 11 *l.* 18 *s.* $1\frac{1}{4}$ *d.* which being added to the old rent 16 *l.* gives 27 *l.* 18 *s.* $1\frac{1}{4}$ *d.* which the Tenant must pay to the end that the *Fine* may be diminished unto 50 *l.*

Quest. 2. There is a *Lease* of certain Lands to be let for 14 years for 250 *l.* *Fine*, and 44 *l.* *Rent per annum*, payable yearly, but the *Tenant* is desirous to pay less *Rent*, *viz.* 20 pounds *per annum*, and to give a greater *Fine*; the question is what *Fine* ought to be paid to bring down the *rent* to 20 *l.* *per annum*, accompting compound interest, at the rate of 6 *per cent. per annum*? *Answer*, 473 *l.* 1 *s.* 7 *d.*

First find the difference between the *Rents*, which will be 24 pounds *per ann.* Then by the help of the preceding *Table* VI. seek what an Annuity or rent of 24 *l.* *per annum*, to contiue 14 years, is worth in ready money at 6 *per centum, per annum*; so will you

find 223 *l.* 1 *s.* 7 *d.* which being added to the first *Fine* 250 pounds, gives 473 *l.* 1 *s*, 7 *d.* which the *Tenant* must pay, to the end the *rent* may be brought down to 20 *l. per annum.*

Quest. 3. There is a *Lease* of certain Lands worth 32 *l, per annum*, more than the *rent* paid to the Lord for it, of which *Lease* seven years are yet in being, and the *Lessee* is desirous to take a *Lease* in *reversion* for 21 years, to begin when his old *Lease* is expired, the question is, what sum of money is to be paid for this *Lease* in *reversion*, accompting compound interest at the rate of 6 *per centum, per annum*? *Answer*, 250 *l.* 7 *s.* 2 *d.*+

First by adding the 7 years of the *Lease* in being, to the 21 years you would have in reversion after those 7 are expired, the sum is 28. Then by the preceding *Table* VI.

The present worth of 1 *l.* Annuity for 28 years at 6 *per centum*, compound interest, is —— ——	13.40616
Likewise the present worth of 1 *l.* Annuity for 7 years is —— ——	5.58233
Therefore, the difference of those present worths, shall be the present value of 1 *l.* Annuity for 21 years in reversion after 7 years ——	7.82383
Which multiplied by 32 (the yearly rent propounded) gives the *Answer* of the question.	250.36256

Otherwise

Otherwiſe thus,

Firſt by the help of the ſaid *Table* VI. find out how much 32 *l.* yearly rent for 21 years is worth in ready money, as if the 21 years were to begin preſently, at the rate of 6 *per centum*, which ready money will be found 376.45024 *l.* Then by *Table* V. find what 376.45024 *l.* due at the end of 7 years to come, is worth in ready money; ſo will it be 250 *l.* 7 *s.* 2 *d.* which agrees with the *Anſwer* before found.

Queſt. 4. One would beſtow 630 *l.* to purchaſe a preſent yearly rent or Annuity of 60 *l.* to be paid by yearly payments, the queſtion is to know how many years the ſaid Annuity muſt continue, compound intereſt at 6 *per centum, per annum*, being allow'd on both ſides. *Anſw.* 17 years, and 23 dayes, very near,

Firſt I divide 630 by 60, ſo the quotient will be 10.5, which ſhews that 10 years purchaſe and an half are given for the Annuity; then ſearching for 10.5 in *Table* VI. in the Column of 6 *per cent.* I find it not exactly, but the neareſt leſs then it, is 10 .47725, ſtanding right againſt 17 years, and the next greater than 10.5 is 10.82760 which is placed againſt 18 years, whence I infer that the Annuity muſt continue 17 years and more, yet leſs then 18 years. Now the proportional part of a year to be added to 17 years, may be found out near enough for uſe, thus, *viz.* ſubtract the ſaid leſſer tabular number 10.47725 from the greater 10,82760, ſo the remainder will be found .35035: Alſo ſubtracting the ſaid 10.47725 from 10.5 (the quoti-

ent

ent first found) the remainder will be .02275; then say by the rule of three in decimals, as .35035 the greater remainder is to .02275 the lesser; so is 1 year (the difference between 17 and 18 years) to .0649 parts of a year, or 23 dayes +) as will appear by the fourth *Rule* of the 26 *Chapter* of the preceding *Book*) therefore the number of years sought by the question is 17 years, 23 dayes.

Quest. 5. If an Annuity of 96 *l.* payable by yearly payments for 14 years be sold for 826 *l.* what rate of interest *per centum*, is implied in that bargain? *Answ.* 7 *l.* 5 *s.* 7½ *d.* near.

First, dividing 826 by 96, the quotient is 8.60416, which shews how many years purchase was given for the Annuity; then searching for 8.60416 in *Table* VI. in a right line passing from 14 years, equidistant to the head line of the *Table*, I find it not exactly, but the nearest less than it, is 8.24423 (which stands in the Column of 8 *per cent.*) and the nearest greater is 8.74546, (which stands in the Column of 7 *per cent.*) whence I infer, that the rate of interest required is between 7 and 8 *per cent.* and the proportional part of 1 *l.* to be added to 7 *l.* may be found out near enough for practice thus, *viz.* subtract the said lesser tabular number 8.24423 from the greater 8.74546, the remainder will be .50123. Also subtract 8.60416 (the quotient first found) from the said greater tabular number 8.74546, so the remainder will be 14130; then say by the *Rule of Three* in decimals, as 50123 the greater remainder, (or difference between the two tabular numbers) is to .14130 the lesser remainder; so is 1 *l.* (the difference between 7 *per cent.* and 8 *per cent.*) to .2819, &c. or 5 *s.* 7 *d.* 2 *f.* which

added

added to 7 *l.* gives 7 *l.* 5 *s.* 7 *d.* 2 *f.* which is near the rate of interest *per cent.* required.

Quest. 6. If a years rent (or one years purchase) be paid as a *Fine*, for renewing or adding 7 years to 14 years yet to come of an old *Lease* for 21 years, and accordingly a new *Lease* be taken for 21 years, to begin presently, (which proportion is ordinarily observed by *Bishops*, *Deans*, and *Chapters*, *Heads* and *Fellows* of *Colledges* in letting *Leases* of their Lands) what rate of interest *per centum* is implied in that Agreement? *Answ.* 11 *l.* 11 *s.* 8 *d.* 1 *f.* and somewhat more.

To solve this Question, first I search in the preceding *Table* VI. to find out two numbers so seated in some one Column of interest, that one of them may stand right against 14 years, and the other against 21 years; and so qualified that the difference between them may be exactly 1 or unity; but not finding any two numbers precisely answering those conditions, I take those numbers that come nearest, which will be found in the Columns of 11 and 12 *per cent.* for the difference between the numbers 6.98186 and 8.07507 which stand in the Column of 11 *per centum*, right against 14 years and 21 years is 1.09321 which exceeds 1 (that is 1 years purchase) by .09321; Also the difference between 6.62816 and 7.56200 which stand in the Column of 12 *per cent.* right against 14 years and 21 years is .93384, which wants .06616 of 1; therefore I divide 1 *l.* (the difference between 11 *l.* and 12 *l.* *per cent.*) into two parts, in such proportion one to the other, as the said decimals .09321 and .06616 are one to the other, so I find the said parts of 1 *l.* to be near .5848 and .4151; or 11 *s.* 8 *d.* 1 *f.* †, and 8 *s.*

3 *d.*

3 *d.*2 *f.*+ ; the former of which being added to 11 *per centum*, or the latter being subtracted from 12 *l. per cent.* gives 11.5848 *l.* or 11 *l.* 11 *s.* 8 *d.* 1 *f.*+, which is very near the rate of interest required by the question.

Quest. 7. What is the present worth of 1 *l. per ann.* payable yearly for 10 years, compound interest being computed at the rate of 11.5848 *l. per cent.* *Answ.* 5 *l.* 15 *s.* 0 *d.* very near, which is found out by the help of the preceding *Table* VI. in this manner, *viz.*

The tabular number for 10 years at 11 *l. per centum* is ————————	5.88923
The tabular number for 10 years at 12 *per centum* is ————————	5.65022
Their difference is ————————	0.23901

Then say by the *Rule of Three* in decimals, as 1 *l.* (the difference between 11 and 12 *per cent.*) is to .5848 *l.* (to wit, the decimal by which the given rate in the question exceeds 11 *per cent.*) so is .23901 (the difference found out as above) to .13977 +, which being subtracted from 5.88923 (the greater of the two tabular numbers above mentioned) there will remain 5.74946, or 5 *l.* 15 *s.* 0 *d.* which is near the present worth of one pound yearly rent to continue 10 years, at the proposed rate of 11.5848 *l. per centum.*

After the same manner the present worth of 1 *l.* yearly rent payable for 21 years, at the same rate of interest, will be found to be 7.77503 *l.*, or 7 *l.* 15 *s.* 6 *d.* very near, from which if you subtract 5.74946 (being the afore-mentioned present worth of 1 *l.* yearly rent for 10 years) there will remain 2.02557

or

or 2 *l*.0 *s*.6 *d*. which is near the present worth of a *Lease* of 1 *l*. rent *per annum*, for 11 years in reversion, to begin after 10 years yet to come in a *Lease* are expired; Hence it is evident, that if a *Tenant* to a Colledge hath 10 years yet to come in a *Lease*, at 1 *l*. rent *per annum*, and desires to have 11 years renewed, or added to those 10, and so take a new *Lease* for 21 years, to begin presently at the same rent, he must give 2 *l*. 0 *s*. 6 *d*. or 2 years purchase and $\frac{1}{40}$ part of a years purchase, very near, (according to the fundamental proportion before assumed in the sixth question.) The like may be done for any other term of years under 30, by the help of the said Table VI.

Concerning the renewing of a Colledge Lease of Lands.

But yet by a *Table* calculated purposely for the said rate of 11.5848 *l*. *per centum*, (according to the fifteenth *Rule* of this *Chapter*) questions of the same kind with the two last, may be more easily answered, and therefore (for that they come often in practice) I shall here insert such a *Table*, as I find it ready calculated to my hand by Doctor *Newton*, in his Scale of Interest lately publish'd, which *Table* is to be used in every respect like to the preceding *Table* VI. and will be very ready and useful, for the proportioning of Fines, in the renewing of *Leases* held from Cathedral Churches and Colledges, as will be manifest by the manner of solving the two following questions.

Quest.

Quest. 8. If a Colledge-Tenant hath 7 years yet to come or unspent in a *Lease* of lands for 21 years, at 1 *l.* yearly rent, and desires to have 14 years renewed or added to those 7 years, and so to take a new *Lease* for 21 years to begin presently, what must he pay for a Fine? *Answ.* 3 *l.* 3 *s.* 0 *d.*

The rule for finding out the answer of the question proposed, and such like, is this; *viz.*

From 7.77507 (being the number which answers to 21 years in this *Table* VIII.) Subtract alwayes the tabular number which belongs to the number of years to come or unspent in the old *Lease*, so the remainder will shew what Fine must be paid for the years to be renewed or added, to make those unspent years in the old *Lease*, to be 21 years compleat again, at 1 *l.* yearly rent.

So to solve the question proposed.

TABLE VIII.

Shewing the present worth of one pound Annuity, for any number of years under 22, at the rate of 11 l. 11 s. 8 d. $1\frac{2}{10}$ f. per centum compound interest.

Years	*present worth*
1	0.90034
2	1.69938
3	2.41922
4	3.06438
5	3.64262
6	4.16088
7	4.62540
8	5.04176
9	5.41496
10	5.74948
11	6.04934
12	6.31819
13	6.55907
14	6.77507
15	6.96868
16	7.14226
17	7.29786
18	7.43737
19	7.56243
20	7.67455
21	7.77507

From

From the preſent worth of 1 *l.* yearly rent for 21 years, which is—	7.77507.
Subtract the preſent worth of the ſame rent for 7 years, (that were unſpent in the old *Leaſe.*)	4.62540
And there will remain the Fine ſought, to wit———	3.14967.

That is to ſay, 3.14967 *l.* or 3 *l.* 3 *s.* 0 *d.* (very near) muſt be paid as a Fine, for renewing or adding 14 years to 7 years, that were unſpent in the old *Leaſe*, the yearly rent being 1 *l.* Alſo the ſaid 3.14967 ſhews that ſuch a renewal is worth 3 years purchaſe, and near $\frac{15}{100}$ parts of a years purchaſe, (whatever the rent be.)

Queſt. 9. If a *Tenant* that hath 17 years yet to come, in a *Leaſe* of lands held of a Colledge for 21 years, at 50 *l.* yearly rent, be deſirous to renew 4 years, and ſo make thoſe 17 years to be 21 years compleat again, at the ſame rent, what muſt he give for a fine? *Anſw.* 23 *l.* 17 *s.* 2 *d.* 1 *f.* For, according to the rule before given,

From the preſent worth of 1 *l.* yearly rent for 21 years ———	7.77507
Subtract the preſent worth of the ſame rent for 17 years (that were unſpent in the old *Leaſe.*)	7.29786
And there will remain ———	0.47721
Which multiplied by the rent ——	50
The Product will be the Fine ſought, to wit, 23 *l.* 17 *ſ.* 2 *d.* 1 *f.*	23|86050

Queſtions

Questions of this nature may be readily solved without the loss of one sixteenth part of a years Purchase by the help of the following Table IX, which I have drawn from the foregoing Table VIII, for the benefit of such as understand not Decimal fractions; for example, if a Colledge-Tenant desireth to have ten years added to eleven years that are yet to come or unspent in a Lease of lands that he may have a new Lease for the term of 21 years to begin presently, he must give for a Fine, one years Purchase and two quarters of a years Purchase, and three quarters of a quarter of a years Purchase, *viz.* one years rent, and half a years rent, and three quarters of a quarter of a years rent: Supposing then the rent to be 48 *l. per annum*, the Fine may be computed thus,

	l.	*s.*	*d.*
One years rent is	48	00	00
Half a years rent is	24	00	00
Three quarters of a quarter of a years rent is	9	00	00
The sum is the Fine required	81	00	00

Whence it appears that the Tenant must give 81 *l.* as a Fine, for adding of 10 years to 11 years that were unexpired in his old Lease, to the end he may have a new Lease for 21 years in being.

In like manner the Fine for renewing or adding seven years to fourteen years that are unspent in a Lease of lands to the end there may be a new Lease for 21 years in being, is valued at one years Purchase precisely, which is the Fundamental proportion assumed in calculating the said Table VIII, as before was said.

TABLE IX.

TABLE IX.

The Fine for renewing or adding	Years		Years		Years Purchase	Quarters of a year	Quarters of a quarter
	1	to	20	is valued at	0	0	1
	2	to	19		0	0	3
	3	to	18		0	1	1
	4	to	17		0	1	3
	5	to	16		0	2	2
	6	to	15	is valued at	0	3	0
	7	to	14		1	0	0
	8	to	13		1	0	3
	9	to	12		1	1	3
	10	to	11		1	2	3
	11	to	10	is valued at	2	0	0
	12	to	9		2	1	1
	13	to	8		2	2	3
	14	to	7		3	0	2
	15	to	6		3	2	1
	16	to	5	is valued at	4	0	2
	17	to	4		4	2	3
	18	to	3		5	1	1
	19	to	2		6	0	1
	20	to	1		6	3	2

The like may be done for renewing any other term of years under 21, at any rent proposed.

Of finding out tabular numbers for any term of years above 30.

But because it may sometimes happen, that the number of years in questions belonging to the preceding 3, 4, 5, 6 and 7 *Tables* may exceed 30, I shall by the five following questions shew how by the help of those *Tables*, the answer to any question of that nature may be found out near the truth, when the term of years is above 30.

Quest. 10. If 340 *l.* be put forth at 4 *per centum* compound interest, and both principal and interest be forborn until the end of 45 years, what will then be due? *Answer,* 1986 *l.* very near.

To resolve this question and the like observe this rule, *viz.* First make choice of any two such numbers that if they be added together will make the number of years proposed in the question, as 17 and 28; or 15 and 30, each of which pairs make 45, then looking into *Table* III. in the Column belonging to 4 *per centum*, you will find right against 17 and 28 years these numbers, 1.94790 and 2.99870 which being multiplyed one by the other will produce 5.84116 + or 5 *l.* 16 *s.* 10 *d.* which shall be the increase of 1 *l*, forborn 45 years at 4 *per centum*, compound interest, therefore multiplying the said 5.84116 by 340, the product will give 1985.994, &c. or 1986 *l.* very near for the Answer of the question.

The reason of the said Rule will be manifest by this Theorem, *viz.* If there be a rank of numbers in Geometrical proportion continued, beginning with

with 1 or unity, as 1, 2, 4, 8, 16, 32, 64, 128, &c. Also if the first term 1 be cast away, and over or under all the rest of the terms there be placed another rank of numbers, beginning at 1 and proceeding according to the natural order of numbers, as 1, 2, 3, 4, 5, 6, 7, &c. which may be called the *Indices* of those in the first rank, after the first term 1 is cast away; I say if any two of those remaining Geometrical proportionals be multiplyed one by the other, the product shall be a proportional, correspondent to that *Index* which is equal to the sum of the *Indices* answering to the two proportionals that were multiplyed one by the other.

Proport.	2 .	4 .	8 .	16 .	32 .	64 .	128
Indices	1 .	2 .	3 .	4 .	5 .	6 .	7

So if 4 and 32, which are the second and fifth proportionals in the upper rank, be multiplyed one by the other, the product is 128, which shall be the seventh proportional, because the sum of the *Indices* 2 and 5, which answer to the said 4 and 32, is 7. In like manner because the sum of the *Indices* 3 and 4 is 7, therefore if the third and fourth proportionals, to wit, 8 and 16, be multiplyed one by the other, the product shall also give the seventh proportional 128. Now forasmuch as the numbers in every one of the Columns, except the first Column of years, in the preceding *Table* III. are continual proportionals whose first term is 1, but 'tis excluded out of the said Columns, as appears by the Construction of that *Table*, and for that the numbers of years 1, 2, 3, 4, 5, &c, are

placed as *Indices* ſhewing the order or ſeat of thoſe proportionals inſerted in the Columns, therefore the rule before given for continuing that Table to any number of years is manifeſt.

Queſt. 11. If one pound be due or payable 50 years hence, what is it worth in ready money, by rebating at 5 *per centum*, *per annum*, compound intereſt? *Anſw.* .08720, &c. or 1 *s.* 9 *d.* + which is found out by the help of *Table* V. in the ſame manner as the Anſwer to the laſt Queſtion; (reſpect being had to the ſecond and third rules of the 26th. Chapter of the preceding Book concerning the multiplication of decimal fractions.)

Queſt. 12. If an Annuity of one pound payable yearly for 40 years, be all forborn until the end of that term, what will it then amount unto, compound intereſt being computed at 5 *per centum*, *per annum*? *Anſw.* 120 *l.* 16 *s.* 0 *d.* thus found out; Firſt, according to the ſecond way of calculating the fourth *Table* in the thirteenth *Section* of this *Chapter*, find out a Principal which may have ſuch proportion to the propoſed Annuity 1 *l.* as 100 *l.* hath to 5, ſaying if 5 *l.* intereſt hath 100 *l.* for a principal, what principal muſt 1 *l.* intereſt have? *Anſwer*, 20 *l.* Secondly, ſeek (after the manner of the preceding tenth queſtion) what 20 *l.* will be augmented unto being forborn 40 years, at the rate of 5 *per centum*, *per annum*, compound intereſt, ſo you will find 140.798 +, from which ſubtracting the ſaid principal 20 *l.* the remainder will be 120.798 +, or 120 *l.* 16 *s.* which is the anſwer of the queſtion.

Queſt. 13. If an Annuity of one pound payable yearly for 37 years, be to be ſold for preſent mo-

ney

ney, what is it worth, compound intereſt being computed on both ſides at 6 *per centum*, *per annum* ? *Anſwer*, 14 *l.* 14 *s.* 9 *d.* which is found out thus: Firſt, according to the ſecond way of calculating the ſixth *Table* in the fifteenth *Section* of this *Chapter*, find out a principal in ſuch proportion to one pound (the propoſed Annuity) as 100 is to 6, ſo will ſuch principal be found 16.66666 +, then after the manner of the preceding eleventh queſtion find out the ready money which is equivalent to 16.66666, due 37 years hence, ſo will ſuch ready money be found to be 1.92988 +(or 1 *l.* 18 *s.* 7 *d.*) which being ſubtracted from the ſaid principal 16.66666, the remainder will be 14.73678 +, or 14 *l.* 14 *s.* 9 *d.* which is the Anſwer of the queſtion propounded.

Queſt 14. What Annuity payable by yearly payments to continue 37 years will one pound purchaſe, at 6 *per centum*, *per annum*, compound intereſt ? *Anſw.* 1 *s.* 4 *d.* near, which is found out thus; Frſt find out the preſent worth of one pound Annuity to continue 37 years, which preſent worth by the laſt queſtion) will be found 14.73678 *l.* Then ſay by the *Rule of Three*, if 14.73678 *l.* will purchaſe an Annuity of one pound, (to continue 37 years) what Annuity to continue the ſame term will 1 *l.* purchaſe ? *Anſw.* .06785 +, or 1 *s.* 4 *d.* which is the anſwer of the queſtion propounded.

CHAP. VI.

A Demonſtration of the Rule of Three, or Rule of Proportion.

I. FOur numbers are ſaid to be proportionals; when the firſt containeth the ſecond, ſo often as the third containeth the fourth; likewiſe when the firſt is ſuch part of the ſecond, as the third is of the fourth: So theſe numbers following are called proportionals, *viz.*

$$4 \times 6 \;.\; 6 :: 4 \times 9 \;.\; 9$$
$$\tfrac{2}{3} \times 12 \;.\; 12 :: \tfrac{2}{3} \times 15 \;.\; 15$$

That is to ſay, 4 times 6 (or 24) is ſaid to have ſuch proportion to 6; as 4 times 9 (or 36) hath to 9. In like manner, $\frac{2}{3}$ of 12 (or 8) hath ſuch proportion to 12; as $\frac{2}{3}$ of 15 (or 10) hath to 15.

II. When four numbers are proportionals, the product ariſing from the multiplication of the two extreams is equal to the product of the two means.

Demonſtration.

By the preceding Definition in 1. theſe four numbers are proportionals, *viz.*

$$\left\{\begin{array}{l} 4 \times 6 \;.\; 6 :: 4 \times 9 \;.\; 9 \\ B \times C \;.\; C :: B \times D \;.\; D \end{array}\right.$$

The

The product of the two extreams is —— $\left.\right\}$ $4 \times 6 \times 9$ / $B \times C \times D$

The product of the two means is —— $\left.\right\}$ $6 \times 4 \times 9$ / $C \times B \times D$

But $\left\{ \begin{matrix} 4 \times 6 \times 9 \\ B \times C \times D \end{matrix} \right\} = \left\{ \begin{matrix} 6 \times 4 \times 9 \\ C \times B \times D \end{matrix} \right.$

Therefore the Prop. is manifest.

Likewise.

By the preceding definition these four numbers are proportionals, *viz.*

$$\tfrac{2}{3} \times 12 \,.\, 12 :: \tfrac{2}{3} \times 15 \,.\, 15$$

The product of the two extreams is —— $\left.\right\}$ $\tfrac{2}{3} \times 12 \times 15$

The product of the two means is —— $\left.\right\}$ $12 \times \tfrac{2}{3} \times 15$

But —— $\tfrac{2}{3} \times 12 \times 15 = 12 \times \tfrac{2}{3} \times 15.$

Wherefore the proposition is every way proved.

III. From the last proposition ariseth the *Rule of Proportion*, commonly called the *Rule of Three*, or *Golden Rule*, which teacheth by three numbers given to find a fourth proportional number, in this manner, *viz. Multiply the second and third numbers mutually one by the other, and divide the product by the first number; so the quotient shall be the fourth proportional number sought, in a direct proportion.* This Rule hath been fully exemplified in the 8th. Chapter of the preceding Book, and the truth of the

ſaid Rule may be thus demonſtrated, *viz.* Let there be three numbers given to find a fourth in direct proportion, *viz.* if 24 give 6, what ſhall 36 give? Or as 24 is in proportion to 6, ſo is 36 to a fourth proportional number ſought, which fourth proportional, (whatſoever it be) we may ſuppoſe to be Q, and then theſe four numbers will be proportionals, *viz.*

$$24 \,.\, 6 :: 36 \,.\, Q$$

Therefore by the ſecond propoſition of this Chapter.

$$24 \times Q = 6 \times 36$$

And becauſe if equal plane numbers be ſeverally divided by one and the ſame number, the quotients will neceſſarily be equal between themſelves, therefore

$$Q = \frac{6 \times 36}{24}$$

Whereby it is manifeſt that the fourth proportional number is equal to the quotient that ariſeth by dividing the product of the multiplication of the ſecond and third proportionals by the firſt, which was to be proved.

Note, that every *Rule of Three inverſe* may be made a *Rule of Three direct*, by making the third term the firſt, and by proceeding forward to the other two terms; therefore one and the ſame demonſtation ſerveth for both rules.

CHAP.

CHAP. VII.

A Demonstration of the Double Rule of Fellowship.

THe *Double Rule of Fellowship*, (commonly called the *Rule of Fellowship with time*) presupposeth two things, *viz.* 1. That the particular Stocks of Merchants in company, have continued unequal spaces of time in the common Stock. 2. That at the end of their Partnership, the total gain or loss is to be divided amongst them, in such manner, that their shares shall have such proportion between themselves, as those sums of interest money have one to another, which at any rate *per centum*, simple interest onely being computed, might be gained by the particular Stocks, within the respective times of their continuance in the common Stock: Now for the effecting of such a proportional partition, the said *Double Rule of Fellowship* gives this direction; *viz.* Divide the total gain or loss into such parts, which shall have the same proportion one to the other, as is between the products arising out of the multiplication of each particular Stock by its correspondent time.

For Example, suppose two Merchants *A* and *B* to be partners in Traffick, for a certain time first agreed

agreed on between them, and that *A* doth permit his Stock of 100 *l.* to be employed in their joynt Traffick three moneths, and that *B* forbears his Stock of 50 *l.* eight moneths; I say (according to the said *Rule of Fellowship* with time) what ever the total gain or loss be, that part thereof which belongs to *A* must have such proportion to the gain or loss of *B* as 100 × 3 (or 300) hath to 50 × 8 (or 400.) This rule hath been fully exemplified in the 13 Chapter of the preceding Book, and the truth thereof, taking the two premised Suppositions for granted, may be thus demonstrated.

1. Supposing 100 *l.* (the stock of A.) to gain in 3 moneths any certain sum of money, as two pounds; I seek how much 50 *l.* (the stock of B) will gain in the same time, and at the said rate, so I find $\frac{2 \times 50}{100}$ *l.* for,

$$100 \,.\, 2 :: 50 \,.\, \frac{2 \times 50}{100}$$

2. Having found what 50 *l.* will gain in 3 moneths, I seek how much the said 50 *l.* will gain in 8 moneths, at the same rate, and so I find $\frac{2 \times 50 \times 8}{100 \times 3}$ *l.* for,

$$\frac{3}{1} \,.\, \frac{2 \times 50}{100} :: \frac{8}{1} \,.\, \frac{2 \times 50 \times 8}{100 \times 3}$$

3. Thus it appears, that if 100 *l.* in 3 moneths doth gain 2 *l.* then 50 *l.* in 8 moneths will gain as the

the ſame rate $\frac{2 \times 50 \times 8}{100 \times 3}$; ſo that the proportion of the gain of A to the gain of B is.

$$As\ 2\ is\ to\ \frac{2 \times 50 \times 8}{100 \times 3}$$

4. If both the terms (to wit, the *Antecedent* and *Conſequent*) of the ſaid proportion be ſeverally multiplied by the ſaid *Denominator* 100 × 3, the products will be in the ſame proportion with the numbers or terms multiplied, (by 17 è 7. *Euclid*) *viz.* the gain of A will be to the gain of B.

As 2 × 100 × 3 is to 2 × 50 × 8

5. Laſtly, becauſe 2 (the ſuppoſititious gain firſt aſſumed) is a Multiplicator as well in the *Antecedent* as in the *Conſequent* of the laſt mentioned proportion, it may be expung'd out of both, and ſo the gain of A. will be to the gain of B, in this proportion (which was to be proved) to wit,

As 100 × 3 is to 50 × 8

CHAP.

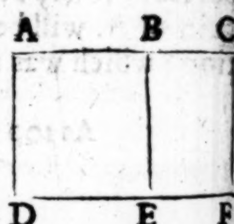
CHAP. VIII.

A Demonstration of the Rule of Alligation alternate, and the use of the said Rule in the Composition of Medicines.

I. IN order to the Demonstration of the said Rule, I shall premise this *Lemma, viz.* if the difference of any two numbers given, be multiplied by a number assigned, the product will be equal to the difference between the products which arise from the multiplication of those two numbers severally by the numbers assigned.

Suppositions.

Two lines or numbers given: AC = 10, BC = 4
Their difference AB = 10—4
A multiplicator assigned: AD = 5



Which suppositions, and the *Diagram* being well viewed, the truth of the said *Lemma* will be evident, *viz.*

$$\underline{AB} \times AD = \underline{AC \times AD}, - \underline{BC \times BE}\ (AD)$$
$$10 - 4 \times 5 = 10 \times 5, - 4 \times 5.$$

II. To

II. To add the more light to the following *Demonstration* of the rule of *Alligation* alternate; I shall propound a question which properly belongs to the said rule, *viz.* Suppose a *Vintner* having *French-wines* at 5 *d.* the *quart*, and at 10 *d.* the *quart*, would make a mixture of them in such manner, that he might sell the mixt quantity at 7 *d.* the *quart*, and so make as much money of the mixture, as if he should sell each quantity of *wine* at its own price; the question is to know what proportion the quantities of both sorts of *wine* in the mixture must bear one to another. Here according to the *Rule of Alligation alternate*, I take the differences between the mean price assigned for the mixture, and the two other given prices, and place those differences alternately, *viz.* the difference between 7 and 10 being 3, I write 3 against 5, likewise 2 being the difference between 7 and 5, I write 2 against 10; so I conclude, that the quantity to be taken of that sort of Wine of 10 *d.* the *quart*, must have such proportion to the quantity of 5 *d.* the *quart*, as 2 to 3. That is to say, if 2 *quarts* at 10 *d.* the *quart* be mixed with 3 *quarts* at 5 *d.* the *quart*, the total mixture 5 *quarts* being sold at 7 *d.* the *quart*, will yield as much money as the said 3 *quarts* at 5 *d.* the *quart*, together with the said 2 *quarts* at 10 *d.* the *quart*; as is evident by the subsequent work.

$$7\left\{\begin{array}{c|c} 10 & 2 \\ 5 & 3 \\ \hline & 5 \end{array}\right.$$

quarts

	quarts		pence		quarts		pence
I.	1.	.	5	::	3	.	15
II.	1	.	10	::	2	.	20
III.	15	+	20	=	7 × 5	=	35.

From the premiſes it appears, that when two things are given to be mixt in ſuch manner as the *Rule* of *Alligation alternate* requires, the propoſition to be demonſtrated will be this, namely,

Three numbers A. B. C. being given in ſuch ſort, that A. is leſs then B. but greater then C. if the difference between A. and B. be multiplied by C. and the difference between A. and C. be multiplied by B. the ſum of thoſe products will be equal to the product ariſing from the multiplication of A. by the ſum of the ſaid differences,

Demonſtration.

mean price	extream prices	alternate differences	Products
A	B	A—C	BA—BC
	C	B—A	CB—CA
		B—C	BA—CA = $\overline{B—C}$ × A

The difference between B. and A. is B—A. which multiplied by C produceth (as is evident by the *Lemma*

Lemma aforegoing in the first Section of this Chapter) CB—CA. Also the difference between A and C is A—C. which multiplied by B produceth BA—BC. Then the sum of those two products is BA—CA, (for + CB and — CB expunge one the other) which sum is manifestly the same with the product arising from the multiplication of A the mean price, by B—C the sum of the aforesaid differences, (to wit, the sum of A—C and B—A) for + A and—A expunge one another.

When more then two things of different prices are given to be mixt as aforesaid, the *Demonstration* will not be otherwise, for if the sum of every two products arising from the multiplication of two alternate differences by their respective prices, be equal to the product of the mean price multiplied by the sum of the said differences; the sum of all the said products will also be equal to the product of the mean price multiplied by the sum of all the differences; as will clearly appear by view of the subsequent work.

Products of alternate differences multiplied by their respective prices.

Products of the mean price multiplied by the several sums of alternate differences.

$$\begin{array}{lcl}
\text{If} & D + E = F & \times\ G \\
\text{and} & H + K = F & \times\ M \\
\text{Then } D + E + H + K = F & & \times\ \overline{G + M}
\end{array}$$

More-

Moreover, because if equal numbers be severally divided by one and the same number, the quotients will be equal between themselves, therefore from the premises, this *Corollary* will arise.

COROLLARY.

In the *Rule of Alligation alternate*, if the aggregate of the products arising from the multiplication of the several alternate differences by their respective prices, be divided by the sum of the said differences, the quotient will be equal to the mean price. This may be a proof of any example of the said *Rule* of *Alligation*.

OF THE COMPOSITION OF MEDICINES.

> *See more of this in Mr.* J. Dee *his Mathematical preface, also* Tom. 2 *of* P. Herigon *and Master* Mores *Arithmetick.*

I. Medicines and Simples in respect of their qualities are considered in some of these five wayes, *viz.* either as they are hot or cold, moist or dry, or as they are temperate; so that such Simples or Medicines which work heat in our bodies are said to be hot, such cold which are the cause of coldness, &c.

II. The mean or middle between the extream qualities of *Heat* and *Coldness*; also between *Dryness* and *Moisture*, is called *Temperate* or the *Temperature*;

perature;

perature; from which, each of the said qualities *hot*, *cold*, *moist* and *dry*, doth differ in four *degrees*, so that a Medicine or Simple is said to be either *temperate* or else *hot*, *cold*, *moist*, or *dry*, in the first, second, third or fourth degree.

III. If the numbers 1,2,3,4,5,6,7,8,9, be placed as you see from A to B, the differences betwee 5 (the middle number) and the superiour numbers 6,7,8,9, will be 1,2,3,4, which may represent the 4 degrees of the qualities hot and dry, likewise the differences between 5 and the inferiour numbers 4,3,2,1, will be 1,2,3,4, which may represent the 4 degrees of the qualities cold and moist, the temperature represented by 0, being the mean or middle from whence the said degrees do swerve.

	Ind.	*Degr.*	
B	9	4	*Qualities hot and dry.*
	8	3	
	7	2	
	6	1	
	5	0	*Temperature.*
	4	1	*Qualities cold and moist.*
	3	2	
	2	3	
A	1	4	
	Ind.	*Degr.*	

IV. Since the *Rule of Alligation alternate* requires, that of two things miscible, the one must exceed the

mean propounded and the other be less, therefore the questions of *Alligation* in this kind are to be wrought with the numbers in the aforesaid Column A B, for by them the degrees and qualities are discovered, being placed as you see in the Column adjacent to A B, and for distinction sake, those numbers in the said Column A B, may be called the *Indices* or *Exponents* of the *degrees*, which *Indices* are to be used in the same manner as the prices of Merchandizes in the questions of *Alligation alternate* in Chapter 14 of the preceding Book; and therefore those examples may be compared with these.

Prop. I.

Having divers Simples whose qualities are known, to make a composition or mixture of them, in such manner that the quality of the Medicine may be some mean amongst the qualities of the simples, and the quantity thereof any quantity assigned.

Example 1. An *Apothecary* hath four sorts of Simples, A, B, C, D, whose qualities are as followeth, *viz.* A is hot in the fourth degree, B is hot in the second, C is temperate, and D is cold in the third degree; the question is to know what quantities of each ought to be taken, to make a Medicine, whose quantity may be 12 ounces, and the quality in the first degree of heat? Seek in the aforesaid column A B, for the *Indices* or *exponents* of the qualities of the Simples given, *viz.* for A which is hot in the fourth degree, take 9, for B which is hot in the second, take 7; for C which

which is temperate, take 5 ; and for D which is cold in the third degree, take 2 ; that done, rank those numbers in the same manner as the prizes of Merchandizes in the questions of the 14 Chapte , viz. descend from the highest degree of heat un o the temperature, and so proceed downwards to t e degrees of cold, setting 6 the *Index* or *exponent* of the mean quality propounded, which is 1 degree of heat, as common to them all: then by crooked lines or otherwise, connect two such *Indices* whereof one may be greater than the mean, and the other less, and proceeding according to the *Rule* of t e fourteenth *Chapter* you will find that to make a Medicine of 9 ounces, and the quality resulting to be in the first degree of heat, you must take 1 ounce of A (being that Simple which was hot i 4) 4 ounces of B, 3 ounces of C, and 1 ounc of D, as will be manifest by the proof:

The Proof.

degr.		*oun.*	*simp.*	
6	9	1	A	$9 \times 1 = 9$
	7	4	B	$7 \times 4 = 28$
	5	3	C	$5 \times 3 = 15$
	2	1	D	$2 \times 1 = 2$
		9		$9)54(6$

Lastly, by the *Rule of Proportion* you may increase the *Medicine* to the quantity of 12 ounces, and yet the quality to continue in the first degree of heat, according to the following operation.

ounc.		ounc.		ounc.		ounc.	
9	.	1	::	12	.	$1\frac{1}{3}$	of A
9	.	4	::	12	.	$5\frac{1}{3}$	of B
9	.	3	::	12	.	4	of C
9	.	1	::	12	.	$1\frac{1}{3}$	of D

The quantity assigned 12 *ounces.*

By other connexions of the qualities, other quantities of each *Simple* would arise, but that hath been sufficiently manifested in the questions of the fourteenth Chapter.

Example 2. Suppose there are five *Simples*, *A*, *B*, *C*, *D*, *E*, whose qualities are as followeth *viz.* *A* is hot in 3°. *B* is hot in 2°. *C* is hot in 1°. *D* is cold in 1°. *E* is cold in 3°. and it is required to mix four ounces of *B*, with such quantities of the rest, that the quality of the *Medicine* may be temperate?

	Degr.		*Oun.*	*Simp.*	*The proof.*
	8	1	1	*A*	8 × 1 = 8
	7	3	3	*B*	7 × 3 = 21
5	6	1	1	*C*	6 × 1 = 6
	4	3 + 1	4	*D*	4 × 4 = 16
	2	2	2	*E*	2 × 2 = 4
			11		11) 55 (5

Proceed

Proceed as before, so will you find that to make a *Medicine* of 11 ounces, and the quality of the *Form* resulting to be temperate, you must take 1 ounce of *A*, 3 ounces of *B*, 1 ounce of *C*, 4 ounces of *D*, and 2 ounces of *E*; then since the quantity of *B*, in the composition propounded is limited; *viz.* 4 ounces, find numbers which may be in such proportion to 4 (the quantity of *B* assigned) as the numbers 1, 1, 4, 2, (the quantities of *A*, *C*, *D*, *E*, in the aforesaid Composition of 11 ounces) are unto 3, (the quantity of *B* in the said Composition) in manner following:

ounc.		ounc.			ounc.		ounc.		
3	.	1	::	4	.	$1\frac{1}{3}$	of *A*.		
3	.	1	::	4	.	$1\frac{1}{3}$	of *C*.	*to be mixed with*	
3	.	4	::	4	.	$5\frac{1}{3}$	of *D*.	*4 ounces of B.*	
3	.	2	::	4	.	$2\frac{2}{3}$	of *E*.		

Prop. H.

A *Medicine* being compounded of divers *Simples* whose qualities and quantities are known, to find the degree of the *Form resulting*, *viz.* the exact temperament of the *Medicine*.

Example 1. Suppose a *Medicine* to be compounded of two *Simples*, *viz.* 6 ounces of *B* hot in 4°. and 3 ounces of *C* hot in 3°. and it is required to find the temperament of the *Medicine*, *viz.* the degree and quality resulting from such mixture? Seek in the aforesaid Column *A B* for the *Indices*

of the respective degrees and qualities of the *Simples* given, and dispose them orderly in ranks right against their respective quantities, then multiply each *Index* by its respective quantity, and divide the sum of the products by the sum of the quantities, so will the Quotient be the Index of the degree and quality of the Medicine.

Index	Quant.		Prod.
$9 \times$	6	$=$	54
$8 \times$	3	$=$	24

$$9)\ 78\ (8\tfrac{2}{3}$$

So in the said example, the Quotient will be found $8\frac{2}{3}$, which is the *Index* of $3\frac{2}{3}$ degrees of heat, and therefore the said *Medicine* is hot in $3\frac{2}{3}$ degrees.

Forasmuch as any two quantities miscible according to the *rule of Alligation alternate*, are in such proportion one to the other, as the respective alternate differences between the mean quality of the mixture and the qualities correspondent unto the said quantities, the demonstration of the aforesaid *rule* will be manifest by the *Corollary* aforegoing in this Chapter.

Example 2. Suppose a *Medicine* to be compounded of 4 *Simples*, whose qualities and quantities are known, *viz.* 2 ounces of *A* hot in 3°, 3 ounces of *B* hot in 2°, 4 ounces of *C* temperate, and 5 ounces of *D* cold in 4°, and let it be required to find

find the mean quality resulting from such mixture. According to the aforesaid *rule*, I multiply each *Index* by its respective quantity, and divide the sum of the products by the sum of the quantities, so the quotient is $4\frac{3}{7}$, which is the *Index* of $\frac{4}{7}$ degrees of cold, (for the difference between 5 the *Index* of the temperature, and $4\frac{3}{7}$ the *Index* found, is $\frac{4}{7}$ degrees of cold) which is the quality of the said *Medicine*.

Ind.		*Ounc.*		*Prod.*
8	×	2	=	16
7	×	3	=	21
5	×	4	=	20
1	×	5	=	5
		14)		62 ($4\frac{3}{7}$

Example 3. Suppose a *Medicine* to be compounded of several *Simples*, whose qualities and quantities are as followeth, *viz.* 4 ounces of a *Simple* which is cold in 2°. and moist in 1°. 5 ounces hot in 3°. and (in respect of dryness and moisture) temperate; 3 ounces hot in 2°. and dry in 2°. 6 ounces hot in 1°. and moist in 4°. 4 ounces cold in 3°. and moist in 2°. the question is to know the temper resulting?

In the resolution of this question there must be two distinct operations, each of them like to that in the last example, *viz.*

 1. Find

1. Find in the same manner as before, the degree and quality resulting from the commixture of the qualities hot and cold, so will you find $5\frac{7}{22}$ which is the Index of $\frac{7}{22}$ degrees of heat (for the difference between 5 the Index of the temperature and $5\frac{7}{22}$ the Index found, is $\frac{7}{22}$ degrees of heat.)

Ind.		*Oun.*		*Prod.*	*Ind.*		*Oun.*		*Prod.*
3	×	4	=	12	4	×	4	=	16
8	×	5	=	40	5	×	5	=	25
7	×	3	=	21	7	×	3	=	21
6	×	6	=	36	1	×	6	=	6
2	×	4	=	8	3	×	4	=	12
		22)		117 ($5\frac{7}{22}$			22)		80 ($3\frac{7}{11}$

2. Find in the same manner, the temper resulting from the mixture of the qualities dry and moist; so will you find $3\frac{7}{11}$ which is the Index of $1\frac{4}{11}$ degree of moisture; so the quality of the said Medicine is $\frac{7}{22}$ degree of heat, and $1\frac{4}{11}$ degree of moisture, as by the operation is manifest.

Prop. III.

To augment or diminish a Medicine in quality according to any degree assigned.

Suppose a Medicine to be compounded as followeth, *viz.* 1 dram of a Simple hot in 4°. 2 drams hot in 3°. 2 drams hot in 2°. 1 dram hot in 1°. 1 dram

dram cold in 1°. and 1 dram cold in 2°. Then will the quality of the said Medicine be in 1½ degree of heat, (as will be manifest by the second Proposition.) Now let it be required to augment the said Medicine in quality, *viz.* to add such a quantity of some one of the Ingredients, (or some other simple) which may raise the quality of the Medicine ½ degree; so that the temperament of the Medicine after it is increased in quantity, may be in 2°. of heat. Make choice of such a simple, the Index of whose quality may exceed the Index of the quality assigned, *viz.* make choice of that simple which is hot in 3°. whose index is 8, then proceed according to the 1 example of the first Proposition; so will you find that if 1 dram of the aforesaid Medicine be mixed with ½ dram of that simple which is hot in 3°. the temper resulting from such mixture will be in 2°. of heat.

Lastly, by the *Rule of Three*, say, if 1 dram require ½ dram, what shall 8 drams (the quantity of the Medicine first given) require?

Answ. 4. drams: So that if 4 drams of a simple which is hot in 3°. be mixed with 8 drams of a Medicine which is hot in 1½ degree, the temper resulting will be in 2°. of heat, as by the operation is manifest.

Ind.

	Ind.	Drams
	$6\frac{1}{2}$	1
7	8	$\frac{1}{2}$

$1 \cdot \frac{1}{2} :: 8 \cdot 4$

The proof.

Ind.		Dra.		Prod.
$6\frac{1}{2}$	×	8	=	52
8	×	4	=	32

$12\frac{1}{2}$) 84 (7

If it be required to diminish a Medicine in quality, you are to make choice of such a Simple, the Index of whose quality may be less than the Index of the quality assigned, and then to proceed as before.

Here observe, that if in questions of this nature, the quantities of the Simples be exprest by weights of divers denominations, they are to be reduced to that weight which is of the lowest denomination in the question, according to the sixth *rule* of the seventh *chapter* of the preceding *book*.

The augmenting or diminishing of a Medicine in respect of quantity; Also the finding of the value of any quantity of a Medicine, the prices of the ingredients being known, will be familiar to such as understand the Rule of Proportion, and therefore I shall not insist upon them.

CHAP.

CHAP. IX.

A Demonstration of the common Rule of False by two Positions.

I. WHat the ordinary *double Rule of False* is, and how to be used in resolving such questions which cannot be readily applied to any of the other rules of *Arithmetick*, hath been fully declared in the 15 and 31 Chapters of the preceding book; it remaineth to shew what kind of operation is presupposed before the said Rule can be applied to the resolution of a question, and then to demonstrate the truth of the Rule it self.

II. In the said *Rule of False*, look what operation the question requires to be performed with the number sought and some given number or numbers; the same kind of operation in every respect is to be made with each of the two feigned numbers, (commonly called Positions) and the said given number or numbers, which threefold process being finisht, (whether it be by any one, or all of these *rules*, to wit, *Addition*, *Subtraction*, *Multiplication*, and *Division*) there will arise three remarkable numbers or *results*, to wit, one resulting from the true number sought, and two others resulting from the

the two feigned numbers; then from these three results, the errors are collected, which are nothing else but the differences between the true result, and each of the two false results.

III. After the said errors or differences are discovered, the *Rule of False* will be of no force unless this Analogy or proportionality doth arise, namely, the first error must have the same proportion to the second; as the difference between the number sought and the first feigned number hath to the difference between the said number sought and the second feigned number; here therefore it may be demanded, what kind of operation will produce the said Analogy? To this I answer, when the question requires the number sought to be increased, lessened, multiplied or divided by some given number, or the number arising from such operation to be increased, lessened, multiplied or divided by some given number; in any of those cases, the aforesaid Analogy will necessarily arise, as I shall here manifest in all the said cases. First, therefore I say, when unto each of three numbers (namely, the number sought by the *Rule of False* and the two feigned numbers) one and the same number is added, the said Analogy will ensue, for in this case the difference between the first sum and the second, will be equal to the difference between the first and second of the said three numbers; likewise the difference between the first sum and the third, will be equal to the difference between the first number and the third, which may be proved in manner following:

Suppositions.

Suppositions.

Let there be three numbers, to wit,

A	B	C
12	7	5

Suppose also that the first number A is greater then either of the numbers B and C.

Suppose also, some number as D (3) to be added to each of the said three numbers, so will the three sums be,

A + D	15
B + D	10
C + D	8

The Proposition to be demonstrated is, that the difference between the first sum and the second is equal to the difference between the first number and the second; also that the difference between the first sum and the third is equal to the difference between the first number and the third.

Demonstration.

The difference between the first number and the second is,

$$A - B$$

The difference between the first sum and the second is,

$$A + D - B - D$$

But

But the latter difference is manifestly equal to the former, (for + D and — D expunge one the other) to wit,

$$A + D - B - D = A - B$$

Therefore the first part of the proposition is proved.

Again, the difference between the first number and the third is,

$$A - C$$

The difference between the first sum and the third is,

$$A + D - C - D$$

But the latter difference is manifestly equal to the former, (for + D and — D expunge one the other) wit.

$$A + D - C - D = A - C$$

Wherefore the proposition is fully proved.

The like property might be proved after the same manner when one and the same number is subtracted from three numbers severally.

Secondly, when three numbers namely the number sought by the *rule of False* and the two feigned numbers) are severally multiplied by one and the same number ; the aforementioned Analogy will likewise ensue, as may be thus proved.

Suppositions.

Let there be three numbers, to wit,

$$A \,.\, B \,.\, C$$
$$2 \,.\, 5 \,.\, 8$$

Sup-

Suppose also that the first number *A* is less than either of the numbers *B* and *C*.

Suppose also, each of those three numbers to be multiplied by one and the same number as *D* (4) and the three products to be these,

DA	12
DB	20
DC	32

The Proposition to be demonstrated is, that the difference between the first product and the second, hath such proportion to the difference between the first product and the third, as the difference between the first number and the second, hath to the difference between the first number and the third, *viz.*

DB — *DA* . *DC* — *DA* : : *B* — *A* . *C* — *A*
8 . 20 : : 2 . 5

Demonstration.

Forasmuch as (by the 17th Prop. of the seventh book of *Euclids Elem.*) If a number (*D*) multiplying two number (*B*—*A* and *C*—*A*) produceth other numbers (*DB*—*DA* and *DC*—*DA*) the numbers produced by the multiplication shall be in the same proportion as the numbers multiplied are, therefore

DB—*DA* . *DC*—*DA* : : *B*—*A* . *C*—*A*

which was to be demonstrated.

Likewise when 3 numbers are divided by one and the same number the *demonstration* will not be otherwise;

otherwise; and because by the second *Section* of this *Chapter*, the errors in the *rule of False* are the differences between the true result and the two false results, therefore from the precedent *demonstrations* it is evident, that the afore-mentioned Analogy or proportionality (namely, when the first error hath such proportion to the second; as the difference between the number sought and the first feigned number, hath to the difference between the said number sought and the second feigned number) will succeed from such operation as is before declared in the beginning of the third *Section* of this *Chapter*.

To know whether a question be resolvable by the Rule of False or not.

IV. Now to discern what kind of operation will not produce the said Analogy, observe this note, *viz.* when a question requires some given number to be divided by the number sought or any part thereof, also when the number sought or some part thereof is to be squared, cubed, &c. Likewise when some parts of the number sought are to be multiplied one by the other; I say from such operations the afore-mentioned Analogy will not arise, and in those cases, the *ordinary rule of False* will be useless; as may partly appear by the two following examples, viz. *What number is that by which if* 360 *be divided the quotient will be* 24? Here if two positions or feigned numbers be taken, and 360 be divided by each of them, the errors will not be in the same proportion with the differences between the true number sought and the two feigned numbers, and therfore the *rule of False* will be used in vain: yet if it be asked what number is that which being multiplied by

by 24, the product will be 360, the *answer* to this latter question is the same with the answer to the former, and may be found by the *rule of False*, but such kind of interpretations and inferences are not alwaies obvious, and therefore since the preparative work of the *rule of False*, (after a number is taken by guess for the number sought) proceeds gradually from one condition in the question to another, it will for the most part be easie to determine whether the ordinary *rule of False* will take place or not, by comparing the conditions of a question with the note before given.

Another Example; a certain person being demanded what number of years he had lived, answered, if $\frac{1}{10}$ of that number were multiplied by $\frac{1}{4}$ of the same number, the product would shew the number, or his age: here it will be in vain to search the number sought (which is 40) by the *rule of False*, for the aforementioned Analogy or proportionality will not succeed, and the question cannot easily be resolved without *Algebra*.

Now from this supposition, that after the preparative work of the *rule of False* is finisht, the errors will be in such proportion as aforesaid, I shall make it manifest that the *Rule of False* will discover the number sought.

V. In the Rule of two false Positions there are 3 cases, *viz.* the errors are either both excesses and noted with +, or else both defects and noted with —, or lastly one of the errors is noted with +, and the other with —.

In the two first cases, the Rule is this, Multiply the Positions or feigned numbers by the altern errors; *viz.* the first Position by the second error,

the second Position by the first error, and reserve those Products; then dividing the difference of the said products by the difference of the said errors, the quotient shall be the number sought by question.

The demonstration of the said Rule here followeth.

Case I. *When the errors are both excesses and noted with* †.

Suppositions.

1. Let some number unknown and sought by the *rule of False* be represented by .... } A
2. Let the first Position (or feigned number) be ..... } B
3. And the second feigned number .... , C
4. Suppose also that B is greater then C, and each of them greater then A.
5. Moreover suppose the error of the first position to be .... } F
6. And the error of the second position to be . } G
7. Suppose also that this Analogy will be found in the said numbers, *viz.*

$$B-A \,.\, C-A :: F \,.\, G.$$

8. The proposition to be demonstrated.

$$A = \frac{FC-GB}{F-G}$$

Demon-

Demonstration.

9. Forasmuch as by supposition in 7°.

$$B-A \,.\, C-A :: F \,.\, G$$

10. Therefore by comparing the rectangle of the extreams to the rectangle of the means,

$$GB-GA = FC-FA$$

11. And by equal addition of FA.

$$FA + GB-GA = FC$$

12. Again, forasmuch as by supposition in 4°.

$$B > C$$

13. And consequently out of 4°. and 12°.

$$B-A > C-A$$

14. Therefore out of 9°. and 13°.

$$F > G$$

15. Therefore

$$FA > GA$$

16. Therefore

$$FA-GA > 0$$

17. Therefore by equal subtraction of GB from the equation in 11*.

$$FA - GA = FC - GB$$

18. Wherefore by dividing both parts of the last equation by F—G, equal quotients will arise, *viz.*

$$A = \frac{FC - GB}{F - G}$$

which was to be demonstrated.

Case II. *When the errors are both defects, and noted with* —

Suppositions.

1. Let some number unknown and sought by the *rule of False* be represented by . . . . . } A
2. Let the first position (or feigned number) be . . . . . . } B
3. And the second position, ... C
4. Suppose also that B is less then C, and each of them less then A.
5. Moreover, suppose the error of the first Position to be . . . . . } F
6. And the error of the second Position . G
7. Suppose also that this Analogy will be found in the said numbers, *viz.*

$$A - B \;.\; A - C :: F \;.\; G$$

8. The

8. The Proposition to be demonstrated.

$$A = \frac{FC - GB}{F - G}$$

Demonstration.

9. Forasmuch as by supposition in 7°.

$$A - B \,.\, A - C :: F \,.\, G$$

10. Therefore by comparing the rectangle of the means to the rectangle of the extreams.

$$FA - FC = GA - GB$$

11. And by equal addition of FC

$$FA = FC + GA - GB$$

12. Again, forasmuch as by supposition in 4°.

$$B < C$$

13. And consequently out of 4°. and 12°.

$$A - B > A - C$$

14. Therefore out of 9°. and 13°.

$$F > G$$

15. Therefore

$$FA > GA$$

16. Therefore

$$FA - GA > 0$$

17. Therefore by equal subtraction of GA from the equation in 11°.

$$FA - GA = FC - GB$$

18. Wherefore by dividing both parts of the last equation by F—G, equal quotients will arise, *viz.*

$$A = \frac{FC - GB}{F - G}$$

which was to be demonstrated.

Case III. *When one of the errors is an excess (to wit, noted by +) and the other a defect) (noted by —)*

In this third Case the *Rule of False* is this, *viz.*

Multiply the Positions by the altern errors, to wit the first Position by the second error, also the second Position by the first error, and reserve those products; then dividing the sum of the said products by the sum of the said errors, the quotient shall be the number sought by the question.

The Demonstration of this latter Rule here followeth,

Suppositions,

1. Let some number unknown and sought by the *rule of False* be represented by . . . . . } A

2. Let the first Position be . . . . . B

3. And

3. And the second Position C

4. Suppose also that B is greater then C, and also greater then A, and that C is less then A.

5. Moreover, suppose the error of the first Position to be . . . . . . } F

6. And the error of the second Position to be . G

7. Suppose also that this Analogy will be found in the said numbers, *viz*.

$$B-A \,.\, A-C :: F \,.\, G$$

8. The Proposition to be demonstrated.

$$A = \frac{GB + FC}{F + G}$$

Demonstration.

9. Forasmuch as by supposition in 7°.

$$B-A \,.\, A-C :: F \,.\, G$$

10. Therefore by comparing the rectangle of the means to the rectangle of the extreams.

$$FA - FC = GB - GA$$

11. And by equal addition of FC and GA to the last equation, this will arise.

$$FA + GA = GB + FC$$

12. Wherefore by dividing both parts of the last

equation by F + G, equal quotients will arise, *viz.*

$$A = \frac{GB + FC}{F + G}$$

which was to be demonstrated.

The learned *Herigonius* (in *cap.* 13. *Tom.* 2. of his *Cursus Mathematicus*) hath delivered another way of resolving the *rule of False*, namely, by the two following rules, *viz.*

When the signs of the Errors are unlike.

Rule I. As the sum of the errors is to the first error, so is the difference of the supposed numbers to a fourth proportional, which being added to the first supposed number, when the said first supposition is less then the second, or subtracted from it when it exceeds the second; the sum or remainder will be the true number sought.

When the signs of the Errors are unlike.

Rule II. As the difference of the errors is to the first error, so is the difference of the supposed numbers to a fourth proportional, which being added to the first supposed number when the signs are — or subtracted from it when the signs are +; the sum or remainder will be the number sought

Both which rules the said *Herigonius* demonstrateth geometrically by lines, upon a supposition of the *Analogy* or *proportionality* before mentioned in the third Section of this Chapter; and the same may likewise be easily demonstrated according to the precedent method by letters.

CHAP.

CHAP. X.

A Collection of pleasant and subtil Questions, to exercise all the parts of Vulgar Arithmetick. To which also are added various practical Questions about the mensuration of Superficial Figures and Solids.

Examples of the Rule of Three mixtly used with other rules.

QUest. 1. If a wedge of Gold weighing $17\frac{3}{7}$ *lb.* of *Troy weight* be worth $679\frac{5}{7}$ *lb. sterling*, what is the value of $1\frac{3}{13}$ grain of that Gold? *Answ.* 2 pence.

$$\text{I. } 1\frac{3}{13}\ (\text{or } \frac{16}{13})\ \text{of } \frac{1}{24}\ \text{of } \frac{1}{20}\ \text{of } \frac{1}{12} = \frac{1}{4680}$$

$$\text{II. } \frac{122}{7} \cdot \frac{4758}{7} :: \frac{1}{4680} \cdot \frac{1}{120}$$

Quest. 2. A man dying gave to his eldest Son $\frac{2}{3}$ of $\frac{1}{4}$ of his estate, to his second Son $\frac{1}{3}$ of $\frac{1}{2}$ of his estate, and when they had counted their Portions, the one had 40 *l.* more then the other, the remainder of the estate was given to the wife and younger children, the question is, what was the portion of the eldest Son, also of the second, and how much did belong to the wife and younger children?

Answ.

Answ. The eldest Sons portion 100 *l.* the second Sons portion 60 *l.* and 440 *l.* for the wife and younger children.

The fractions being reduced, it will be manifest that the eldest Son had $\frac{1}{6}$, *and the second* $\frac{1}{10}$ *also the difference of the said fractions is* $\frac{1}{15}$, *then say,*

$$\frac{1}{15} \cdot \frac{40}{1} :: \frac{1}{10} \cdot \frac{60}{1}$$

	l.
The second Sons portion	60
The difference of their portions	40
The eldest Sons portion	100

$$\frac{1}{15} \cdot \frac{40}{1} :: \frac{1}{1} \cdot \frac{600}{1}$$

Lastly, 600 — 160 = 440 *for the wife and younger children.*

Quest. 3. A young man received $66\frac{2}{3}$ *l.* which was $\frac{2}{3}$ of $\frac{1}{2}$ of his elder brothers portion, and $3\frac{1}{2}$ times of his elder brothers portion was $1\frac{1}{4}$ times of his fathers estate; the question is, what was the fathers estate? *Answ.* 560 *l.*

$$\frac{1}{3} \cdot 66\frac{2}{3} :: 1 \cdot 200$$

$$200 \times 3\frac{1}{2} = 700.$$

$$1\frac{1}{4} \cdot 700 :: 1 \cdot 560$$

Quest. 4. If A can finish a work in 20 dayes, and B in 30 dayes; in what time will that work be finished by A and B working together? *Answer* 12 dayes.

First find what quantity of the work will be done

done by each workman in one and the same time ; then it will be, as the sum of those quantities is in proportion to the said time, so is 1 or the whole work to the time wherein such work will be finished by both workmen working together.

dayes	*work*		*dayes*	*work*
30	. 1	::	20 .	$\frac{2}{3}$
			add	1
			sum	$1\frac{2}{3}$

Hence it appears that A and B working together 20 dayes, will finish that work once, together with $\frac{2}{3}$ of the same work ; therefore say again by the *Rule of Three*,

work	*dayes*		*work*	*dayes*
$1\frac{2}{3}$ ..	20	::	1 .	12

Quest. 5.

Æreus adsto leo, tubuli mihi lumina bina
Osque etiam, dextri sic quoque planta pedis.
Binis dextro oculo, ternis lacus iste diebus
Impletur lævo, sed pede bis geminis.
Ori sufficiunt sex horæ. Dic simul ergo
Quo spatio os, oculi, pesque replere valent.

The sense is this. A brazen Lyon being placed in an artificial fountain, conveyeth water into a Cistern by two streams issuing from his eyes, also by one from his mouth, and by another at the bottom of his right foot. Now the Pipes through which these streams pass, are of different capacities, in

in such sort, that by the right eye set open alone, the rest of the streams being stopt, the Cistern will be filled in two dayes; (the length of a day being supposed to be 12 hours) by the left eye alone in three dayes; by the foot alone in four dayes, and by the mouth alone in six hours. The question is, to find in what time the Cistern will be filled, if all those streams be set open at once?

Answer, $\frac{12}{37}$ day.

dayes		*Cist.*		*dayes*		*Cist.*
2	.	1	::	3	.	$1\frac{1}{2}$
4	.	1	::	3	.	$0\frac{3}{4}$
$\frac{1}{2}$	.	1	::	3	.	6
				add		1.

The sum is $9\frac{1}{4}$ *Cisterns that will be filled in 3 dayes by all the four streams running together: Then say by the rule of Three.*

Cist.		*Dayes*		*Cist.*		*day*
$9\frac{1}{4}$	.	3	::	1	.	$\frac{12}{37}$

*Quest.*6. A Cistern in a certain Conduit is supplied with water by one pipe of such bigness, that if the cock *A* at the end of the pipe be set open, the Cistern will be filled in $\frac{1}{2}$ hour; moreover at the bottom of the Cistern two other cocks *B* and *C* are placed, whose capacities are such, that by the Cock *B* set open alone (all the rest being stopt) the Cistern supposed to be full) will be emptied in $1\frac{4}{7}$ hour; also by the cock *C* set open alone the Cistern will be emptied in $2\frac{1}{3}$ hour: now because more water will be infused by the cock *A*, then can be

be expelled by both the cocks *B* and *C* in one and the ſame time; the queſtion is to find in what time the Ciſtern will be filled if all the ſaid three cocks be ſet open at once? *Anſw.* $1\frac{9}{61}$ hour.

After the manner of the fourth queſtion of this chapter, find how many times the ciſtern will be emptied in one and the ſame ſpace of time, by the cocks *B* and *C* running together; alſo how much of the ciſtern will be filled by *A* in the ſame time, then will the difference ſhew how much of the ciſtern is gained by the filling cock in the ſaid time: Laſtly, as the ciſterns or parts gained are in proportion to the correſpondent time; ſo is the whole ciſtern, to the time wherein it will be gained or filled.

hou. ciſt. hou. ciſt.

$$\text{I.}\quad 2\tfrac{1}{3} \;.\; 1 \;::\; 1\tfrac{3}{7} \;.\; \left.\begin{array}{r} \frac{30}{49} \\ \textit{add } 1 \\ \hline \textit{ſum } 1\frac{30}{49} \end{array}\right\} \textit{emptied by} \left\{\begin{array}{l} C \\ B \\ B \;\&\; C \end{array}\right.$$

in $1\frac{3}{7}$ *hour.*

hou. ciſt. hou.

$$\text{II.}\quad \tfrac{1}{2} \;.\; 1 \;::\; 1\tfrac{3}{7} \;.\; \begin{array}{l} (2\frac{6}{7} \textit{ filled by } A \\ \hline 1\frac{12}{49} \textit{ gained by } A \end{array}$$

ciſt. hou. ciſt. hou.

$$\text{III.}\quad 1\tfrac{12}{49} \;.\; 1\tfrac{3}{7} \;::\; 1 \;.\; 1\tfrac{9}{61}$$

Queſt. 7. Suppoſe a Dog, a Wolf, and a Lion, were to devour a Sheep, and that the Dog could eat up the ſheep in an hour, the Wolf in $\frac{3}{4}$ hour, and the Lion in $\frac{1}{2}$ hour; now if the Lion begin to eat $\frac{1}{8}$ hour before the other two, and afterwards all three

three eat together, the question is, in what time the sheep would be devoured? *Answ.* $\frac{31}{104}$ hour.

		hou.	sh.		hou.	sh.
I.	If	$\frac{1}{2}$	1	: :	$\frac{1}{8}$	$\frac{1}{4}$

Thus it appears that $\frac{1}{4}$ of the sheep would be eaten by the Lion, before the Dog and Wolf began to eat.

II. Proceed according to the fourth question, so will you find the remaining $\frac{3}{4}$ to be eaten by them all in $\frac{9}{52}$ hour, which added to $\frac{1}{8}$ gives $\frac{31}{104}$ hour, in which time the sheep would be devoured.

Quest. 8. If $120\frac{1}{3}$ *l.* be to be distributed amongst three persons A, B, C, in such sort, that as often as A takes 5, B shall take 4, and as often as B takes 3, C shall take 2; what shall be the share of each?

Answ. A $51\frac{4}{7}$ *l.* B $41\frac{9}{35}$ *l.* C $27\frac{53}{105}$ *l.*

Find three Numbers which may express the proportions of their shares, by the *Rule of Three*, or (to avoid fractions) thus,

$$\begin{array}{ccc} 5 & \ldots\ldots\ldots & 4 \\ 3 & \ldots\ldots\ldots & 2 \\ \hline \end{array}$$

$$\underbrace{15 \;.\; 12 \;.\; 8}_{\textit{thus found}}$$

$5 \times 3 = 15$
$3 \times 4 = 12$
$4 \times 2 = 8$

$$35 \;.\; 120\tfrac{1}{3} :: \begin{cases} 15 \;.\; 51\tfrac{4}{7} \\ 12 \;.\; 41\tfrac{9}{35} \\ 8 \;.\; 27\tfrac{53}{105} \end{cases}$$

Quest. 9. A Governour of a certain Garrison, being desirous to know how much money the Port or passage of the Garrison did amount unto in certain moneths, made choice of a loyal servant, giving him order to receive of every coachman passing with a coach 4 *d.* of every horseman 2 *d.* and of every footman $\frac{1}{2}$ *d.* Now at the years end, the servant making his accompt to the Governour, giveth him 94 *l.* 15 *s.* 10 *d.* and lets him know that as often as 5 passed with coaches, 9 passed on horseback; and as often as 6 passed on horseback, 10 passed on foot; the question is, how many coaches, horsemen, and footmen passed? *Answer*, 2500 coaches, 4500 horsemen, 7500 footmen.

Find 3 proportional numbers after the manner of the eighth question, which will be 5, 9, 15, then proceed as followeth,

		d.
5	*Coaches* ..	20
9	*Horsemen*	18
15	*Footmen* .	$7\frac{1}{2}$

$$\text{If } 45\tfrac{1}{2} \,.\, 22750 :: \begin{cases} 5 \,.\, 2500 \\ 9 \,.\, 4500 \\ 15 \,.\, 7500 \end{cases}$$

Quest. 10 A Factor would exchange 780 *l. sterling* for double Ducats, Dollars, and French Crowns, the Ducats at 7 *s.* 6 *d.* the piece, the Dollars at 4 *s.* 4 *d.* and the French Crowns at 6 *s.* the piece; to be in such proportion, that $\frac{1}{2}$ of the number of Ducats may be equal to $\frac{1}{3}$ of the number of Dollars, and $\frac{1}{4}$ of the Dollars equal to $\frac{3}{16}$ of the Crowns: the question is, how many pieces of each coin he shall receive for his 780 pounds.

Answ. 600 Ducats, 900 Dollars, 1200 Crowns.

Find three proportional Numbers (after the manner of the eighth question) which will be 6, 4, 3.

$$\begin{array}{ccccc} \frac{1}{2} & \cdots\cdots\cdots & \frac{1}{3} & & \\ & \frac{1}{4} & \cdots\cdots\cdots & \frac{3}{16} & \\ \hline \frac{1}{8} & . & \frac{1}{12} & . & \frac{2}{32} \\ 6 & . & 4 & . & 3 \end{array}$$

Thus it appears that six times the number of Ducats must be equal to four times the number of Dollars, also equal unto three times the number of Crowns. Then make choice of 3 numbers to answer those proportions, such are these, 2, 3, 4, (for

(for $6 \times 2 = 4 \times 3 = 3 \times 4$) with which numbers proceed as followeth :

		l.
2	*ducats* ...	$\frac{3}{4}$
3	*dollars* .	$\frac{13}{20}$
4	*crowns* ..	$1\frac{1}{5}$

$$\text{ſay if} \ldots 2\tfrac{3}{5} \;.\; 780 \;::\; \begin{cases} \frac{3}{4} \;.\; 225 \\ \frac{13}{20} \;.\; 195 \\ 1\frac{1}{5} \;.\; 360 \end{cases}$$

l.			l.		
	ducat				
$\frac{3}{8}$	. 1	::	225	.	600 *ducats.*
	doll.				
$\frac{13}{60}$	. 1	::	195	.	900 *dollars.*
	crown				
$\frac{3}{10}$	. 1	::	360	.	1200 *crowns.*

Quest. 11. Twenty Knights, 30 Merchants, 24 Lawyers, and 24 Citizens, ſpent at a dinner 64 pound, which was divided amongſt them in ſuch manner, that 4 Knights paid as much as 5 Merchants, 10 Merchants as much as 16 Lawyers ; and 8 Lawyers as much as 12 Citizens ; the queſtion is, to know the ſum of money paid by all the Knights, alſo by the Merchants, Lawyers and Citizens ?

Anſwer; The 20 Knights paid 20 pounds, the 30 Merchants 24 pounds, the 24 Lawyers 12 pounds, and the 24 Citizens 8 pounds.

Find four numbers to expreſs the proportions of their payments, by the *Rule of Three*, or, (to avoid fractions) in manner following, ſo will the proportional numbers be 4, 5, 8, 12, *viz.* 4 Knights paid as much as 5 Merchants, or 8 Lawyers, or 12 Citizens,

4 5
10 16
8 12

---

320 . 400 . 640 . 960
4 . 5 . 8 . 12

thus found,

$$4 \times 10 \times 8 = 320$$
$$10 \times 8 \times 5 = 400$$
$$8 \times 5 \times 16 = 640$$
$$5 \times 16 \times 12 = 960$$

Then presupposing that a Knight is to pay 4 *s.* proceed as followeth, *viz.*

		l.
20	*Knights* ...	4
30	*Merchants* ..	$4\frac{4}{5}$
24	*Lawyers* ..	$2\frac{2}{5}$
24	*Citizens* ...	$1\frac{3}{5}$

say, if $12\frac{4}{5}$. 64 :: $\begin{cases} 4 & . \; 20 \\ 4\frac{4}{5} & . \; 24 \\ 2\frac{2}{5} & . \; 12 \\ 1\frac{3}{5} & . \; 8 \end{cases}$ *l.*

64

Quest. 12. A certain man with his wife did usually drink out a vessel of Beer in 12 dayes, and the husband found by often experience, that his wife being absent, he drank it out in 20 dayes; the question is, in how many dayes the wife alone could drink it out? *Answer*, 30 dayes.

Note,

Note, it is to be supposed that the husband in 12 of the 20 dayes wherein he drank alone, did drink as much as in the 12 dayes wherein he drank with his wife; hence it followeth, that in the remaining 8 of the said 20 dayes, he drank as much as his wife did in 12 dayes. Therefore by the *Rule of Three* say, If 8 give 12, what 20? *Answ.* 30. view the following form of the work.

From 20
Subtract 12

Then if 8 . 12 : : 20 . 30

Quest. 13. If a house be to be built by three Carpenters, A, B, C, working in such sort, that A alone will finish it in 30 dayes, B in 40 dayes, and A, B, C together in 15 dayes, in what time could C alone build the house? *Answ.* 120 *dayes.*

I. After the manner of the fourth question, find in what time A and B working together will finish the house; *Answ.* $17\frac{1}{7}$ dayes.

dayes	*work*		*dayes*	*work*
40 .	1	: :	30 .	$\frac{1}{4}$
			add	1
			sum	$1\frac{1}{4}$

work	*dayes*		*work*	*dayes.*
$1\frac{1}{4}$.	30	: :	1 .	$17\frac{1}{7}$

II. Supposing the work of A and B to be performed by one person, as D, the house will be built by D in $17\frac{1}{7}$ dayes, but by D and C together in 15 dayes; Then find (according to the 12th question)

ſtion) in what time C will build the ſame; *Anſw.* 120 *dayes.*

From $17\frac{1}{7}$
Subtract 15

Then if $2\frac{1}{7}$. 15 :: $17\frac{1}{7}$. 120

The proof may be wrought according to the fourth or fifth queſtions.

Queſt. 14. Two travellers A and B perform a journey to one and the ſame place in this manner, *viz.* A travels 14 miles every day, and had travelled 8 dayes before B began; upon the ninth day B ſets forward, and travels 22 miles every day; the queſtion is, to find in what time B ſhall overtake A? *Anſw.* at the end of 14 dayes.

I. Find how many miles A had travelled before B ſet forward, *Anſw.* 112 miles; For

day		*miles*		*dayes*		*miles*
1	.	14	::	8	.	112

II. Find how many miles B gains of A in a day; *Anſw.* 8 miles; For

$$22 - 14 = 8$$

		miles		*day*		*miles*		*dayes.*
III.	If	8	.	1	::	112	.	14

Queſt. 15. There is an Iſland which is 36 miles in compaſs. Now if at the ſame time, and from the ſame place, two footmen A and B ſet forward to

to travel round about the said Island, and follow one another in such manner that A travelleth every day 9 miles, and B 7 miles; the question is to find in what space of time they will again meet; also how many miles, and how many times about the Island each footman will then have travelled?

Answer, They will meet at the end of 18 dayes from their first parting, and then A will have travelled 162 miles (or $4\frac{1}{2}$ times the compass of the Island) and B will have travelled 126 miles (or $3\frac{1}{2}$ times the compass of the Island.)

	miles					
From ...	9					
Subtract	7					
			day	*miles*		*dayes*
	2	.	1 ::	36	.	18

mult.	18		*mult.*	18
by	9		*by*	7
36)	162 ($4\frac{1}{2}$		36)	126 ($3\frac{1}{2}$

Quest. 16. Two footmen A and B depart at the same time from *London* towards *York*, travelling at this rate, *viz.* A goeth 8 miles every day, B goeth 1 mile the first day, 2 miles the second day, 3 miles the third day, and in that progression he goeth forward, travelling in every following day one mile more then in the preceding day; the question is, to know in how many dayes B will overtake A?

Answer, 15 *dayes.*

To resolve this and such like questions, double 8 (the number of miles which A travelleth daily)

which makes 16, from which subtract 1, the remainder is 15 the number of dayes sought.

Quest. 17. If *Exceter* be distant from *London* 140 miles, and that at the same time one footman A departed from *London* towards *Exceter*, travelling every day 8 miles; and another B from *Exceter* towards *London*, travelling every day 6 miles; the question is, in how many dayes they will meet one another, and how many miles each footman will have then travelled?

Answer, They will meet at the end of 10 dayes; and then A will have travelled 80 miles, and B 60 miles.

add. { 8 miles travelled daily by A.
6 miles travelled daily by B. }

sum 14 miles which A and B together did travel daily.

m.	*da.*	*miles*	*da.*	
14 .	1 : :	140 :	10	in which time A and B will meet each other.

10 × 8 = 80 miles travelled by A.
10 × 6 = 60 miles travelled by B.

Quest. 18. A certain footman A departeth from *London* towards *Lincoln*, and at the same time another footman B departeth from *Lincoln* towards *London*; also A travelleth every day $2\frac{1}{2}$ miles more then B. Now supposing those two Cities to be 100 miles distant one from the other, and that those two footmen do meet one another at the end of 8 dayes after the beginning of their journeys; the question is, how many miles each will have then travelled,

travelled, as also how many miles each travelled daily?

Answer, A 60 miles, B 40 miles. Also A travelled $7\frac{1}{2}$ miles every day, and B 5 miles.

day	*miles*	*dayes*	*miles*
1 .	$2\frac{1}{2}$ ::	8 .	20

Hence it appears that at the time of their meeting A had travelled 20 miles more than B, which 20 miles being subtracted from 100 miles leaves 80 miles, whereof the half is 40 miles which B had travelled, therefore A had travelled 60 miles.

Now to find how many miles each travelled daily, say,

dayes	*miles*	*day*	*miles*
8 .	40 ::	1 .	5

Therefore $\left\{\begin{matrix}A\\B\end{matrix}\right\}$ travelled $\left\{\begin{matrix}7\frac{1}{2}\\5\end{matrix}\right\}$ *miles* daily.

Quest. 19. There is an Island which is 134 miles in compass; now at the same time, and from the same place, two footmen A and B begin a journey round about the said Island, but they travel towards contrary parts, at this rate, *viz.* *A* travelleth 11 miles in every 2 dayes, and *B* 17 miles in 3 dayes: the question is to find in what space of time *A* and *B* will meet one another, and how many miles each will then have travelled?

Answer, They will meet at the end of 12 dayes, and then *A* will have travelled 66 miles, and *B* 68 miles.

After the manner of the fourth question of this chapter the time sought will be found 12 dayes.

dayes	*miles*		*dayes*	*miles*					
2 .	11	::	3 .	$16\frac{1}{2}$					
			add	17					
					dayes		*miles*		*dayes*
				$33\frac{1}{2}$.	3	::	134 .		12

The miles travelled by each will be found in this manner.

dayes	*miles*		*dayes*	
2 .	11	::	12 .	66 miles travelled by *A*.
3 .	17	::	12 .	68 miles travelled by *B*.

Quest.20. If a Clock hath two Indices (or hands) one of which (to wit *A*) is carryed twice round the whole circumference of the Dyal in one day; and the other (*B*) once in 30 dayes, and that both at once shewing the same point begin to be moved; the question is, in what time they will be again conjoyned?

Answer, $\frac{30}{59}$ day or $12\frac{12}{59}$ hours.

day	*circum.*		*dayes*	*circum.*
1 .	2	::	30 :	60
			subtract	1
				59

Hence it appears, that in 30 dayes *A* will have run through 60 circumferences, and *B* one circumference only in the same time, therefore *A* gains of

of *B* 59 circumferences in 30 dayes; therefore say,

circum.		*dayes*		*circum.*		*day*
59	.	30	::	1	.	$\frac{30}{59}$

Quest. 21. If 6 *lb.* of Sugar be equal in value to 7 *lb.* of Raisins; 5 *lb.* of Raisins to 2 *lb.* of Almonds; 3 *lb.* of Almonds to 5 *lb.* of Currans; 2 *lb.* of Currans to 18 *d.* how many pence are the value of 3 *lb.* of Sugar? *Answ.* 21 *d.*

mult. these contin.	6	S.	=	7	R.	*mult. these contin.*
	5	R.	=	2	A.	
	3	A.	=	5	C.	
	2	C.	=	18	*d.*	
	?	*d.*	=	3	S.	
	180			3780	(21	

Quest. 22. If 3 dozen pair of Gloves be equal in value to 2 pieces of Ribbon; 3 pieces of Ribbon to 7 dozen of Points; 6 dozen of points to 2 yards of Flanders lace; and 3 yards of Flanders lace to 81 shillings; how many dozen pair of Gloves may be bought for 28 shillings?
Answ, 2 dozen pair of Gloves.

mult. these contin.	3	G.	=	2	R.	*mult. these contin.*	
	3	R.	=	7	P.		
	6	P.	[illegible]	2	L.		2268) 4536 (2
	3	L.	=	81	S.		
	28	S.	=	?	G.		
	4536			2268			

Quest. 23.

Quest. 23. Suppose a *Greyhound* to be coursing a *Hare*, in such sort that the *Hare* takes five leaps for every four leaps of the *Greyhound*, and that the *Hare* is one hundred of her own leaps distant from the *Greyhound*; now if three of the *Greyhounds* leaps be equal to four leaps of the *Hares*, the question is to know how many leaps the *Greyhound* must take before he obtain his prey?

Answer, 1200 *leaps*.

I. If 3 . 4 :: 4 . $5\frac{1}{3}$

Thus it appears, that 4 of the *Greyhounds* leaps are equal to $5\frac{1}{3}$ of the *Hares* leaps, and because by the question the *Greyhound* takes 4 leaps for every 5 of the *Hares*, therefore the *Greyhound* in every four of his leaps gains $\frac{1}{3}$ of one of the *Hares* leaps; therefore say by the *Rule of Three*,

II. If $\frac{1}{3}$. 4 :: 100 . 1200

Quest. 24. There is a certain room whose Basis is a long square, which is in circuit $50\frac{1}{2}$ feet, and the height of the walls or sides of the room is $8\frac{1}{4}$ feet; all which walls of the room except a space taken out for a window in the form of a long square, whose height is five feet, and breadth four feet, are to be furnished with Hangings of ell-broad stuff at 3 *s*. 4 *d*. the yard, the question is to know how much money the stuff will cost?

Answer, 5 *l*. 17 *s*. $6\frac{1}{4}$ *d*.

$$50\tfrac{1}{2} \times 8\tfrac{1}{4} = 416\tfrac{5}{8}$$ *square feet.*
$$5 \times 4 = 20$$ *subtract*

$$396\tfrac{5}{8}$$

$3\tfrac{3}{4} \times 3 = 11\tfrac{1}{4}$ *square feet in one yard of stuff.*

	feet	*d.*		*feet*	*d.*
If	$11\tfrac{1}{4}$	. 40	: :	$396\tfrac{5}{8}$	. $1410\tfrac{5}{9}$

Quest. 25. There is a certain Walk which is a long square, whose length is 40 yards, and breadth 7 yards, to be paved with stones, each of which being in the form of a long square is 28 inches in length, and 24 inches in breadth, the question is to know how many such stones will be requisite to pave the said Walk?

Answer, 540.

Inches		*Inches*		
1440	×	252 = 362880		*square Inches.*
	28 × 24 =		672	*square Inches.*
672 . 1 : : 362880 . 540 *Stones.*				

Quest. 26. Suppose a piece of *Tapestry* to be $5\tfrac{1}{8}$ *yards English* in length, and $3\tfrac{7}{8}$ *yards* in breadth, the question is, how many square *ells Flemish* are contained in that piece of *Tapestry*, when the length of 1 *ell Flemish* is equal to $\tfrac{3}{4}$ of a *yard English*?

Answer, $37\tfrac{1}{36}$ *square ells Flemish.*

$5\tfrac{1}{8} \times 3\tfrac{7}{8} = \tfrac{1333}{64}$ *square yards.*

Then

Then because $\frac{2}{16}$ of a ſquare yard is equal to 1 ell ſquare of *Flemiſh* meaſure (for $\frac{1}{4} \times \frac{2}{4} = \frac{2}{16}$) ſay,

$$\text{If } \frac{2}{16} \;.\; 1 \;::\; \frac{1333}{64} \;.\; 37\frac{11}{36}$$

Queſt. 27. A Workman hath performed a piece of Tiling bearing the form of a long ſquare, whoſe length is 273 feet, 7 inches; and breadth 21 feet 5 inches; now when Tiles are ſold at the rate of 11 *s*. $10\frac{3}{4}$ *d*. for 1000 tiles, and every ſquare of tiling conſiſting of 10 feet as well in length as in breadth doth take up 1000 tiles, what doth the ſaid piece of tiling amount unto?

Anſwer, 34 *l*. 17 *s*. $0\frac{4001}{57600}$ *d*.

I. $273\frac{7}{12} \times 21\frac{5}{12} = \frac{843731}{144}$. *ſquare feet*

II. $100 \;.\; \overset{d.}{142\frac{3}{4}} \;::\; \frac{843731}{144} \;.\; \overset{d.}{8364\frac{4001}{14400}}$

Queſt. 28. A Merchant would beſtow 220 *l*. in Cloves, Mace and Nutmegs, the Cloves being at 5 *s*. the pound, the Mace at 11 *s*. the pound, and the Nutmegs at 6 *s*. the pound; now he would have of each ſort an equal quantity, the queſtion is how many pounds he may have of each ſort?

Anſwer, 200 *lb*.

s.
5
11
6

22 . 1 :: 4400 . 200

The

The Proof.

lb.		s.		l.
200	at	5	*amounts unto* ...	50
200	at	11	*amounts unto* ...	110
200	at	6	*amounts unto* ...	60
				2 0

*Quest.*29. A Factor is to receive a ſum of money, and is offered Dollars at 4 *s*. 4 *d*. which are worth but 4 *s*. 3 *d*. or French Crowns at 6 *s*. $1\frac{1}{2}$ *d*. which are worth but 6 *s*. the queſtion is by which coin he ſhall ſuſtain the leaſt loſs?

Anſwer, the Dollars.

$$\begin{array}{ccccccc} d. & & d. & & d. & & d. \\ 52 & . & 1 & :: & 73\frac{1}{2} & . & 1\frac{43}{104} \end{array}$$

That is, in receiving the Dollars every 6 *s*. $1\frac{1}{2}$ *d* loſeth $1\frac{43}{104}$ *d*. but in receiving the Crowns 6 *s*. $1\frac{1}{2}$ *d*. looſeth $1\frac{1}{2}$ *d*. which is a greater loſs than $1\frac{43}{104}$ *d*.

*Queſt.*30. A Butcher agrees with a Graſier, for the feeding of 20 Oxen, during the ſpace of 12 equal moneths, but at 2 moneths end, the Butcher adds 5 Oxen more, and $6\frac{1}{5}$ moneths after that, he added 10 Oxen more, and then it is agreed between them, that the Graſier ſhall feed them all, ſo long time as will be equivalent to the keeping of the firſt twenty during 12 moneths; the queſtion is, how long time he ſhall feed them all, after the putting in of the laſt 10?

Anſwer, 1 moneth.

Conſider, that as he receives more Oxen to feed, he ought to keep them all the leſs time; therefore

work

work as the question imports by the *Rule of Three inverse*.

	mon.		*Oxen*			
	12		20			
Oxen	2		5		*mon.*	*Oxen*
If 20 .	10	: :	25 .		(8	25
					$6\frac{1}{5}$	10
			If 25		$1\frac{1}{5}$ . .	35 (1 *mon.*

Examples of the Rule of Fellowship.

Quest. 31. Two Merchants, *viz.* A and B, have entered Company; A puts in 500 *l.* and at 4 moneths end takes out a certain sum, leaving the remainder to continue 8 moneths longer; B puts in 250 *l.* and at five moneths end puts in three hundred pounds more, and then his whole sum continues seven moneths longer. Now at the making of their Accompt, A findeth that he hath gained $106\frac{2}{3}$ pounds, and B gained $133\frac{1}{3}$ pounds; the question is to know how much A took out of the bank at 4 moneths end?

Answer, 240 *l.*

$250 \times 5 = 1250$
add 300

$550 \times 7 = 3850$

5100

$133^{1} . 5100 :: 106\frac{2}{3} . 4080$
$500 \times 4 = 2000$ *subtract*

8) 2080 (260

Lastly, 500 — 260 = 240 *taken out by A.*

The

The Proof.

	l.		*mon.*		
	500	×	4	=	2000
Subtract	240				
	260	×	8	=	2080
					4080

Quest. 32. Five Merchants, *viz.* *A*, *B*, *C*, *D*, and *E*, have gained 2025 *l.* which they divide in ſuch ſort, that $\frac{1}{2}$ of the ſhare of *A* is equal ſeverally to $\frac{1}{4}$ of the ſhare of *B*. $\frac{1}{5}$ of *C*. $\frac{1}{6}$ of *D*. $\frac{1}{8}$ of *E*. the queſtion is, what was the ſhare of each Merchant?

Anſwer, *A* 162 *l.* *B* 324 *l.* *C* 405 *l.* *D* 486 *l.* *E* 648 *l.*

Divide a number at pleaſure into ſuch parts which may be in ſuch proportion as the ſhares required, and proceed according to the ſubſequent operation.

A 2
B 4
C 5
D 6
E 8

				l.						
If 25	. 2025 ::	2.	(	162	*for*	*A*	*whereof*	$\frac{1}{2}$	*is*	81
		4.	(	324	*for*	*B*	*whereof*	$\frac{1}{4}$	*is*	81
		5.	(	405	*for*	*C*	*whereof*	$\frac{1}{5}$	*is*	81
		6.	(	486	*for*	*D*	*whereof*	$\frac{1}{6}$	*is*	81
		8.	(	648	*for*	*E*	*whereof*	$\frac{1}{8}$	*is*	81
				2025						

Quest.33. Two Merchants A and B are in company, the sum of their stocks is 300 *l*. the money of A continuing in company 9 moneths, the money of B 11 moneths, they gain 200 *l*. which they divide equally; the question is to know how much each Merchant did put in?

Answer, A 165 *l*. B 135 *l*.

Divide 300 into two such parts which may be in proportion as 11 to 9, so will the greater part be the stock of A, and the lesser the stock of B, which stocks being multiplied by their respective times, the products will be equal.

$$\begin{array}{l} 11 \\ \underline{\;\;9} \\ 20\ .\ 300 :: \end{array} \left\{ \begin{array}{l} 11\ .\ 165 \textit{ for A} \\ \\ 9\ .\ 135 \textit{ for B} \end{array} \right.$$

Quest.34. Two Merchants, *viz.* *A* and *B* are in company, *A* did put in 325 *l*. more then *B*, and the stock of *A* continued in company $7\frac{1}{2}$ moneths; *B* put in a certain sum which is unknown, and it continued in company $10\frac{1}{4}$ moneths, after a certain time they divide the gain equally; the question is what each Merchant did put in?

Answer, *B* 750 *l*. and *A* 1075 *l*.

Divide the product of the difference of their stocks multiplied by the time of *A*, by the difference of their times, so will the quotient be the stock of *B*, which added to 325 *l*. gives the stock of *A*.

325

$$325 \times 7\tfrac{1}{2} = 2437\tfrac{1}{2}$$

$3\tfrac{1}{4}$) $2437\tfrac{1}{2}$ (750 *stock of B*
add 325

1075 *stock of A*

Examples of the Rule of Alligation. How the fineness of gold and silver is estimated, vide pa. 111.

Quest. 35. A Goldsmith hath some Gold of 24 Carects, others of 22 Carects, and another sort of 18 Carects fine; he would so mix these together that the mass mixed might be 60 *lb.* and that the whole mixture might bear 20 Carects fine. How much of each sort must he take?

	lb.		
Answer,	12	*of* 24	*Carects.*
	12	*of* 22	*Carects.*
	36	*of* 18	*Carects.*

$$20\begin{cases}24\\22\\18\end{cases}\qquad \begin{array}{c|c} 2 & 2\\ 2 & 2\\ 4+2 & 6\\ \hline \end{array}$$

$$10\ .\ 60 :: \begin{cases}2\ .\ 12\\2\ .\ 12\\6\ .\ 36\end{cases}$$

Note, some may think that questions of *Alligation* are capable only of so many several answers as there are different wayes to connect the mean rate or price with the extream rates or prices; yet it is most certain, that any ordinary

See Bachets *Commentary on the 41 quest. of the 4 book of* Diophantus.

dinary question of Alligation, where three or more things are propounded to be mixt in such manner as that rule requires, is capable of infinite answers if fractions be admitted, and sometimes of many answers in whole numbers, which are not discoverable by the common rule of *Alligation*: so albeit to the last mentioned question, the said rule of *Alligation* can find but one answer only, which is before given, yet there are eight other answers in whole numbers, which are these that follow (the invention whereof I shall, if God spare life, make manifest in another place.)

Of 24 *Carects*	18	16	14	10
Of 22 *Carects*	3	6	9	15
Of 18 *Carects*	39	38	37	35

Of 24 *Carects*	8	6	4	2
Of 22 *Carects*	18	21	24	27
Of 18 *Carects*	34	33	32	31

See chap. 8. *of this Appendix.*

Quest. 36. An Apothecary hath several Simples: *viz*. *A* hot in 3°. *B* hot in 2°. *C* temperate, *D* cold in 2°. and *E* cold in 4°. Now he desires to make a Medicine of those Simples, in such sort that the temper thereof in respect of quality may be in 1°. of heat, and the quantity $8\frac{1}{2}$ Drams, the demand is what quantity of each Simple he must take?

Answer, $4\frac{1}{2}$ Drams of *A*, $\frac{1}{2}$ Dram of *B*, $1\frac{1}{2}$ Dram of *C*, 1 Dram of *D*, and 1 Dram of *E*.

Indices			*Drams*	
6	8	1, 3, 5	9	*A.*
	7	1	1	*B.*
	5	2, 1	3	*C.*
	3	2	2	*D.*
	1	2	2	*E.*
			17	

		Drams	
17 . $8\frac{1}{2}$ ::		9 . $4\frac{1}{2}$	*A.*
		1 . $0\frac{1}{2}$	*B.*
		3 . $1\frac{1}{2}$	*C.*
		2 . 1	*D.*
		2 . 1	*E.*
		$8\frac{1}{2}$	

Examples of the Rule of False Position.

Quest. 37. A Merchant buyeth 2 sorts of Clothes, *viz.* of blacks and of whites for 68 *l.* 2 *s.* after the rate of 21 *s.* the yard for the blacks, and 12 *s.* the yard for the white, and he taketh so much of each sort, that $\frac{1}{6}$ of the number of yards of the black, are equal to $\frac{2}{8}$ of the white; the demand is, how many yards he bought of each sort?

Answer, 42 yards of black, and 40 yards of white.

Quest. 38. A certain person A payeth unto the use of B for ever 2500 *l.* in present money, upon this condition, that B shall pay unto A an Annuity or yearly rent to be continued four years, the equality of their agreement being thus grounded, *viz.* the said 2500 *l.* is supposed to be put forth at

interest for a year, (to commence from the time of their agreement) at the rate of 8 *per centum, per annum*. Then from the sum of that principal and interest, (arising due at the years end) the first payment of the Annuity being subtracted, the remainder is likewise supposed to be put forth at the same rate of interest for the second year; then from the composed of this principal and interest, (due at the second years end) the second payment of the *Annuity* being subtracted, the remainder is likewise supposed to be put forth at the same rate of interest for the third year; then from this principal and interest the third payment of the *Annuity* being subtracted, the remainder is in like manner supposed to be put forth at the same rate of interest for the fourth year: lastly, from this principal and interest the fourth and last payment of the *Annuity* being subtracted, there must be nothing left: the question is, what sum of money must be yearly paid to satisfie those conditions?

Answer, $754\frac{14117}{17602}$ *l*, as will be manifest by the subsequent proof.

I. 100 . 108 : : 2500 . 2700

Subtract the first payment $754\frac{14117}{17602}$

$1945\frac{3485}{17602}$

II. 100 . 108 : : $1945\frac{3485}{17602}$. $2100\frac{14325}{17602}$

Subtract the second payment $754\frac{14117}{17602}$

$1346\frac{208}{17602}$

III. 100

III. $100 \,.\, 108 :: 1346\frac{208}{17602} \,.\, 1453\frac{12194}{17602}$

Subtract the third payment $754\frac{14117}{17602}$

$698\frac{15679}{17602}$

IV. $100 \,.\, 108 :: 698\frac{15679}{17609} \,.\, 754\frac{14117}{17602}$

Subtract the last payment ... $754\frac{14117}{17602}$

000

Quest. 39.

Mulæ, Asinæque duos imponit servulus utres
Impletos vino ; segnemque ut vidit Asellam
Pondere defessam vestigia figere tarda,
Mula rogat : quid chara parens cunctare, gemisque ?
Unam ex utre tuo mensuram si mihi reddas,
Duplum oneris tunc ipsa feram ; sed si tibi tradam
Unam mensuram, fient æqualia utrique
Pondera : mensuras dic docte Geometer istas ?

The sense is this. A Mule and an *Ass* carried two unequal quantities of Wine, each consisting of a certain number of measures, in such sort, that if the *Ass* imparted one of her measures to the *Mule*, then the *Mules* number of measures so increased would be the double of those which the *Ass* had remaining ; but if the *Mule* gave one measure to the *Ass*, then the *Asses* measures with that increase would be equal to the *Mules* remaining measures. The question is, how many measures each carried ?

Answer, the *Mule* 7, and the *Ass* 5.

Queſt. 40.

Æs, ferrum, ſtannum miſcens, aurique metallum,
Sexaginta minas penſantem finge coronam.
Æs aurumque duos ſimul efficiunto trientes.
Ternos quadrantes ſtanno mixtum impleat aurum.
At totidem quintas auri vis addita ferro,
Ergo age dic fulvi quantum tibi conjicis auri
Miſcendum, dic quantum æris ſtannique requiras?
Dic quoque ſufficiant duri quot pondera ferri.
Præſcriptam ut valeas rite efformare coronam.

The ſenſe is this, Suppoſe a Crown that ſhall weigh 60 *lb.* is to be made of Gold, Braſs, Iron, and Tin mixed together in ſuch proportion that the weight of the Gold and of the Braſs together may be 40 *lb.* the joynt weight of the Gold and of the Tin 45 *lb.* and the joynt weight of the Gold and of the Iron 36 *lb.* The queſtion is how much of every one of thoſe four metals muſt be taken?

	l.	
Anſwer,	$30\frac{1}{2}$	*of Gold.*
	$9\frac{1}{2}$	*of Braſs.*
	$5\frac{1}{2}$	*of Iron.*
	$14\frac{1}{2}$	*of Tin.*

Queſt. 41. One being demanded what was the preſent hour of the day, anſwered, that the time then paſt from noon was equal to $\frac{4}{5}$ of $\frac{2}{3}$ of the time remaining until midnight. The queſtion is, what a clock it was? (ſuppoſing the time between noon and midnight

midnight to be divided into twelve equal parts o hours.)

Answer, $\frac{16}{43}$ hour after noon.

*Quest.*42. A Factor delivers 6 French Crowns and 2 Dollars for 45 ſhillings ſterling ; alſo at another time he delivers 9 French Crowns and 5 Dollars (at the ſame rate with the former) for 76 ſhillings. The queſtion is to know the value of a French Crown, alſo of a Dollar ?

Answer, A Crown was valued at 6 *s.* 1 *d.* and a Dollar at 4 *s.* 3 *d.*

*Quest.*43. A certain Uſurer received 36 Dollars for the ſimple intereſt of 186 *l.* lent for a certain time unknown; alſo he received 90 Dollars for the gain of 360 *l.* at the ſame rate of intereſt for a certain time unknown ; now the ſum of the moneths wherein both the ſaid numbers of Dollars were gained was twenty moneths. The queſtion is to know in what time aſwell the 36 Dollars as the 90 Dollars were gained ?

Answer ; The 36 Dollars were gained in 8 moneths, and the 90 Dollars in $11\frac{3}{11}$ moneths, a may be proved by the *Double Rule of Three.*

Which anſwer may be diſcovered by the following Canon found out by the *Algebraick art.*

Multiply the Dollars firſt gained, the latter Principal, and the given time, according to the rule of continual Multiplication, for a dividend; then multiply the firſt principal by the Dollars laſt gained, alſo multiply the latter Principal by the Dollars firſt gained, and reſerve the ſum of theſe two laſt products for a Diviſor; laſtly, divide the Dividend firſt found, by the ſaid Diviſor, ſo ſhall the quotient be the time wherein the firſt number of Dollars

was gained, which subtracted from the time given in the question, discovers the time wherein the latter number of Dollars was gained.

$$\frac{36 \times 360 \times 20 = 259200}{186 \times 90, + 300 \times 36, = 29700} = 8\tfrac{8}{11}$$

And consequently . . . . $20 - 8\tfrac{8}{11} = 11\tfrac{3}{11}$

Examples of the Extraction of roots.

Quest.44. If 3481 Souldiers are to be placed in a square battel, how many are to be set in rank or in File?

Answer,59; (for the square root of 3481 is 59)

Quest. 45.If 4050 Souldiers are to be set in battel in a figure, which beareth the form of a long square in such manner,that the number in File may be to the number in Rank as 1 to 2; how many Souldiers are to be placed in rank and how many in File?

Answer,90 in rank and 45 in File (found by this Canon or general rule) *viz*.

As the greater term of the proportion given is to the lesser,so is the number of men to be placed in battel to a fourth proportional, whose square root is the lesser number sought (whether it be for the rank or File)also as the lesser term of the given proportion is to the greater; so is the number of men to be set in battel to a fourth proportional, whose square root is the greater number sought (whether it be for the rank or File.)

I. 2

I.	2 . 1 :: 4050 . 2025
II.	$\surd$q . 2025 = 45 (*men in File.*
III.	1 . 2 :: 4050 . 8100
IV.	$\surd$q . 8100 = 90 (*men in Rank.*

The proof.

$$45 \times 90 = 4050$$

Also 45 . 90 :: 1 . 2

Or when one of the numbers ſought (whether it be for the rank or File) is found, the other may be diſcovered by *Diviſion*, viz.

$$45\ )\ 4050\ (\ 90$$
$$90\ )\ 4050\ (\ 45$$

Queſt. 46. Suppoſe the wall of a Garriſon to be in height 21 feet, and the breadth of the Moat ſurrounding the ſaid wall to be 28 feet; the queſtion is, what length muſt a ſcaling ladder have to reach from the outermoſt ſide of the Moat to the top of the Wall?

Anſwer, 35. (to wit, the ſquare root of the ſum of the ſquares of 21 and 28.)

$$21 \times 21 = 441$$
$$28 \times 28 = 784$$

$$\surd q.\ 1225 = 35$$

Queſt. 47. If 100 *l.* being put forth for intereſt at a certain rate, will at the end of two years be augmented

augmented unto $112\frac{36}{100}$ *l.* (compound intereſt, or intereſt upon intereſt being computed) what principal and intereſt will be due at the firſt years end?

Anſwer, 106 *l.* (compos'd of 100 *l.* principal and 6 *l.* intereſt) which 106 is a mean Geometrically proportional between 100 and 112.36 (and may be found by the eighteenth rule of the fifth Chapter of this Appendix.)

$$100 \times 112.36 = 11236 \quad (106$$

Queſt. 48. If 100 *l.* being put forth for intereſt at a certain rate, will at the end of three years be augmented unto 115.7625 *l.* (compound intereſt being computed) what principal and intereſt will be due at the firſt years end?

Anſwer, 105 *l.* (compoſed of 100 *l.* Principal, and 5 *l.* intereſt) which 105 is the firſt of two mean proportional numbers between 100 and 115.7625 *l.* (See the nineteenth rule of the fifth Chapter of this Appendix.)

Various Practical Queſtions to exerciſe Decimal Arithmetick, in the menſuration of Superficial Figures and Solids.

See the ſecond Section of the 23. chapter of the preceding Book.

Queſt. 49. If the ſide of a ſquare Superficies be 3 feet, what is the Area or content of that Superficies? Or (which is the ſame thing) how many ſquares, each of which is a foot ſquare, are contained in that Superficies?

Anſwer,

Answer, 9 square feet, which content is found out by multiplying the given side 3 by it self, *viz.* 3 multiplyed by 3 produceth 9.

In like manner, if the side of a square pavement of stone be 15.7 feet, the superficial content of that pavement will be 246.49 feet, that is 246 feet and an half very near, (for 15.7 multiplyed by it self produceth 246.49.)

Likewise, a square piece of Wainscot whose side is 3.24 yards, will be found to contain 10.49 + yards, or 10 yards and an half almost; for, 3.24 multiplyed by it self, to wit, by 3.24 will produce 10.49 +.

Also if the side of a square piece of Land be 37.25 perches, the content in square perches (neglecting the fraction in the product) will be found 1387, which being reduced (according to the seventh *Tablet* in *Rule* 4, *chapter* 7. of the preceding *book*) will give 8 acres, 2 roods, and 27 perches for the content of that square piece of land.

Quest. 50. If a long square be 8 feet in length, and 5 feet in breadth, what is the superficial content?

Answer, 40 feet; which content is found out by multiplying the length by the breadth, *viz.* 8 multiplyed by 5 produceth 40. So if one of the lights of a glass window supposed to be in the form of a long square, hath for its length 3.06 feet, and breadth 1.47 feet, the content of that glass will be 4.4982 feet, or 4 feet and an half almost, (for 3.06 multiplyed by 1.47 produceth 4.4982.)

In like manner if there be a piece of Wainscot, Plaistring, or any other superficies in the form of

a

a long ſquare, which is in length 6.325 yards, and in breadth 3.214 yards, the ſuperficial content will be found 20.32 + yards, that is 20 yards, one quarter of a yard, and ſomewhat more: for, 6.325 multiplyed by 3.214 produceth 20.32 +.

Likewiſe a piece of Tiling in the form of a long ſquare whoſe length is 18.5 feet, and breadth 11.7 feet will be found to contain 216.45 ſquare feet, which will be reduced to 2.1645 ſquares of Tiling, by allowing (according to cuſtom) 100 ſquare feet to one ſquare of Tiling.

Alſo if a piece of land in the form of a long ſquare be 48.75 perches in length, and 36.25 in breadth, the area or content in perches will be found 1767.18 + which 1767 perches being reduced will give 11 acres and 7 perches for the content of that piece of ground.

Queſt. 51. If it be required to ſet forth in a Meadow one acre of graſs to lye in the faſhion of a long ſquare, and that the length thereof be limited or agreed to be 20 perches, what muſt the breadth be?

Anſwer, 8 perches: which breadth is found out by dividing 160 (the number of ſquare perches contained in an acre) by the given length 20. If two acres were required, then 320 (to wit, twice 160) muſt be divided by the given ſide, whether it be the length or breadth; ſo if 7.25 perches be preſcribed for the breadth of two acres, the length muſt be 44.13 + perches.

In like manner, if the breadth of a Board be 1.32 foot, and it be demanded how far one ought to meaſure along the ſide thereof to have a ſuperficial foot, or a foot ſquare of that Board: divide

1 by

1 by the given breadth, so you will find in the quotient this decimal fraction .757 + which represents three quarters of a foot or nine inches and somewhat more, and so much in length ought to be measured along the side of that Board to make a superficial foot. Likewise if the breadth of a board be given in inches, then 144 (the number of square inches contained in a superficial foot square) being divided by the given breadth, the quotient will shew how many inches ought to be measured along the side of that board to make a superficial foot; so the breadth of a board being 9 inches, the length forward to make a superficial foot will be found 16 inches.

Quest. 52. If the three sides of a piece of land that lyes in the form of a triangle be 15 perches, 14 perches, and 13 perches, what is the area or number of square perches contained in that triangle?

Answer, 84 perches, or half an acre and four perches, which content is found out by this Rule, *viz.*

From half the sum of the three sides of any plane triangle, subtract each of the three sides severally, & note the three remainders, then multiply the said half sum and those three remainders one into the other, (according to the rule of continual Multiplication:) that done, extract the square root of the last product, so shall such square root be the area or content of the triangle.

The

	Perches
The 3 ſides of a triangle	15 14 13
The ſum of the 3 ſides	42
The half of that ſum	21
The 3 remainders found out by ſubtracting each ſide from the half ſum	6 7 8
The product ariſing from the continual multiplication of the four laſt numbers	7056
The ſquare root of which product is the content required, to wit,	84

Another Example.

	Perches	
The 3 ſides of a triangle	120 112 90	5 6 3
The ſum of the 3 ſides	323	4
The half of that ſum	161	7
The 3 remainders found by ſubtracting each ſide from the half ſum	41 49 71	2 1 4
The product ariſing from the continual multiplication of the four laſt numbers	23355380	1096
The ſquare root of that product	4832	7 +

Wherefore

Wherefore I conclude that the content of a plane triangle whose three sides are 120.5 perches, 112.6 perches, and 90.3 perches, is 4832.7 + perches, which reduced give 30 acres and 32 perches, (the fraction of a perch being neglected.)

Now forasmuch as every irregular piece of ground may be divided into triangles, for a four-sided field will be divided into two triangles by one imaginary straight line leading overthwart from corner to corner called a *Diagonal line*; a five-sided field into three triangles by two *Diagonals*; a six-sided ground into four *Triangles* by three *Diagonals*, &c. the rule before given will be of excellent use to find out the Contents of large fields, especially if the land be of a dear value, as also when any controversie ariseth by reason of the different admeasurements of Surveyors of land: for if the sides of those *Triangles* be measured in the field, and their lengths be agreed on, all Artists to whom the reason of the rule before given is known, will agree in one and the same content. But yet this way of measuring presupposeth that there is no obstacle, as Water, Wood, or other impediment, to hinder the measuring of the sides of those *Triangles* into which the field is divided as aforesaid.

Quest. 53. If the diameter of a Circle be 28.25, what is the circumference?

Answer, 88.749 +: for as 113 is in proportion to 355; or as 1 is to 3.14159, so is the diameter to the circumference: Therefore multiplying alwayes the diameter given by the said 3.14159 the product shall be the circumference required.

Quest. 54. If the diameter of a Circle be 28.25 what

what is the superficial content of that Circle?

Answer, 626,79 + : for as 1 is in proportion to .78539, so is the square of the diameter to the superficial content, Therefore multiplying alwayes the said decimal fraction .78539 by the square of the given diameter (which square is the product of the multiplication of the diameter by it self) the product shall be the superficial content required.

Quest. 55. If the diameter of a Circle be 28.25, what is the side of a square which may be inscribed within the same Circle?

Answer, 19.975 + for the square root of half the square of the diameter, or the square root of the double of the square of the semidiameter, shall be the side of the inscribed square sought. Otherwise, as 1 is to .707106, so is the diameter to the side required. Therefore if you multiply (alwayes) the said .707106 by the diameter given, the product will be the side of the inscribed square required.

Quest. 56. If the circumference of a Circle be 88.75 what is the diameter?

Answer, 28.249 + for as 355 is to 113, or as 1 is to .318309, so is the circumference to the diameter. Therefore if .318309 be multiplyed alwayes by the given circumference, the product shall be the diameter required.

Quest. 57. If the circumference of a Circle be 88.75 what is the superficial content of that Circle?

Answer, 626.801 + for as 1 is to .079578, so is the square of the circumference to the superficial content. Therefore if .079578 be alwayes multiplyed by the square of the given circumference, the product shall be the superficial content sought.

Quest.

Quest. 58. If the circumference of a Circle be 88.75, what is the side of a square that may be inscribed within the same Circle?

Answer, 19.975 + for as 1 is to .225078, so is the circumference to the side required. Therefore if .225078 be alwaies multiplied by the circumference given, the product will be the side of the inscribed square sought.

Quest. 59. If the superficial content of a Circle be 626.8, what is the diameter?

Answer, 28.25 + for as 1 is to 1.27324, so is the content to the square of the diameter. Therefore multiplying alwayes 1.27324 by the given content, the square root of that product shall be the diameter required.

Quest. 60. If the superficial content of a circle be 626.8, what is the circumference?

Answer, 88.75 + for as 1 is to 12.5664, so is the content to the square of the circumference. Therefore if 12.5664 be alwaies multiplied by the given content, the square root of the product shall be the circumference required.

Quest. 61. If the superficial content of a Circle be 626.8, what is the side of a square equal to the same Circle?

Answer, 25.035 + for the square root of the given content is the side of the square required.

Quest. 62. If the side of a Cube be 12 inches; how many cubical inches are contained in that Cube?

Answer, 1728. What a Cube is may be well represented by a Dye, which is a little cube it self, being a rectangular or square solid that hath an equal length, breadth, and depth, and is compre-

hended under ſix equal ſquares ; now if the ſide of one of thoſe equal ſquares (which is alſo the ſide of the Cube) be 12 inches, the ſuperficial content of that ſquare will be 144 ſquare inches, (for according to the preceding 49th queſtion, 12 multiplied by 12 produceth 144) which multiplied by the depth 12 inches, produceth 1728 cubical inches, and ſuch is the ſolid content of that Cube whoſe ſide is 12 inches : ſo that by one foot of timber or ſtone in whatſoever kind of ſolid it be found, is underſtood a Cube, containing 1728 cubical or dye-ſquare inches, and conſequently half a foot ſolid contains 864 cubick inches, and a quarter of a foot ſolid contains 432 cubick inches.

In like manner, if the ſide of a Cube of ſtone be 2.53 feet, the ſolid content of that Cube will be found 16.194 + feet, for 2.53 being multiplied by it ſelf produceth 6.4009 ſuperficial feet, which product being multiplied by the ſaid 2.53 will produce 16.194 + ſolid feet.

Alſo if the ſide of a Cube of ſtone or wood be 6 inches, or .5 foot, the ſolid content will be found 216 cubick inches, or .125 parts of a foot ſolid (for 6 multiplied cubically produceth 216, likewiſe .5 multiplied cubically produceth .125) whence it may be infer'd, that 8 little cubes of ſtone or wood, each of which is half a foot or 6 inches ſquare, are contained in a foot of ſtone or timber ; for 8 times 216 produceth 1728, (being the number of cubick inches contained in a foot ſolid) likewiſe 8 times .125 produceth 1, (to wit, one entire foot ſolid.)

Queſt. 63. If the breadth of a ſquared piece of timber, ſuppoſed to be ſtraight and terminated at both

both ends by two equal squares, be 1.55 foot; th depth also 1.55 foot and the length 17.33 feet how many cubick feet are contained in that piec of timber:

Answer, 41.635 feet, that is, 41 feet and an half and about half a quarter of a foot. Which solid content is found out by this rule, *viz.* multiply the breadth 1.55 by the depth 1.55 the product will be 2.4025 superficial feet, which is the content of th Base, (that is, the Area of either of the two equa squares at the ends of the piece) lastly multiplying the said Base 2.4025 by the length 17.33 the product will be 41.635 +, which is the solid content required.

In like manner if the breadth of a squared piece of timber, supposed to be straight and terminated at both ends by two equal long squares, (which are called the Bases) be 2.34 feet, the depth 1.61 foot, and the length 17.58 feet, the solid content will be 66.23 + feet; for (as before) multiplying the breadth by the depth, and that product by the length, the last product shall be the solid content required.

Quest. 64. If the breadth, as also the depth of a squared piece of timber having equal square bases, be 1.55 foot, how far ought one to measure along the length of that piece of timber to make a foot solid?

Answer, .416 parts of a foot, or 5 inches very near; which decimal is thus found, *viz.* First find the superficial content of the base, which will be 2.4025: (for, 1.55 multiplied by 1.55 produceth 2.4025.) Then dividing 1, (to wit 1 solid foot) by the Base 2.4025 the quotient will be .416 +

or $\frac{416}{1000}$ parts of a foot, or five inches almoſt, and ſo far ought to be meaſured along the length of the piece to make a foot ſolid. In like manner, if the breadth be 2.34 feet, and the depth 1.61 feet, the length forward along the piece to make one ſolid foot will be found .265 parts of a foot, or three inches and almoſt $\frac{1}{2}$ part of an inch.

Queſt. 65. If a ſtraight ſquared piece of timber be terminated by unequal Baſes, whereof one contains 1.92 ſuperficial foot, the other .85 foot, and the length of that piece of timber be 17.4 feet, what is the ſolid content, or how many Cubical feet are contained in that piece of timber?

Anſwer, 23.474 + feet (found out by one of Mr. *Oughtreds* Rules for meaſuring a ſegment of a Pyramid in *Problem* 21. *Chapter* 19. of his *Clavis Mathemat.*) The Rule is this,

Multiply the greater Baſe by the leſs, and extract the ſquare root of that product, then multiply the ſum of the two Baſes and that ſquare root by one third part of the length of the ſolid propounded, ſo ſhall the laſt product be the ſolid content required.

Example

Example.

The greater Base	1 . 9
The lesser Base	0 . 8
The product of the multiplication of those two Bases	1 . 6;20
The square root of that product	1 . 2774
The sum of that square root and the two Bases	4 . 0474
One third part of the length is	5 . 8
The product of the multiplication of the two last numbers is the solid content required	23 . 474 †

Quest. 66. A Pyramid is a solid comprehended under plane surfaces, and from a triangular, quadrangular, or any multangular Base, diminisheth equally less and less till it finish in a point at the top; now if the superficial content of the Base of a Pyramid be 5.756 feet, and the height thereof 14.25 feet, (which height is the length of the perpendicular line that falleth from the top of the Pyramid to the Base) what is the solid content of that Pyramid?

Answer, 27.341 † feet: for if the Area of the Base of a Pyramid, be multiplied by one third part of the height thereof, the product shall be the solid content of the Pyramid; therefore 5.756 × 4.75 = 27.341 feet = the solidity of the Pyramid propounded.

Note, If a Pyramid be cut into two segments by a Plane parallel to the Base, one of those segments will be a Pyramid, and the other will have two unequal Bases, for the measuring of which latter segment,

ment, a rule hath been already given in the ſixty fifth queſtion, the Area of each Baſe being known.

Queſt.67. A Cone is a ſolid, which hath a Circle for its Baſe, from whence it grows equally leſs and leſs (like a round Steeple of a Church) till it finiſh in a point at the top; now if the Area of the Baſe of a Cone be 5.756 feet, and the height thereof be 14 25 feet what is the ſolid content of that Cone?

Anſwer, 27.341 feet: for if the Area of the Baſe of a Cone be multiplied by one third part of the height thereof, the product ſhall be the ſolid content of the Cone.

Note, If a Cone be cut into two ſegments by a Plane parallel to the Baſe, one of thoſe ſegments will be a Cone, and the other ſegment will have two unequal Baſes which are Circles, the ſolidity of which latter ſegment may be found out by the rule before given in the 65 queſtion, the Area of each Baſe (or circle) being known.

Queſt. 68. A Cylinder is a ſolid which may be well repreſented by a Stone-roll, ſuch as are uſed in Gardens for the rolling of Walks. Now if the circumference of a Cylinder be 4.57 feet, and the length 3.25 feet, what is the ſolid content of that Cylinder?

Anſwer, 5.4+ feet, thus found out: Firſt by the help of the given circumference 4.57, find out the ſuperficial content of that Circle, (being the Baſe of the Cylinder) which content (by the preceding 57th queſtion) will be found 1.6619 + foot, then multiplying the ſaid 1.6619 by the given length 3.25, the product will be 5.4008 which is the ſolid content required.

Queſt.

Quest. 69. If the Base of a Cylinder be 1.6619 foot, how much in length of that Cylinder will make a foot solid?

Answer, .601 parts of a foot; For 1 (to wit, 1 solid foot) being divided by the base 1.6619, gives in the quotient the decimal .601 + for the length required.

*Quest.*70. A Globe is a perfect round body contained under one Plane; in the middle of the Globe there is a point called the Center, from whence all straight lines drawn to the outside are of equal length, and called Semidiameters, the double of any one of which is equal to the Diameter of the Globe; now if the Diameter of a Globe of Stone be 1.75 feet, how many feet solid are contained in that Globe?

Answer, 2.807 + feet, for as 21 is in proportion to 11, or as 1 is to .5238, so is the Cube of the Diameter to the solid content of the Globe: Therefore, multiplying alwayes the Cube of the Diameter by the said decimal .5238, the product shall be the solid content required: So the Diameter 1.75 being first multiplied by it self, the product will be 3.0625, which multiplied by the said 1.75, gives in the product 5.359375, to wit, the cube of the diameter which being multiplied by .5238, the product thence arising will be 2.807 +, which is the solidity of the Globe propounded.

Quest. 71. What is the Diameter of a Globe of stone which contains 4 cubical or solid feet?

Answer, 1.96 + foot, for as 11 is in proportion to 21, or as 1 is to 1.9090909 so is 4 (the solid content given) to a fourth proportional, to wit, 7.636363 + whose cubick root is 1.96 + the diameter required.

Concerning the gaging of Vessels.

The easiest and aptest wayes for practice in gaging, are those which are perform'd by the help of Tables, or Gaging rods purposely compos'd: Neverthelesse to give the Reader of this Treatise some light in this matter, I shall here insert one rule to find out the number of Gallons contained in a full Tun, Pipe, Hogshead, Barrel, or such like vessel, according to Mr. *Wingate's* way of reducing a Vessel to a Cylinder. The Rule is this;

Having found the difference of the two diameters at the bongue and head of the Vessel, take $\frac{7}{10}$ of that difference and add it to the lesser diameter; then square that sum and reserve the product; that done, if the content be required in Wine gallons, multiply the product reserved, this decimal fraction .0034, and the length of the vessel, one into the other, (according to the *Rule of continual Multiplication*) so shall the last product be the number of Wine gallons required; but if the content be required in Ale gallons, multiply the product before reserved, this decimal fraction .0027, and the length of the vessel, one into the other continually, so shall the product be the content in Ale-gallons: This Rule I shall first explain by two questions, and then shew how it is raised.

Quest. 72. If the diameter at the bongue of a vessel be 32 inches, the diameter at the head 28.2 inches, and the length 39 inches, (which dimensions are

are said to agree very near with those of an Eng-lish vessel called a Pipe) what is the content of that vessel in Wine gallons?

Answer, 126.278 Wine-gallons, that is 126 Wine-gallons and about a quart more (found out by the rule above given, as will be manifest by the following operation.)

Explication.

The Diameter at the bongue	32 . 0
The diameter at the head	28 . 2
Their difference	3 . 8
Which multiplied by $\frac{7}{10}$, that is,	0 . 7
The product will be	2 . 66
Which added to the lesser diameter gives the mean diameter	30 . 86
Which mean diameter being squared (that is, multiplied by it self) produceth	952 . 3396
Which product multiplied by	0 . 0034
The product thence arising will be	3 . 2379+
Which multiplied by the length of the vessel	39 . 0
The product is the number of Wine gallons sought, *viz.*	126 . 278+

Quest. 73. If the diameter at the bongue of a barrel be 23 inches, the diameter at the head 19.9 inches, and the length 27.4 inches; what is the content of that barrel in Ale-gallons?

Answer, 36.031 Ale-gallons, that is 36 Gallons and about a quarter of a Pint more (found out by the preceding Rule.)

Explication

Explication.

The diameter at the bongue	23 . 0
The diameter at the head	19 . 9
Their difference	3 . 1
Which multiplied by $\frac{7}{10}$, that is	0 . 7
The product will be	2 . 17
Which added to the lesser diameter gives the mean diameter	22 . 07
Which mean diameter being squared (that is, multiplied by it self) produceth	487 . 0849
Which product multiplied by	0 . 0027
The product thence arising is	1 . 315 +
Which multiplied by the length of the vessel	27 . 4
The product is the number of Ale-gallons sought, to wit	36.031 +

The reason of the Rule.

Two things are taken for granted in the said Rule, *viz.* First, it is supposed that if $\frac{7}{10}$ of the difference of the two diameters at the bongue and head, be added to the lesser diameter, the sum shall be an equated or mean diameter, (near enough for practical use though it be not exact) *viz.* If there be a Cylinder whose diameter is equal to that mean diameter, and whose length is equal to the length of the vessel, that Cylinder shall be equal to the capacity of the vessel very near. Secondly, the

the ſaid Rule preſuppoſeth that 231 cubick inches are equal to a Wine-gallon, and 282 equal to an Ale-gallon; concerning which equalities (eſpecially the latter) Artiſts differ ſomewhat in their experiments; but according to any equality which in that particular ſhall be agreed on, from this that follows a rule may be framed, and Tables thence calculated for gaging a full veſſel without conſiderable error.

Taking then thoſe two things above mentioned for granted, we may rightly infer that if a Cylinder hath for its Baſe a Circle whoſe ſuperficial content is 231 inches, every inch in length of that Cylinder will contain 231 cubick inches, or one intire Wine-gallon; Now foraſmuch as all Circles are in ſuch proportion one to the other as the ſquares of their diameters, it ſhall be as 294.11844, (to wit, the ſquare of the diameter of that Circle whoſe ſuperficial content is 231) is to 1; (to wit, the ſuperficial content 231 conſidered as the Baſe of one Wine-gallon) or as 1 is to .0034; So is the ſquare of the equated (or any other) diameter, to the ſuperficial content of that Circle in Wine gallons and parts of a gallon, which content multiplied by the length of the veſſel will produce its ſolidity or capacity in Wine-gallons: Therefore the firſt part of the preceding rule for finding of the number of Wine-gallons contained in a full veſſel is manifeſt: And after the ſame manner, ſuppoſing as before 282 cubick inches are equal to an Ale-gallon, the decimal .0027 preſcribed in the ſaid rule will be found out.

Upon thoſe grounds Mr. *Wingate* compos'd his Gaging rod; Mr. *Oughtred* alſo in his Circles of Proportion

Proportion hath delivered another rule for Gaging from whence his Gaging rod is deduced; but the particular constructions of those rods, and likewise the making of Tables for the same purpose, being handled by several Artists, I shall not insist upon them.

Now if the industrious and more curious Arithmetician, after he is well exercis'd in vulgar Arithmetick, shall yet desire a further knowledge in finding out the Answers of subtle Questions about numbers, his best Guide will be the admirable *Algebraical* Art, which discovers rules for the solving of *Problems*, as well Arithmetical as Geometrical, that are above the reach of any of the rules of common Arithmetick, or practical Geometry, as may partly appear by the two rules in the aforegoing 52 and 65 Questions, as also by the two following Questions, with which I shall conclude this chapter.

Quest. 74. To find two numbers in a given proportion, (suppose the lesser to the greater as 2 to 3) and such, that if the lesser number be added to the square of the greater, also if the greater number be added to the square of the lesser, the two sums shall be square numbers whose roots are expressible by rational or true numbers: (fractions being admitted for numbers.)

Answer, $\frac{1}{10}$ and $\frac{3}{20}$.

The

The proof.

The square of $\frac{3}{20}$ (the greater number) is ————————	$\frac{9}{400}$
To which adding the lesser number ——	$\frac{1}{10}$
The sum in its least terms will be ——	$\frac{49}{400}$
Which is a square number whose root is ————————	$\frac{7}{20}$
Again, the square of $\frac{1}{10}$ (the lesser number) is ————————	$\frac{1}{100}$
To which adding the greater number ————————	$\frac{3}{20}$
The sum in its least terms will be ——	$\frac{4}{25}$
Which is a square number whose root is ————————	$\frac{2}{5}$

Also the said numbers $\frac{1}{10}$ and $\frac{3}{20}$ are one to the other as 2 to 3, wherefore the question is solved. Which numbers $\frac{1}{10}$ and $\frac{3}{20}$ are found out by this following

Theoreme.

If the fraction $\frac{1}{4}$ be divided into any two parts, either of those parts being increased with the square of the other part shall give a fraction having a rational square root.

Wherefore by dividing $\frac{1}{4}$ into the two fractions $\frac{1}{10}$ and $\frac{3}{20}$, which are in the prescribed proportion of 2 to 3, those fractions will satisfie the conditions in the question propounded.

Likewise these two fractions $\frac{722}{10080}$ and $\frac{1083}{10080}$ will answer the question, and are found out without extracting any root, but the manner of finding them out excels the skill of a vulgar *Arithmetician*.

Quest. 75.

Quest. 75. To find three numbers, such that the square of any one of them being added to the other two numbers, the sum of such addition shall be a square number, whose root is a rational number.

Answer, 1, $\frac{8}{3}$, and $\frac{16}{3}$.

The proof.

First, the square of the first number 1 is————————	1
To which adding the second and third numbers $\frac{8}{3}$ and $\frac{16}{3}$, the sum will be-	9
Which is a square number whose root is————————	3
Secondly, the square of the second number $\frac{8}{3}$ is————————	$\frac{64}{9}$
To which adding the first and third numbers 1 and $\frac{16}{3}$, the sum in its least terms will be————————	$\frac{121}{9}$
Which is a square number whose root is————————	$\frac{11}{3}$
Thirdly, the square of the third number $\frac{16}{3}$ is————————	$\frac{256}{9}$
To which adding the first and second numbers 1 and $\frac{8}{3}$ the sum in its least terms will be————————	$\frac{289}{9}$
Which is a square number whose root is————————	$\frac{17}{3}$

Wherefore it is manifest that the three numbers 1, $\frac{8}{3}$ and $\frac{16}{3}$ will satisfie the conditions in the question, which may be solved also by other numbers, but the manner of finding them out I leave as an exercise to the *Algebrician.*

CHAP.

CHAP. XI.

Of Sports and Pastimes.

Probl. I.

To discover a number which any one shall have in his mind, without requiring him to reveal any part of that or any number whatsoever.

AFter any one hath thought upon a number at pleasure, bid him double it, and to that double bid him add any such even number which you please to assign, then from the sum of that addition let him reject one half, and reserve the other half: Lastly, from this half bid him to subtract the number which he first thought upon; then may you boldly tell him what number remaineth in his mind after that subtraction is made, for it will alwayes be half the number which you assigned him to add.

For example, suppose he thought upon 6, the double thereof is 12, to which bid him add some even number at your pleasure, suppose 4, so will the sum be 16, whereof the half is 8, from which if he subtract 6, (the number first thought on) the remainder is 2, (to wit, half the number 4, which was by you assigned to be added) which remainder you discover, notwithstanding all the operation was performed in his mind, without his making known of any number whatsoever. Note that the adding of an even number as aforesaid is not of necessity, but only to avoid a fraction which will arise by taking the half of an odd number.

The

The reason of the Rule.

If to the double of any number (which number for diſtinction ſake I call the firſt) a ſecond number be added, the half of the ſum muſt neceſſarily conſiſt of the ſaid firſt number, and half the ſecond; therefore if from the ſaid half ſum the firſt number be ſubtracted, the remainder muſt of neceſſity be half of the ſecond number which was added.

Probl. II.

Two numbers, the one even and the other odd, being propounded unto two perſons, to the end they may (out of your ſight) ſeverally chuſe one of thoſe numbers; to diſcover which of theſe numbers each perſon ſhall have choſen.

Suppoſe you have propounded unto *Peter* and *John* two numbers, the one even and the other odd, as 10 and 9, and that each of thoſe perſons is to chuſe one of the ſaid numbers unknown to you. Now to diſcover which number each perſon ſhall have choſen, you muſt take two numbers, the one even and the other odd, as 2 and 3; then bid *Peter* multiply that number which he ſhall have choſen, by 2; and cauſe *John* to multiply that number which he ſhall have choſen by 3; that done, bid them add the two products together, and let them make known the ſum to you, or elſe demand of them whether the ſaid ſum be even or odd, or by any other way more ſecret endeavour to diſcover it, by bidding them to take the half of the ſaid ſum,

for by knowing whether the said sum be even or odd, you do obtain the principal end to be aimed at, because if the said sum be an even number, then infallibly he that multiplied his number by your odd number (to wit, by 3) did chuse the even number, (to wit 10) but if the said sum happen to be an odd number, then he whom you caused to multiply his number by your odd number, (to wit, by 3) did infallibly chuse the odd number, (to wit 9.)

For example, if *Peter* had made choice of 10, and *John* 9, suppose you willed *Peter* to multiply his number 10 by 2, and *John* to multiply his number 9 by 3; the products will be 20 and 27, whereof the sum is 47, which being an odd number, you may thence conclude that *John* whom you caused to multiply his number by 3 did chuse the odd number 9, and therefore *Peter* did chuse 10. But if you had willed *John* to have multiplied his number 9 by 2, and *Peter* to have multiplied his number 10 by 3, the products would have been 18 and 30, whereof the sum is 48, which is an even number, from whence you may infer that he that multiplied his number by 3 did chuse the even number, and therefore *Peter* had chose 10, and *John* 9.

Demonstration.

The reason of the said rule is very easie, and dependeth principally upon the 28 and 29 propositions of the 9th book of *Euclid*; for one may infer from the 21 of the same book, that an even number multiplied by any number whatsoever produceth an even number, but an odd number is of a different nature, for if it be multiplied by an even number

ber the product is an even number (by the said 28 proposition) and if it be multiplied by an odd number the product is odd; (by the said 29 proposition.) Therefore if in making this sport it happeneth that the even number be multiplied by your odd number, both the products shall be even, and consequently the sum shall be infallibly an even number (by the said 21 proposition.) But if it happen that you cause the odd number to be multiplied by your odd number, that product will be odd, and the other product even, therefore the sum of these two products shall be an odd number, (as *Clavius* hath demonstrated upon the 23. of the 9th of *Euclid.*)

Probl. 3.

A certain number of distinct things being propounded, to dispose them in such an order, that casting away alwayes the ninth, or the tenth, or any other that shall be assigned, unto a certain number, those remaining may be such as were first intended to be left.

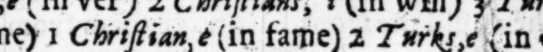
This Problem is usually propounded in this manner, *viz.* fifteen *Christians* and fifteen *Turks* being at Sea in one and the same Ship in a terrible storm, and the Pilot declaring a necessity of casting the one half of those persons into the Sea, that the rest might be saved; they all agreed that the persons to be cast away should be set out by lot after this manner, *viz.* the thirty persons should be placed in a round form like a Ring, and then beginning to count at one of the Passengers, and proceeding circularly, every ninth person should be cast into the Sea, until of the thirty persons there

there remained only fifteen. The question is, how those thirty persons ought to be placed, that the lot might infallibly fall upon the fifteen *Turks* and not upon any of the fifteen *Christians*? For the more easie remembring of the rule to resolve this question, I shall presuppose the five vowels, *a, e, i, o, u,* to signifie five numbers, to wit, *(a)* one, *(e)* two, *(i)* three, *(o)* four, and *(u)* five; then will the rule it self be briefly comprehended in these two following verses,

From numbers aid and art,
Never will fame depart.

In which verses you are principally to observe the vowels, with their correspondent numbers before assigned, and then beginning with the *Christians*, the vowel *o* (in from) signifieth that four *Christians* are to be placed together; next unto them, the vowel *u* (in num.) signifieth that five *Turks* are to be placed; In like manner *e* (in bers) denoteth 2 *Christians*. *a* (in aid) 1 *Turk*, *i* (in aid) 3 *Christians*, *a* (in and) 1 *Turk*, *a* (in art) 1 *Christian*, *e* (in ne) 2 *Turks*, *e* (in ver) 2 *Christians*, *i* (in will) 3 *Turks*, *a* (in fame) 1 *Christian*, *e* (in fame) 2 *Turks*, *e* (in de) 2 *Christians*, *a* (in part) 1 *Turk*.

The invention of the said Rule, and such like, dependeth upon the subsequent demonstration, *viz.* if the number of persons be thirty, let thirty figures or cyphers be placed circularly, or else in a right line as you see,



That done, begin to count from the first, and

mark the ninth (or what other ſhall be aſſigned) by putting a point or croſs over it, then count forward from that which you have marked, and place another point over the next ninth, and continue to do the ſame, beginning again when you ſhall be at the end, (if the cyphers are placed in a right line) and paſſing over thoſe, which you ſhall have already marked, until you have marked the number required, as in the example propounded, until you have marked fifteen, for then all the cyphers marked ſhall be thoſe which muſt be caſt away, and the others, thoſe which ſhall remain. Hence it is evident, that if you obſerve how thoſe cyphers which are marked, are diſpoſed amongſt thoſe which are not marked, you will eaſily make a rule for any number whatſoever.

By this invention (as ſome do conjecture) the famous Hiſtorian *Joſephus* the *Jew*, preſerved his life very ſubtilly, in the Cave to which himſelf and forty of his Countreymen had fled, from the furious and conquering *Romans*, at the Seige of *Jotapata*, for his ſaid Countrymen having moſt wickedly reſolved to kill one another, rather then yield to their enemies, he at length (when no arguments that he could uſe would diſſwade them from ſo horrid an act) prevailed with them to execute their tragical deſign by lot; and ſo by the help of the aforeſaid artifice, (as we may ſuppoſe) himſelf with one other perſon only remaining alive, after the reſt were inhumanely murthered, they agreed to put an end to the lot, and thereby ſave their lives. This ſtory you may ſee at large in the fourteenth Chapter of the third book of the Hiſtory of *Joſephus* of the Warrs of the *Jews*.

Probl.

Probl. 4.

Many numbers which proceed from 1 or unity in a progression, according to the natural order of numbers, (such as these, 1, 2, 3, 4, 5, 6, &c.) being placed in a round form like a Ring; to discover which of those numbers any one shall have thought upon.

Let any multitude of numbers in the aforesaid progression, suppose these 10, to wit, 1.2.3.4.5.6.7.8.9.10. be written upon 10 ivory counters (or for want thereof upon 10 small pieces of paper) which may be represented by these 10. letters, A. B.C.D.E.F.G.H.K.L. *viz.* suppose 1 to be writte upon the counter A, 2 upon B, 3 upon C, &c. Then having placed those counters circularly as you see (with their blank faces uppermost, and the figures underneath, that the subtilty of the spo

A
1
L 10   2 B
K 9   3 C
H 8   4 D
G 7   5 E
6
F

may the better be concealed) let any one think upon any number of unities which doth not exceed 10; that done, bid him touch one of those counters at pleasure, and to the number on the backside of the counter touched (which you cannot be ignorant of, having noted well the place of 1 or

A.) add secretly in your mind, the just number of all the counters, and reserve the sum; then bid him imagine in his mind the counter touched to be the number which he thought, and from that counter to count backwards, until he shall have made up the aforesaid sum, which you reserved, so will his computation infallibly end upon the counter upon which the number thought upon is written.

For example, suppose that he thought 7 or G, and that he touched B to wit, 2. Add to 2 the number of all the counters, to wit, 10, so the sum will be 12, then bid him to count unto 12, beginning at B and going backwards, and esteeming B to be the number thought, to wit 7, so will 8 fall upon A, 9 upon L, 10 upon K, 11 upon H, and lastly, 12 upon the counter G, which being turned up will shew 7 the number thought.

The reason of this rule is not difficult to be apprehended, two principles being presupposed, the one is this, to wit, many counters or things whatsoever being disposed orderly one after the other, in one continued line, whether it be right or circular, if you value or name the first counter to be some number of unities at pleasure, and continue to count forward according to the natural order of numbers, until another number be named which falleth upon the last counter; or if you imagine or name the last counter, to be the same number of unities as before you put upon the first, and continue to count backwards unto the first counter; I say, that the same number will be named at the end of both those computations: for example, in these 9 letters A.B.C.D.E.F.G.H.K. if the letter A be esteemed

esteemed to be 4, and from thence you count forwards unto K, according to the natural order of numbers, the letter K will fall upon the number 12. In like manner, if you esteem K to be 4, and count backwards from K to A, the letter A will likewise fall upon 12.

4.	5.	6.	7.	8.	9.	10.	11.	12
A.	B.	C.	D.	E.	F.	G.	H.	K
12.	11.	10.	9.	8.	7.	6.	5.	4

The other principle is this, to wit, many counters being disposed in a round manner like a Ring, if you esteem any one of those counters to be some number at pleasure, and then from that counter if you count circularly, until you end upon the counter where you began, the number last named will be equal to the sum of the number of all the counters, and of the number which you put upon the first counter: for example; if D be one of 10 Letters placed in a circumference, and that imagining D to be 7, you begin with it, and count round the whole circumference, according to the natural progression of numbers, till

A

1

L 10 2 B

K 9 3 C

H 8 4 D

G 7 5 E

6

F

Kk 4 yo[u]

you end with D where you began; the number 17 which is composed of 10 and 7 will necessarily fall upon D; for 9 (which is the number of letters in the circumference besides D) being added to 7 (which was first put upon D) makes 16, to which 1 being added (because D doth end as well as begin the circumference) the sum is 17.

Now these two principles being presupposed, it will not be difficult to apprehend the reason of the aforesaid rule in all cases that can happen; for imagine that one hath thought upon 7, or the counter G, then that counter which he shall touch must either be the same counter G or some other that precedeth or followeth G.

First therefore supposing the counter or number touched to be the same with the number thought, the truth of the rule will be then evident, for by the rule given, he shall begin to count from the same G unto 17, putting 7 upon G, therefore by the second presupposition the number 17 will fall upon G.

Secondly, imagine that he touched a counter or number following G the number thought, as L or 10, then according to the rule adding 10 (the multitude of all the counters placed circularly) unto 10 or L, (the counter touched) bid him count backwards unto 20 by beginning at L and esteem L to be 7. Now because by beginning to count at G which is 7, and proceeding to count forward, the number 10 will fall upon L, therefore by the first presupposed principle if we esteem L to be 7 and count backwards, the number 10 will infallibly fall upon G, and then the number 20 shall also fall upon the same G by the second presupposed principle.

Lastly,

Lastly, imagine he touched some number or counter which precedeth 7 the number sought, as B or 2; then adding 10 to 2, you are to bid him count unto 12, he having first imagined B to be the number thought 7, and going backwards to A, L, K, &c. Now because by proceeding to count at B, which is 2, and beginning to count forward to C, D, &c. the number 7 falleth upon G. Therefore if one imagine that G is 2, and from thence count backwards towards F. E. &c. the number 7 will fall upon B, (by the first presupposed principle) therefore when one assumeth B to be 7, and counteth towards A. L. &c. to any assigned number, it is in effect as much as when one imagineth G to be 2, and counteth towards F. E. &c. unto the said assigned number, for each of those computations will end in the same point; but it is manifest (by the second presupposed principle) that esteeming G to be 2, and counting towards F. E. D. &c. round the whole circumference, the number 12 will fall upon the same G. And because G being supposed to be 2, and counting on the same coast as before, the number 7 falls upon B, therefore if the computation be continued on the same coast from B 7 unto 12, the number 12 will fall upon the same G. So that the practice of this sport in all its Cases is fully demonstrated.

Note, that to the number of the counter touched, you may not only add the number of all the counters once (as the Rule directs) but twice, thrice or more times: for example, B being touched, you may cause him to count unto 12, or unto 22, or to 32. 42. &c. the reason whereof is evident from the second presupposed principle.

Probl.

Probl. 5.

Many numbers being shewed by pairs, to wit, two by two, unto any one, that he may think upon any one of those pairs at pleasure; to discover the pair that was thought upon.

Let 20 numbers, suppose these, 1.2.3.4.5.6.7.8.9.10.11.12.13.14.15.16.17.18.19.20. be written upon Ivory counters, (or for want thereof upon small pieces of paper) to wit, 1 upon one counter, 2 upon another, 3 upon a third, &c. Then dispose them into pairs as you see; *viz.* suppose 1 and 2 to be one pair, 3 and four to be another

1.	2
3.	4
5.	6
7.	8
9.	10
11.	12
13.	14
15.	16
17.	18
19.	20

pair, &c. and of these pairs let any one think upon which pair he pleaseth. That done, you are to distribute the said 20 numbers in ranks, in the form of a long square, until there be 5 numbers in length, and 4 in breadth, after this manner, *viz.* lay

lay the three first numbers 1, 2. and 3 in a rank (as you see in the second figure) from A towards B, then place 4 underneath 1, and 5 after 3 (in the said rank A B.) Again place 6 under 4, and 7 after 5 (in the said rank A B.) Then place 8 under 6, also 9. 10. 11 on the right hand of 4 in the rank C D. Again place 12 under 9, and 13 on the right hand of 11 in the rank C D. and 14 under 12. Moreover place 15. 16. 17 on the right hand of 12 in the rank E F. Lastly, place 18. 19. 20 on the right hand of 14 in the rank G H, so will all the numbers be ranked, as you see in the Table. That done, you are to demand of him that thought upon two numbers as aforesaid, in what rank or ranks the said numbers do happen to be found, *viz.*

A	1	2	3	5	7	B
C	4	9	10	11	13	D
E	6	12	15	16	17	F
G	8	14	18	19	20	H

in which of the ranks A B, C D, E F, G H, or in which two of the said ranks: now if he answer that the two numbers which he first thought upon are in the first rank A B, then 1 and 2 shall be the numbers thought upon; if in the second C D, then 9 and 10 shall be the numbers thought; if in the third rank E F, then 15 and 16 shall be the numbers thought; if they are in the fourth rank G H; then 19 and 20 shall be the numbers thought; but if he shall say, that the numbers thought are in different ranks, then you are heedfully to mark the said numbers 1 and 2, 9 and 10, 15 and 16, 19 and 20, which

which may be called the keys of the ſport, in regard they ſerve not only to diſcover the two numbers thought, when they are both in one and the ſame rank (as aforeſaid) but alſo when they are in two different ranks, for in this latter caſe as ſoon as it hath been declared to you in which two ranks the two numbers thought are placed, you muſt take the key of the higheſt of thoſe two ranks, and deſcending in a down-right line from the firſt number of that key unto the lower of the ſaid two ranks, you ſhall there find one of the two numbers thought, and upon the right hand of the ſecond number of the ſaid key, at the ſame diſtance ſidewiſe from the ſecond number of the key, as one of the numbers thought was diſtant from the firſt number of the key, you ſhall find the other number thought.

For example, ſuppoſe the two numbers thought are 7 and 8, and that it ſhall be declared unto you that they are in the firſt and fourth ranks, take then the key of the higheſt of theſe two ranks, to wit of the firſt, which is 1 and 2, and deſcending down-right from 1 unto the fourth rank, you ſhall there find 8 one of the numbers thought, then ſeek ſide-wiſe on the right hand of 2 (the ſecond number of the key) a number as far ſeparated from 2, as 8 is diſtant from 1, and you will find 7 the other number thought.

Again, ſuppoſe he ſaith that the numbers thought are in the ſecond and third ranks; take then the key of the ſecond rank which is 9 and 10, and deſcending down-right from 9 to the third rank, you ſhall there find 12 which is one of the numbers thought; then ſeek ſidewiſe on the right

hand of 10 (the second number of the key) a number as far distant from 10 as 12 is from 9, and you shall find 11 which is the other number thought.

The reason of this will be apparent from a serious consideration of the placing of the numbers according to the rules before given, for it is thereby evident that of the two numbers coupled two by two, there can never be found more then one pair in one and the same rank, and of all the other pairs one number is alwayes found in one rank, and the other number in another rank.

Note also, that this sport may be practiced with divers persons at once, and not only with 20 numbers, but with any such multitude of numbers which is produced by the multiplication of any two numbers which differ by 1 or unity; as 30, which is the product of 5 multiplyed by 6, and 42 which is the product of the multiplication o[f] 6 and 7. That which is chiefly to be regarded is the placing of the numbers in ranks according to the directions before given, and for the more ea[s]ie comprehending of that order, I have in the following Table ranked 30 numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer illustration than can be exprest by many words.

1	2	3	5	7	9
4	11	12	13	15	17
6	14	19	20	21	23
8	16	22	25	26	27
10	18	24	28	29	30

Probl.

Probl. 6.

Three jealous husbands with their wives, being ready to pass by night over a river, do find at the river side a boat which can carry but two persons at once, and for want of a boatman they are necessitated to row themselves over the river at several times: the question is how these six persons shall pass two by two, so that none of the three wives may be found in the company of one or of two men unless her husband be present.

They must pass in this manner, *viz.* First two women pass, then one of them bringeth back the boat and repasseth with the third woman; that done, one of the three women bringeth back the boat, and sitting down upon the ground with her husband permitteth the other two men to pass over to find their wives; then one of the said men with his wife bringeth back the boat, and placing her upon the ground he taketh the other man and repasseth with him; lastly, the woman which is found with the three men entreth into the boat, and at twice goeth to fetch over the other two women.

Probl. 7.

Two merry companions are to have equal shares of 8 Gallons of wine, which are in a vessel containing exactly 8 Gallons, now to make this equal partition they have only two other empty vessels, whereof one containeth 5 Gallons, and the other 3; the question is, how they shall exactly divide the wine by the help of those three vessels only.

First, from the vessel which containeth 8 gallons

and is full of wine,let 5 gallons be poured into the empty veſſel of 5, and from this veſſel ſo filled let 3 be poured into the empty veſſel of three,ſo there will remain 2 gallons within the veſſel of 5. Then let the three gallons which are within the veſſel of 3 be poured into the veſſel of 8, which will now have 6 gallons within it, that done, let the 2 gallons which are in the veſſel of 5, be put into the empty veſſel of 3, then of the 6 gallons of wine which are within the veſſel of 8 fil again the five, and from thoſe 5 pour out 1 gallon into the veſſel of 3,which wanted only 1 gallon to fill it,ſo there will remain exactly 4 gallons within the veſſel of 5, and 4 gallons within the other two veſſels. This queſtion may be reſolved in another way, but I leave that as an exerciſe to the wit of the ingenious Reader.

Now albeit at firſt ſight it may be thought by ſome, that the two laſt mentioned *Problems* cannot be reſolved by any certain rule, but only by many trials, yet by infallible argumentation and diſcourſe, the ſolution of thoſe queſtions may be found out or elſe the impoſſibility of them, if by chance they ſhould have been propounded impoſſible; as the moſt ingenious *Gaſper Bachet* hath manifeſted in a little Book in the *French* Tongue, intituled *Problemes plaiſans & delectables qui ſe font par les nombres*, from which Book I have extracted the Contents of this Chapter.

Soli Deo Gloria.

ERRATA.

ERRATA.

Page.	Line.	Faults.	Amendments.
219	25	as you	you
247	2	Multiply	*XIX*. Multiply
278	14	$\surd 1441$	$\surd 144$
302	10	$\frac{3}{7}$ C.	$\frac{3}{2}$ C.
416	12	numbers	number
448	23	$\frac{4}{83}$	$\frac{3}{48}$